Hohmann Transfer

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What is the need?

- Interplanetary missions are very costly as we require huge amounts of fuel to provide the required energy.
- Hohmann Transfer helps us by simplifying that process and finding a fuel efficient way.

Plan

1. Introduction
2. Computation
3. Analysis
4. Analysis
5. Conclusion

Introduction

History

- A Hohmann Transfer is an orbital maneuver that transfers a spacecraft from one orbit to another.
- In 1925, Walter Hohmann wrote about this in his book **'Die Erreichbarkeit der Himmelskörper'**

Figure: The Attainability of Celestial Bodies

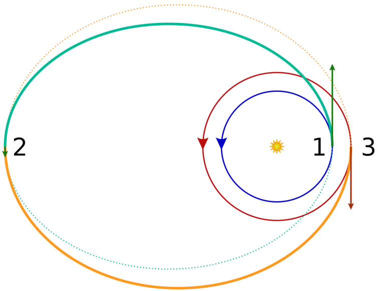
Introduction

History

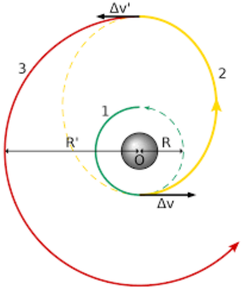
- Walter Hohmann born on 18 March,1880, was a German Scientist who was a trained Civil Engineer.
- In 1912, he read a book on Astronomy which sparkled him to study about spaceflight.

Figure: Sir Walter Hohmann

Introduction



Bi-elliptical Transfer Orbit



Hohmann Transfer Orbit

Introduction

Advantages of Hohmann over Bi-elliptical

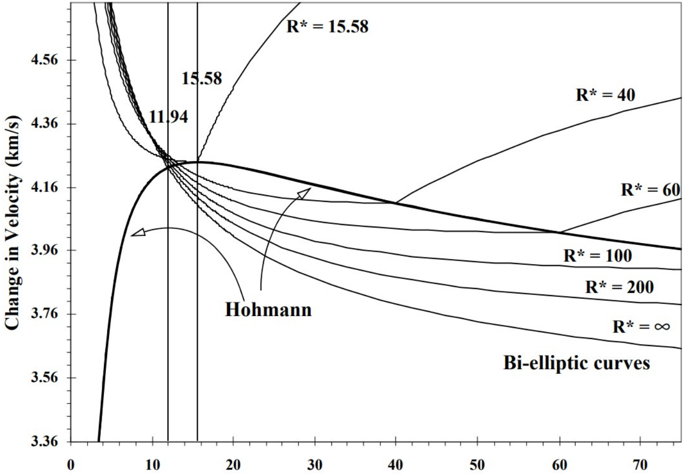
- The bi-elliptic transfer consists of two half-elliptic orbits, and they require one extra engine burn than a Hohmann transfer.
- While Bi-Elliptical Transfer requires more Δv than Hohmann when :

$$\frac{R_{Destination}}{R_{Origin}} \leq 11.94$$

  Hence here Hohmann Transfer uses less fuel and energy here than bi-elliptical transfer.
- While Bi-Elliptical Transfer requires less Δv than Hohmann when :

$$\frac{R_{Destination}}{R_{Origin}} \geq 11.94$$

  Hence here Hohmann Transfer uses more fuel and energy here than bi-elliptical transfer


R* = 15.58
15.58
11.94
R* = 40
R* = 60
R* = 100
R* = 200
R* = ∞
Hohmann
Bi-elliptic curves
Change in Velocity (km/s)
4.56
4.36
4.16
3.96
3.76
3.56
3.36
0
10
20
30
40
50
60
70

Introduction

Shortest path

- Though shortest path required to reach any planet is a straight line, to achieve it one has to continuously fire the rocket till it reaches the destination.
- While Hohmann transfer just fires at two different location and then uses Sun's gravitational force to reach the planet, hence consuming less energy.

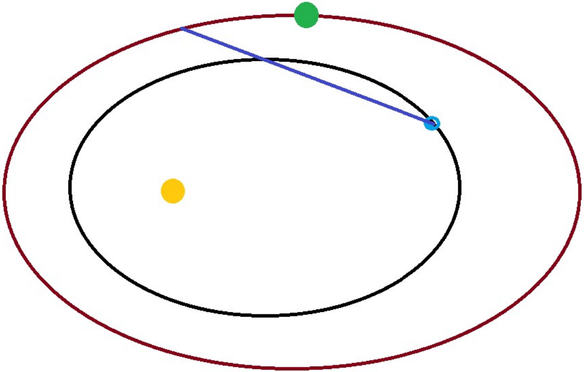


Figure: Direct Transfer

Introduction

Data

Important Data				
Planet	Orbital Radius (in AU)	Time Period (in Earth Years)	Eccentricity (e)	Mass(in $10^{-6} X M_{Sun}$)
Mercury	0.387	0.240	0.205	0.166014
Venus	0.722	0.616	0.007	2.08106272
Earth	1.000	1.000	0.017	3.003486962
Mars	1.520	1.880	0.094	0.3232371722
Jupiter	5.200	11.860	0.049	954.7919
Saturn	9.580	29.440	0.057	285.885670
Uranus	19.200	83.960	0.046	43.66244
Neptune	30.100	164.770	0.011	51.51384

