Assignment No 9

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1 Aim

In this assignment, we continue our analysis of signals using Fourier Transforms. This time, we focus on finding transforms of non periodic functions.

2 Examples

The worked examples in the assignment are given below: Spectrum of $sin(\sqrt{2}t)$ is given below

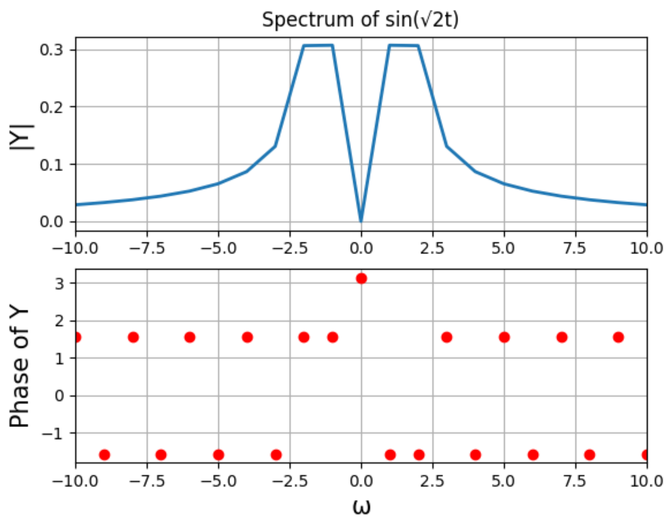


Figure 1: Spectrum of $\sin(\sqrt{2}t)$

Original function for which we want the DFT: As the DFT is computed

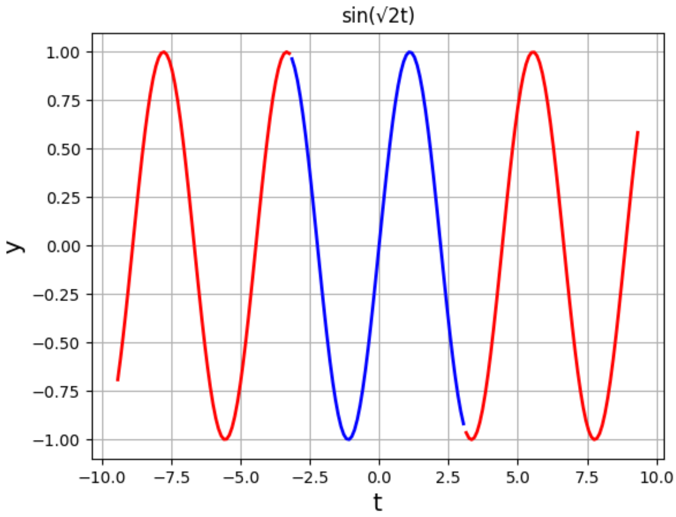


Figure 2: $\sin(\sqrt{2}t)$

over a finite time interval, we have actually plotted the DFT for this function

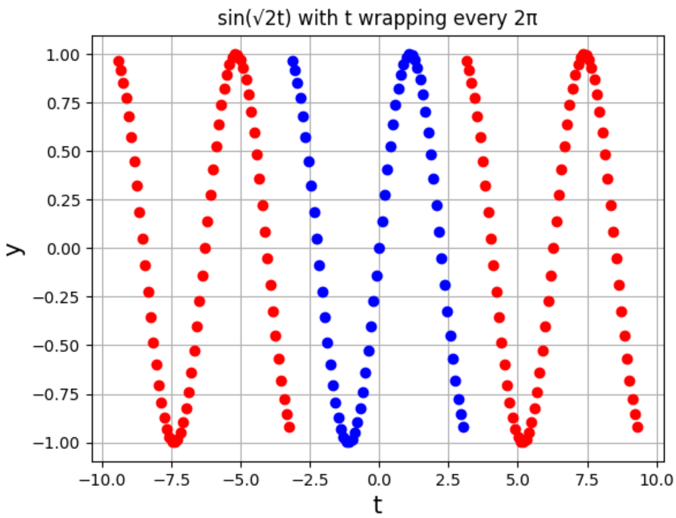


Figure 3: Spectrum of $\sin(\sqrt{2}t)$

These discontinuities lead to non harmonic components in the FFT which decay as $\frac{1}{\omega}$. To confirm this, we plot the spectrum of the periodic ramp.

