Model Question Paper-II with effect from 2021-22 (CBCS Scheme)

USN										

18MAT753

Seventh Semester B.E.(CBCS) Examination

ADVANCED MATHEMATICAL METHODS

(Open Elective)

Time: 03 Hrs **Max. Marks: 100**

Note: Answer any FIVE full questions, choosing at least ONE question from each module.

Module-1

1. (a) Transform the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ into tri-diagonal form by Given's method. Using Strum's sequence, obtain exact eigenvalues or the interval of unit length each containing one eigenvalue. **(12 marks)**

(b) Perform two iterations of the Birge-Vieta method to find a real root of the equation $x^4 + x^3 + 5x^2 + 4x + 4 = 0$ Use the initial approximation $p_0 = 1$ and also find the deflated polynomial. **(8 marks)**

OR

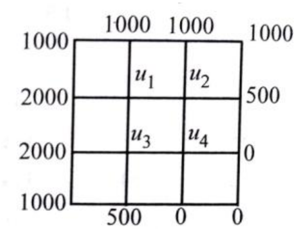
2. (a) Transform the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$ to tri-diagonal form by Householder's method. Using Strum's sequence, obtain exact eigenvalues or the interval of unit length each containing one eigenvalue. **(12 marks)**

(b) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + 5x^3 + 3x^2 - 5x - 9 = 0$. Use the initial approximations as $p_0 = 3 \; and \; q_0 = -5$ **(8 marks)**

Contd...2

Module-2

3. a) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ for the square mesh of the figure with boundary values as given below:



Carryout three iterations. **(10 marks)**

b) Solve the Poisson's equation $u_{xx} + u_{yy} = -81xy$ where $0 < x < 3,\ 0\ y < 3$ given that $u(0, y) = 0, u(x, 0) = 0, u(3, y) = 100, u(x, 3) = 100\ and\ h = 1$ **(10 marks)**

OR

4. a) Solve by Crank-Nicholson method: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0, u(4, t) = 0$ and $u(x, 0) = \frac{x}{3}(16 - x^2)$ Find $u(i, j)$ where $i = 0,1,2,3,4\ \ and\ \ j = 0,1,2$ **(10 marks)**

b) Obtain the solution of $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ by taking $h = 0.2$ where $0 \leq x \leq 1$ subjected to the initial conditions $u(x, 0) = \sin \pi x$, $u_t(x, 0) = \sin \pi x$ and the boundary conditions $u(0, t) = 0, u(1, t) = 0$ by using explicit method. **(10 marks)**

Module-3

5. a) Find the Taylor's expansion of $f(z) = \frac{1}{(z+1)^2}$ in the region $z = -i$ **(06 marks)**

b) Find the Residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles and hence evaluate $\oint_c f(z)dz$ where 'c' is the circle $|z| = 2.5$. **(07 marks)**

c) Evaluate $\int_0^{2\pi} \frac{cos3\theta}{5-4cos\theta} d\theta$ for the unit circle $|z| = 1$, using contour integration. **(07 marks)**

Contd...3

OR

6. a) Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)(z-2)}$ about the point $1 < z+1 < 3$

(06 marks)

b) Evaluate $\oint \frac{z-3}{(z^2+2z+5)} dz$, where C is the circle $|z+1-i| = 2$ **(07 marks)**

c) Evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$, by using contour integration. **(07 marks)**

Module-4

7. a) Two independent samples of eight and seven items respectively had the following values of the variable:

Sample 1 : 9 11 13 11 15 9 12 14

Sample 2 : 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly at 5% level of significance ($F_{0.05}$=4.21) . **(10 marks)**

b) Set up an analysis of variance (ANOVA) table to assess the significance of possible variation in performance in a certain test between the convent schools of a city. A common test was given to a number of students taken at random from the fifth class of the three schools concerned the results given below:

A	B	C
9	13	14
11	12	13
13	10	17
9	15	7
8	5	9

Construct the analysis of variance for the given data. **(10 marks)**

Contd...4

OR

8. a) The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find (i) 95% (ii) 99% confidence limits for mean of the maximum loads of all cables produced by the company. **(10 marks)**

b) Two sample sizes nine and eight gave the sums of squares of deviations from their respective means equal to 160 and 91 respectively. Can they be regarded as drawn from the same normal population? ($F_{0.05}$=3.73) **(10 marks)**

Module-5

9. a) Find the unique fixed probability vector for the regular stochastic matrix: **(06 marks)**

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

b) Each year a man trades his car for a new car in 3 brands of the popular company Maruthi Udyog limited. If he has a 'Standard' he trades it for 'Zen'. If he has a 'Zen' the trades it for a 'Esteem'. If he has a 'Esteem' he is just as likely to trade it for a new 'Esteem' or for a 'Zen' or a 'Standard' one. In 1996, he bought his first car which was Esteem. Find the probability the he has (i) 1998 Esteem (ii) 1998 Standard (iii) 1999 Zen and (iv) 1999 Esteem.

In the long run, how often will he have a Esteem? **(07 marks)**

c) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. Find the average number of persons waiting in the system.

Also, (i) what is the probability that a person arriving at the booth will have to wait in the queue?

Contd...5

(ii) What is the probability that it will take him more than 10min. altogether to wait for the phone and complete his call. Further, estimate the fraction of the day when the phone will be in use. **(07 marks)**

OR

10. a) Explain (i) Absorbing State (ii) Transient State (iii) Recurrent State **(06 marks)**

b) Patients arrive at a clinic according Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponentially with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic?
(ii) What is the probability that an arriving patient will not wait?

What is the expected waiting time until a patient is discharged from the clinic?

(07 marks)

c) A Man's eating food habit as follows: if he eats vegetarian one week, he switches to non- vegetarian food the next week with the probability 0.2 On the other hand, if he eats non- vegetarian food one week there is probability of 0.7 that he will eats non-vegetarian food the next week as well. In the long run how often does he eats vegetarian food. **(07 marks)**

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