Introduction

Working

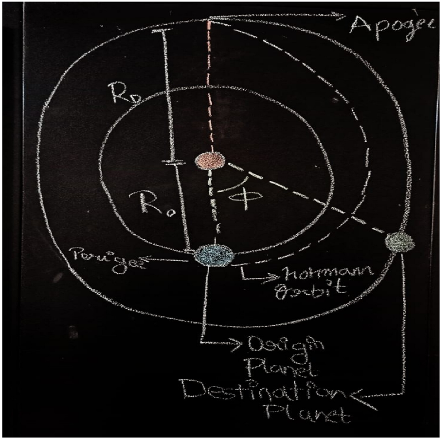


Fig 1 : Diagrammatic Representation of Hohmann Transfer

Introduction

Assumptions

- All planets are in same plane.
- Gravitation effect of Sun is only dominant in the entire solar system.
- The positions of planets in the solar system is random and is fixed at their eccentric anomaly. So for example if we are doing a transfer from Earth to Mars, the position for these two planets is fixed while of the others is random and this can be seen when we do the execution many times.

Introduction

Formula

$$\mathbf{V}_{\mathbf{Origin}} = \sqrt{\frac{\mathbf{GM}_{\mathbf{Sun}}}{\mathbf{R}_{\mathbf{Origin}}}} \tag{1}$$

$$\mathbf{V}_{\mathbf{Destination}} = \sqrt{\frac{\mathbf{GM}_{\mathbf{Sun}}}{\mathbf{R}_{\mathbf{Destination}}}} \tag{2}$$

$$\mathbf{V}_{\mathbf{Perigee}} = \sqrt{\frac{\mathbf{2GM}_{\mathbf{Sun}}\mathbf{R}_{\mathbf{Destination}}}{\mathbf{R}_{\mathbf{Origin}}(\mathbf{R}_{\mathbf{Origin}} + \mathbf{R}_{\mathbf{Destination}})}} \tag{3}$$

$$\mathbf{V}_{\mathbf{Apogee}} = \sqrt{\frac{\mathbf{2GM}_{\mathbf{Sun}}\mathbf{R}_{\mathbf{Origin}}}{\mathbf{R}_{\mathbf{Destination}}(\mathbf{R}_{\mathbf{Origin}} + \mathbf{R}_{\mathbf{Destination}})}} \tag{4}$$

Introduction

Formula

$$\Delta v_1 = \sqrt{\frac{GM_{Sun}}{R_{Origin}}}\left(\sqrt{\frac{2R_{Destination}}{R_{Origin} + R_{Destination}}} - 1\right) \tag{5}$$

$$\Delta v_2 = \sqrt{\frac{GM_{Sun}}{R_{Destination}}}\left(1 - \sqrt{\frac{2R_{Origin}}{R_{Origin} + R_{Destination}}}\right) \tag{6}$$

$$T = (0.5)\left(\frac{R_{Origin} + R_{Destination}}{2}\right)^{\frac{3}{2}} \tag{7}$$

$$\frac{T}{T_{Destination}} = \frac{180^\circ - \phi}{360^\circ} \tag{8}$$

Introduction

Dimensionless form

$$V'_{Origin} = \sqrt{\frac{1}{R_{Origin}}} \tag{9}$$

$$V'_{Destination} = \sqrt{\frac{1}{R_{Destination}}} \tag{10}$$

$$V'_{Perigee} = \sqrt{\frac{2R_{Destination}}{R_{Origin}(R_{Origin} + R_{Destination})}} \tag{11}$$

$$V'_{Apogee} = \sqrt{\frac{2R_{Origin}}{R_{Destination}(R_{Origin} + R_{Destination})}} \tag{12}$$

Introduction

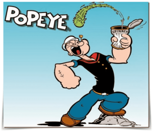
Dimensionless form

$$\mathbf{\Delta v_1'} = \sqrt{\frac{\mathbf{1}}{\mathbf{R_{Origin}}}}\left(\sqrt{\frac{\mathbf{2R_{Destination}}}{\mathbf{R_{Origin}} + \mathbf{R_{Destination}}}} - \mathbf{1}\right) \tag{13}$$

$$\mathbf{\Delta v_2'} = \sqrt{\frac{\mathbf{1}}{\mathbf{R_{Destination}}}}\left(\mathbf{1} - \sqrt{\frac{\mathbf{2R_{Origin}}}{\mathbf{R_{Origin}} + \mathbf{R_{Destination}}}}\right) \tag{14}$$

$$\frac{\mathbf{T}}{\mathbf{T_{Destination}}} = \frac{\mathbf{180}^\circ - \phi}{\mathbf{360}^\circ} \tag{15}$$