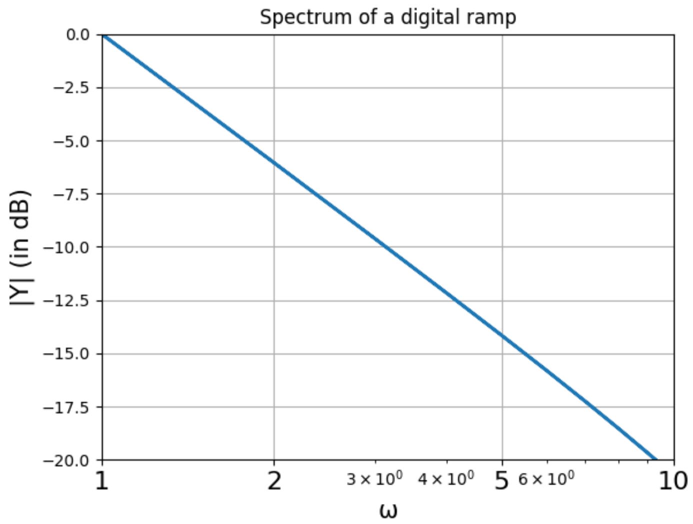


Figure 4: Spectrum of $\sin(\sqrt{2}t)$

2.1 Windowing

The hamming window removes discontinuities by attenuating the high frequency components that cause the discontinuities. The hamming window function is given by

$$x[n] = 0.54 + 0.46cos(\frac{2\pi n}{N-1}) \tag{1}$$

We multiply our signal with the hamming window and periodically extend it. The discontinuities nearly vanish.

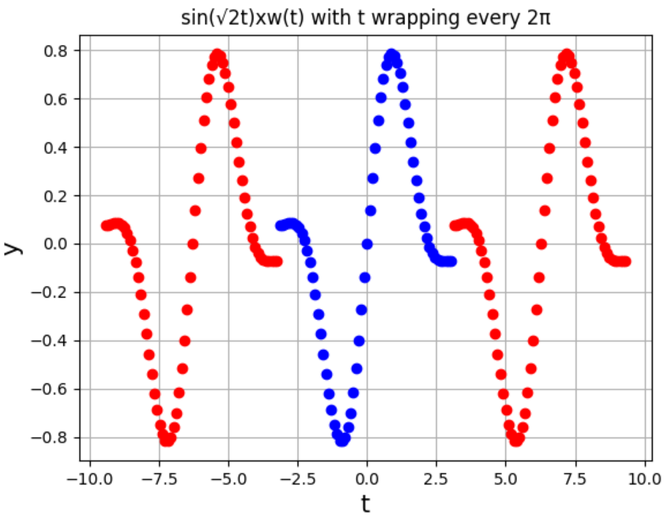


Figure 5: Spectrum of $\sin(\sqrt{2}t) * w(t)$

The spectrum that is obtained with a time period 2π is given below:

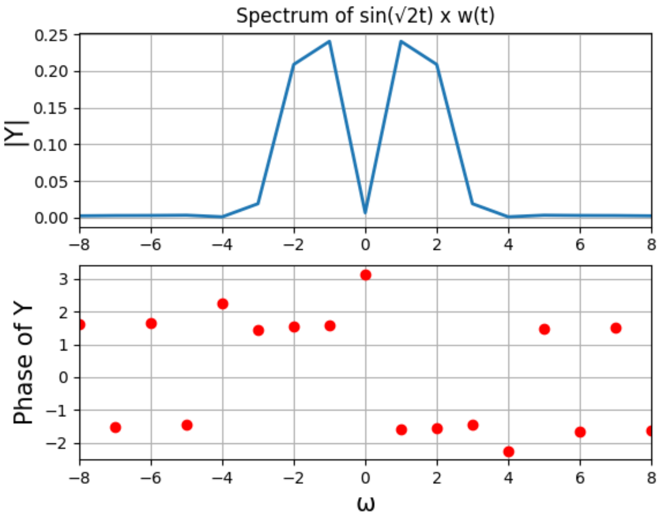


Figure 6: Spectrum of $\sin(\sqrt{2}t) * w(t)$

The spectrum that is obtained with a time period 8π has a sharper peak and is given below:

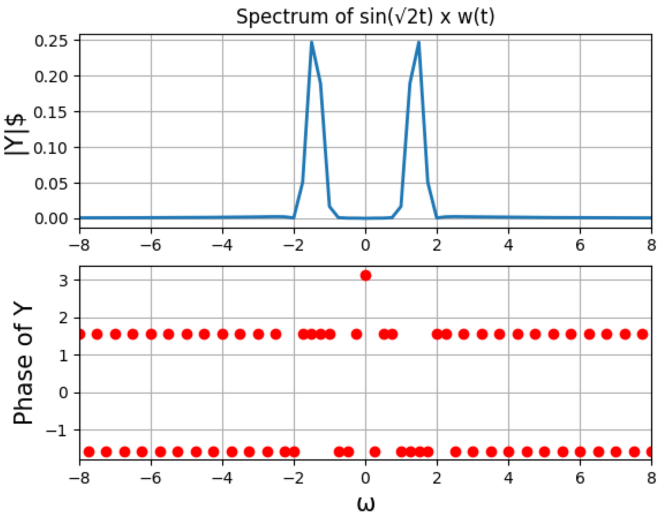
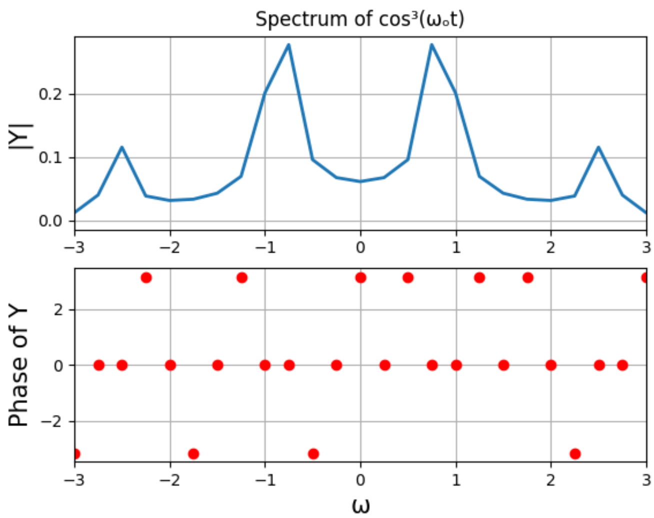


