1. A manufacturer of light bulbs claims that on the average 2% of the bulbs manufactured by his firm are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of the this sample, can you support the manufacturer's claim at 5% LOS?

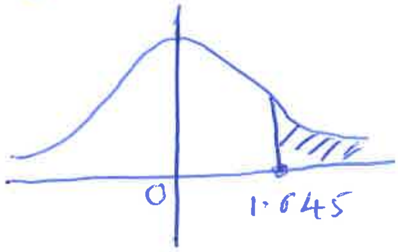
H_0: claim supported : P (proportion of defective bulbs) $= \frac{2}{100} = 0.02$

H_1: claim not supported: $P > 0.02$ (Right tailed)

Test statistic: p (proportion of defective bulbs in sample)

$$z = \frac{p - P}{\sqrt{PQ/n}}$$

$= 13/400 = 0.0325$



$$= \frac{0.0325 - 0.02}{\sqrt{\frac{0.02 \times 0.98}{400}}} = 1.7857 \in \text{critical region}$$

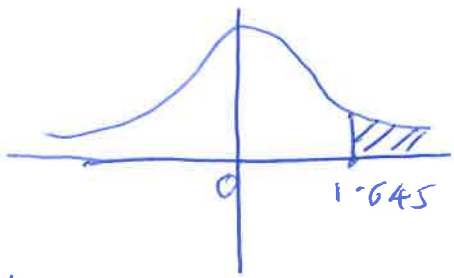
$\therefore$ H_0 is rejected. Manufacturer claim that 2% of bulbs are defective cannot be accepted

2. 100 people were affected by cholera and out of them only 90 survived. Would you reject the hypothesis that the survival rate, if affected by cholera, is 85% in favour of the hypothesis that it is more at 5% LOS?

H_0: $P = 0.85$

H_1: (Survival rate is more) : $P > 0.85$ (Right tailed)

$$p = \frac{90}{100} = 0.90$$



Test statistic:

$$z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.9 - 0.85}{\sqrt{\frac{0.85 \times 0.15}{100}}} = 1.4 \notin \text{critical region}$$

$\therefore$ H_0 is accepted

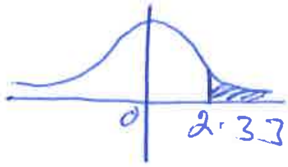
$\therefore$ Survival rate cannot be taken as more than 85%.

3. A random sample of 64 articles produced by a machine contained 14 defectives. Is it reasonable to assume that only 10% of the articles produced by the machine are defective? If not, find the 99% confidence limits for the percentage of defective articles produced by the machine.

$H_0 : P = 10\% = 0.1$

H_1 : (More than 10% are defective) : $P > 10\% = 0.1$ (Right tailed)

Test statistic: $p = \frac{14}{64} = 0.219$



$$z = \frac{p - P}{\sqrt{PQ/n}} = 3.17 \in \text{critical region}$$

$\therefore$ H_0 is rejected. Hence, more than 10% of articles are defective.

confidence interval (99%):

$$CI = p \pm \sqrt{\frac{pq}{n}} \times 2.58 = 0.219 \pm 0.517 \times 2.58 = (0.0856, 0.352)$$

$\therefore$ CI (in %) = (8.56% 35.2%)

4. During a countrywide investigation, the incidence of TB was found to be 1%. In a college with 400 students, 5 are reported to be affected whereas in another college of 1200 students, 10 are found to be affected. Does this indicate any significant difference? @ 5% LOS

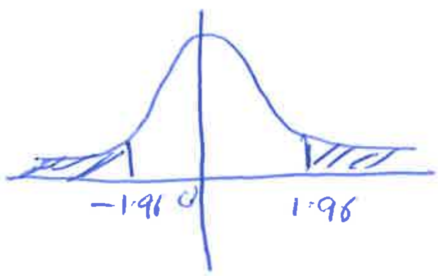
H_0 : (No significant difference) : $P_1 = P_2 \; (= P = 0.01)$

H_1 : (There is significant difference) : $P_1 \neq P_2$ (Two tailed)

$p_1 = 5/400 = 0.0125$

$p_2 = 10/1200 = 0.0083$

Test statistic.



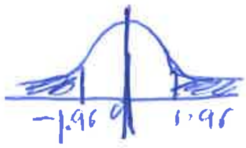
$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(1/n_1 + 1/n_2\right)}}$$

$= 0.721 \notin$ critical region

$\therefore$ Ho may be accepted.

Hence there is no significant difference between two samples.

5. A random sample of 600 men chosen from a certain city contained 400 smokers. In another sample of 900 men chosen from another city, there were 450 smokers. Do the data indicate that the cities are significantly different with respect to smoking habit among men?



H_0: (Not significantly different): ~~$P_1 \neq P_2$~~ $P_1 = P_2$

H_1: (significantly different): $P_1 \neq P_2$ (Two tailed)

$p_1 = \frac{400}{600} = 2/3 \quad p_2 = \frac{450}{900} = 1/2$

Test statistic:

$$Z = \frac{p_1 - p_2}{\sqrt{PQ(1/n_1 + 1/n_2)}} = 6.4 \in \text{critical region}$$

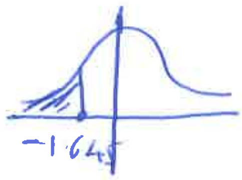
∴ Ho is rejected

∴ Cities are significantly different w.r.t smoking habits.

