Model Question Paper-I with effect from 2021-22 (CBCS Scheme)

USN										

18MAT753

Seventh Semester B.E.(CBCS) Examination

ADVANCED MATHEMATICAL METHODS

(Open Elective)

Time: 03 Hrs **Max.Marks: 100**

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ into tri-diagonal form by Given's method. Using Strum's sequence, obtain exact eigenvalues or the interval of unit length each containing one eigenvalue. **(12 marks)**

(b) Perform two iterations of the Birge-Vieta method to find a real root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $p_0 = 0.5$ and also find the deflated polynomial. **(08 marks)**

OR

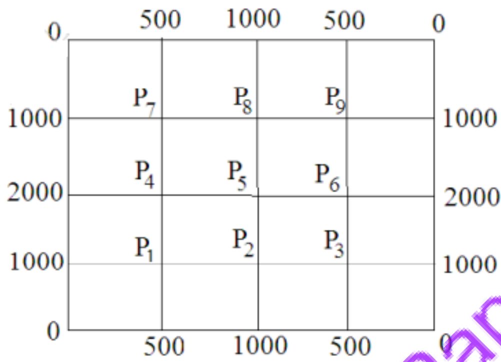
2. (a) Transform the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ to tri-diagonal form by Householder's method. Using Strum's sequence, obtain exact eigenvalues or the interval of unit length each containing one eigenvalue. **(12 marks)**

(b) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use the initial approximations $p_0 = -0.9, q_0 = 0.9$. **(08 marks)**

Contd...2

Module-2

3. a) Solve the Laplace equation $u_{xx}+u_{yy}=0$ for the following mesh with boundary conditions as shown below:

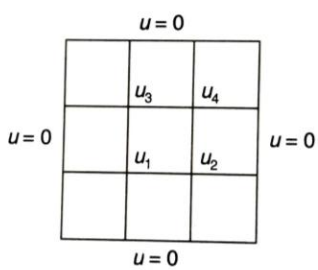


Carryout two iterations. **(10 marks)**

b) Use the Bender-Schmidt formula to solve the heat equation $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(x,0)=4x-x^2$ and $u(0,t)=u(4,t)=0$. **(10 marks)**

OR

4. a) Solve the Poisson equation $u_{xx}+u_{yy}=8x^2y^2$ in the domain of the figure. **(10 marks)**



b) Use the Crank-Nicolson scheme to solve the heat equation problem $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with the conditions: $u(x,0)=\sin x,\ 0\le x\le \pi$, $u(0,t)=0=u(\pi,t), t>0$. **(10 marks)**

Contd...3

Module-3

5. a) Find the Taylor's expansion of $f(z) = \frac{2z^3+1}{z^2+z}$ in the region $z = i$. **(06 marks)**

b) Evaluate $\oint \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)^2(z-2)}dz$, where C is the circle $|z| = 3$ **(07 marks)**

c) Evaluate $\int_0^\infty \frac{\cos ax}{x^2+1}dx.$ by using contour integration. **(07 marks)**

OR

6. a) Find the Laurent's expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ about the point $1 < z + 1 < 3$.

(06 marks)

b) Evaluate $\oint \frac{3z^2+2}{(z-1)(z^2+9)}dz$, where C is the circle $|z - 2| = 2$ **(07 marks)**

c) Showthat $\int_0^\infty \frac{x^{p-1}}{1+x}dx = \frac{\pi}{\sin px}, 0 < p < 1.$ using contour integration. **(07 marks)**

Module-4

7. a) Measurements on the length of a copper wire were taken in 2 experiments A and B as under:

A's measurements	12.29	12.25	11.86	12.13	12.44	12.78	12.77	11.90	12.47
B's measurements	12.39	12.46	12.34	12.22	11.98	12.46	12.23	12.06	

Test whether B's measurements are more accurate than A's at 5% level of significance.

(The readings taken in both cases being unbiased) **(10 marks)**

b) Set up an analysis of variance (ANOVA) table for the following per acre production data for three varieties of wheat each grown on four plots (See page 4) and state if the variety differences are significant at 5% level of significance: **(10 marks)**

Contd...4

Plot of land	Per acre production		
	Variety of Wheat		
	A	B	C
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

OR

8. a) From the following data test whether the two samples, drawn from two normal populations have the same variance at 5% level of significance: **(10 marks)**

Sample 1	60	65	71	74	76	82	85	87	
Sample 2	61	66	67	85	78	63	85	86	88

b) The three drying techniques for curing a glue were studied and the following times were observed:

Formula A	13	10	8	11	8	
Formula B	13	11	14	14		
Formula C	4	1	3	4	2	4

Test the hypothesis that the average times for the three formulae are same at 1% level of significance. **(10 marks)**

Module-5

9. a) Find the unique fixed probability vector for the regular stochastic matrix: **(06 marks)**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

b) Three boys, A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probability A has the ball for the fourth throw. **(07 marks)**

Contd...5

c) Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 min. Customers spend an average of 10 min in the barber's chair.

(i) What is the expected number of customers in the barber shop and in the queue?

(ii) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait

(iii) How much time can a customer expect to spend in the barber's shop? **(07 marks)**

OR

10 a) Explain: (i) Absorbing state (ii) Transient state (iii) Recurrent state **(06 marks)**

b) A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(07 marks)**

c) Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers per hour or at one of two channels in parallel with mean service rate of 11 customers per hour for each of the two channels. Assume both the queues to be Poisson type. **(07 marks)**

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