Figure 7: Spectrum of $\sin(\sqrt{2}t) * w(t)$

3 Questions

3.1 Question 2

In this question, we shall plot the FFT of $cos^3(0.86t)$ The FFT without the hamming Window:



We notice that a lot of the energy is stored in frequencies that aren't a part of the signal. After windowing, these frequencies are attenuated and hence the peaks are sharper in the windowed function.

3.2 Question 3

We need to estimate ω and δ for a signal $\cos(\omega t + \delta)$ for 128 samples between $[-\pi, \pi)$. We estimate omega using a weighted average. We have to extract the digital spectrum of the signal and find the two peaks at $\pm\omega_0$, and estimate ω and δ.

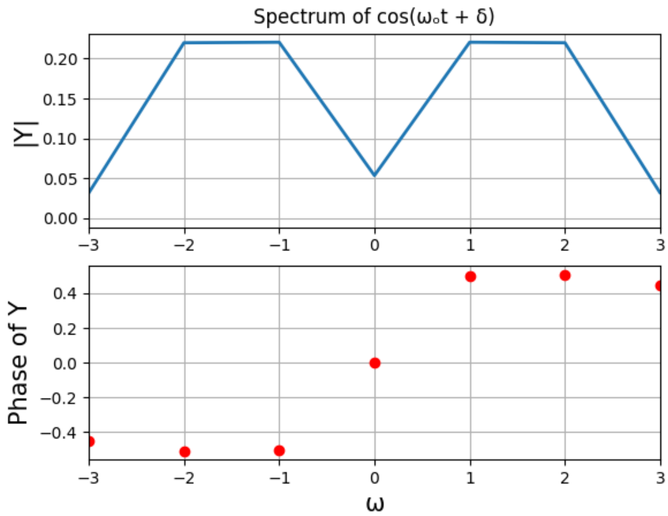


Figure 8: Fourier transform of $cos(1.5t + 0.5)$

We estimate omega by performing a Mean average of ω over the magnitude of $|Y(j\omega)|$. For delta we consider a widow on each half of ω (split into positive and negative values) and extract their mean slope.

3.3 Question 4

In this question we analyze a chirp signal which is an FM signal where frequency is directly proportional to time. A chirp signal we shall consider is given by

$$f(t) = cos(16t(1.5 + \frac{t}{2\pi})) \tag{2}$$

The FFT of the chirp is given by: We note that the frequency response is spread between 5-50 rad/s. A large section of this range apears due to Gibbs phenomenon. On windowing, only frequencies between 16 and 32 rad/s remain.

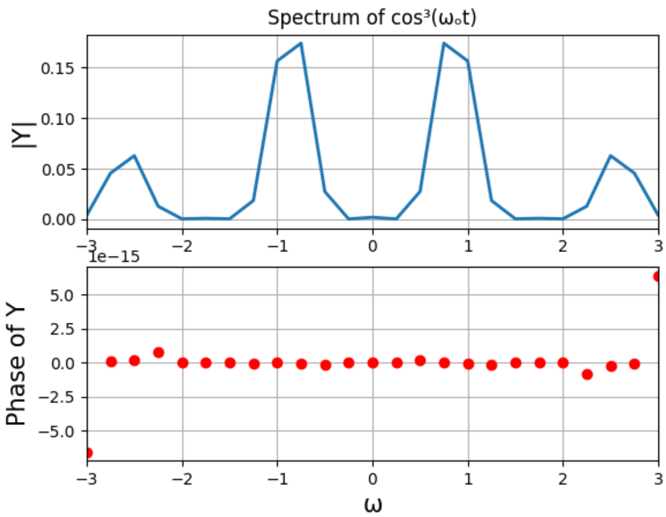


Figure 9: Chirp function fourier transform, windowed

4 Conclusion

The DFT was obtained using a 2 periodic extension of the signal, and thus the spectrum was found to be erroneous for a non periodic function. The spectrum was rectified by the using a windowing technique, by employing the Hamming window. Given a vector of cosine values in the a time interval, the frequency and phase were estimated from the DFT spectrum, by using the expectation value of the frequency and a parameter grid search for optimum values. The DFT of a chirped signal was analysed and its time-frequency plot showed the gradual variation of peak frequency of the spectrum with time.