6. A sample of 300 spare parts produced by a machine contained 48 defectives. Another sample of 100 spare parts produced by another machine contained 24 defectives. Can you conclude that the first machine is better than the second? @ 5% LOS

H_0: (Both machines are same): $p_1 = P_2$

H_1: (1st machine is better than 2nd): $p_1 < p_2$ ⊛ (Left tailed)



p_1 = ~~part~~ proportion of defective parts $= \frac{48}{300} = 0.16$

$p_2 = \frac{24}{100} = 0.24 \qquad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.18$

Test statistic:

$$Z = \frac{p_1 - p_2}{\sqrt{PQ(1/n_1 + 1/n_2)}} = -1.8 \in \text{critical region}$$

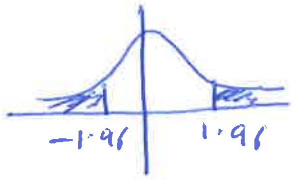
∴ Ho is rejected

∴ first machine is better than 2nd

7. A sample of 900 items is found to have a mean of 3.47 cm. Can it be reasonably regarded as a simple sample from a population with mean 3.23 cm and SD2.31 cm ? @ 5% LOS

H_0: Population mean $\mu = 3.23$

H_1: $\mu \neq 3.23$ (Two tailed)



Test statistic:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{3.47 - 3.23}{2.31/\sqrt{900}} = 3.11 \in \text{critical region}$$

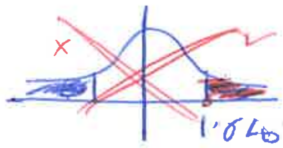
$\therefore$ H_0 is rejected.

Hence population mean cannot be taken as 3.23.

Or it cannot be regarded as the sample is from population ~~with~~ with mean 3.23.

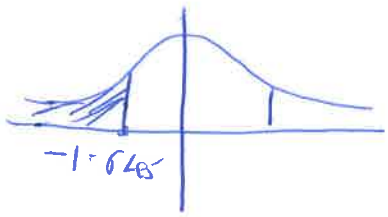
8. A manufacturer claims that, the mean breaking strength of safety belts for air passengers produced in his factory is 1275kgs. A sample of 100 belts was tested and the mean breaking strength and SD were found to be 1258 and 90 kg respectively. Test the manufacturer's claim at 5% LOS.

H_0: $\mu = 1275$



H_1: (Mean breaking strength is less than claimed): $\mu < 1275$ (Left tailed)

Test statistic



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1258 - 1275}{90/\sqrt{100}}$$

$= -1.88 \in$ critical region

$\therefore$ H_0 is ~~accepted~~ rejected.

$\therefore$ Manufacturer claim is not true.

9. An IQ test was given to a large group of boys in the age group of 18-20 years, who scored an average of 62.5 marks. The same test was given to a fresh group of 100 boys of the same age group. They scored an average of 64.5 marks with a SD 12.5 marks. Can we conclude that the fresh group of boys have better IQ?

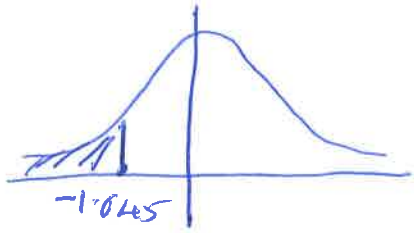
$\mu_1 \rightarrow$ Avg IQ of 1st group
$\mu_2 \Rightarrow$ Avg IQ of 2nd group.

$H_0 : \mu_1 = \mu_2$

H_1 : (Fresh group have better IQ) : $\mu_1 < \mu_2$ (Left tailed)

$\bar{x}_1 = 62.5, \quad n_1 \rightarrow$ large $(1/n_1 \rightarrow 0)$

$\bar{x}_2 = 64.5 \quad n_2 = 100 \quad \sigma = 12.5$



Test statistic

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{1/n_1 + 1/n_2}} = \frac{62.5 - 64.5}{12.5\sqrt{0 + 1/100}} = -1.6 \notin \text{critical region}$$

$\therefore H_0$ may be accepted

$\therefore$ we cannot conclude that fresh group have better IQ

10. A random sample of 100 students gave a mean weight of 58 kg with a SD of 4 kg. Find the 95% and 99% confidence limits of the mean of the population.

$n = 100, \quad \bar{x} = 58, \quad \sigma = 4$

95% confidence limits

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}} \times 1.96 = 58 \pm \frac{4}{\sqrt{100}} \times 1.96 = 58 \pm 0.784.$$

$CI = (57.2, \ 58.8)$

99% confidence limits

$$\bar{x} \pm \frac{\sigma}{\sqrt{n}} \times 2.58 = 58 \pm \frac{4}{\sqrt{100}} \times 2.58 = 58 \pm 1.032$$

$CI = \cancel{(58.1/2} \ (57, \ 59)$

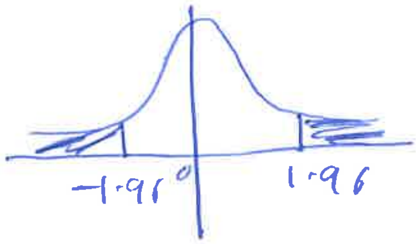
11. The means of two simple samples of 1000 and 2000 items are 170 cm and 169 cm respectively. Can the samples be regarded as drawn from the same population with SD 10 at 5% LOS?

H_0 : (samples drawn from same population) : $\mu_1 = \mu_2$

H_1 : (samples are from different population) : $\mu_1 \neq \mu_2$

(Two tailed)

<u>Test statistic</u>



$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{1/n_1 + 1/n_2}}$$

$$= \frac{170 - 169}{10\sqrt{1/1000 + 1/2000}} = 2.58 \in \text{critical region}$$

$\therefore$ H_0 is rejected

$\therefore$ It cannot be concluded that samples are drawn from same population.