"After doing these changes, our equations changed like how Popeye changes after eating Spinach"



Computation

Leapfrog Integration

Newton's Second Law

$$\frac{d^2y}{dt^2} = a(y)$$

which can be written as

$$a = \frac{dv}{dt}$$

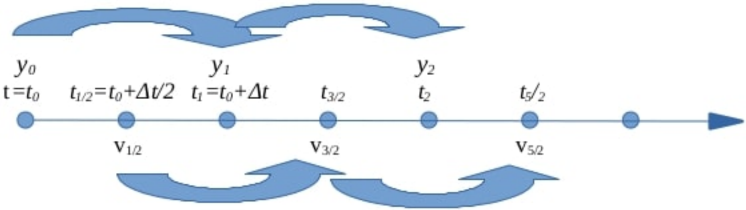
$$v = \frac{dy}{dt}$$

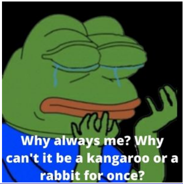
Now our aim is to calculate position y(t) and velocity v(t) by solving two coupled first order ordinary differential equations. For this, first of all, we descretize the time domain

Computation

Leapfrog Integration

To achieve the second order accuracy, in the Leapfrog method, the position y is evaluated at the end-points of the time points (at t_0, t_1, t_2 ...) and velocity v is evaluated at the mid-points of the time points (at $t_{1/2}, t_{3/2}, t_{5/2}$ ...), i.e. y and v are staggered in such a way that they "leapfrog" over each other as shown





Computation

Leapfrog Integration

Now, according to equations of motion, we know that

$$v = u + at$$

and

$$y = y_0 + vt$$

Using these we can derive advances from y_i to y_{i+1} and from $v_{i+1/2}$ to $v_{i+3/2}$:

$$y_{i+1} = y_i + v_{i+1/2}\Delta t$$

$$v_{i+1/2} = v_{i-1/2} + a_i\Delta t$$

Computation

Leapfrog Integration

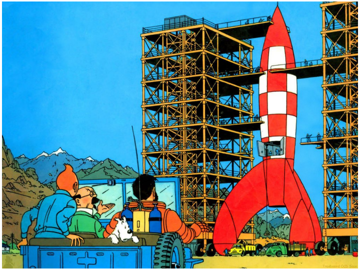
The Leapfrog scheme is second order accurate. Hence, in terms of accuracy, the Leapfrog scheme is better than the Euler's scheme but inferior to the Runge-Kutta 4^{th} order scheme. Yet we chose Leapfrog. Why?

- Time-reversibility i.e. we can calculate the solution in forward n time steps, and then reverse the direction of integration and can obtain the solution in backwards n time steps to arrive at the same initial conditions.
- Leapfrog scheme conserves the energy of dynamical systems. This strength becomes crucial when computing orbital dynamics. In comparison, many other integration schemes such as the Runge-Kutta 4^{th} order, do not conserve energy.

Analysis

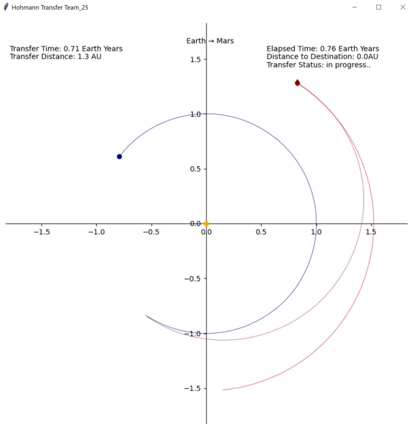
Simulation

Let's be a Martian!!!!



Analysis

Simulation



Conclusion

- It was a successful Bon-Voyage to different planets.
- As we observed, we only needed to add velocity at 2 different points.
- I hope we were successful in making this seemingly difficult concept simple for you to understand.

Mission Success!!

Thank You for your Patience!!

Acknowldegment

- We would like to thank **Prof.A.E.Roy** whose paper on **Astrodynamics (for teachers)** published in 1977, in *Physics Education, Volume 12, Number 7* for inspiring us to take up this project.
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- We would also like to thank the **'The Physics Society'** of **St.Stephen's College** for giving us a chance to showcase our project as a part of '**Annual Meera Memorial Paper Reading Competition**'

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The End

We are open to Questions!!

- For more information on the project:
  `https://github.com/SuhasAdiga/Hohmann.git`
- Scan this QR Code or use this link for watching the video involving derivations of the equations used.
  `https://tinyurl.com/ThHohmann`