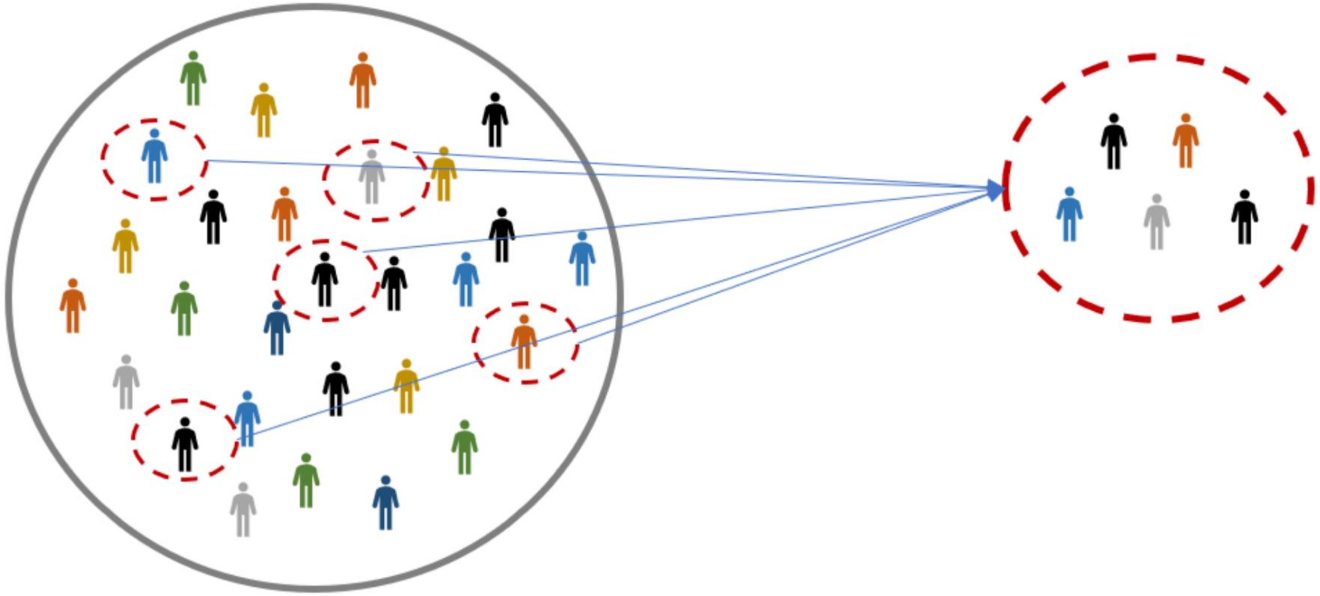
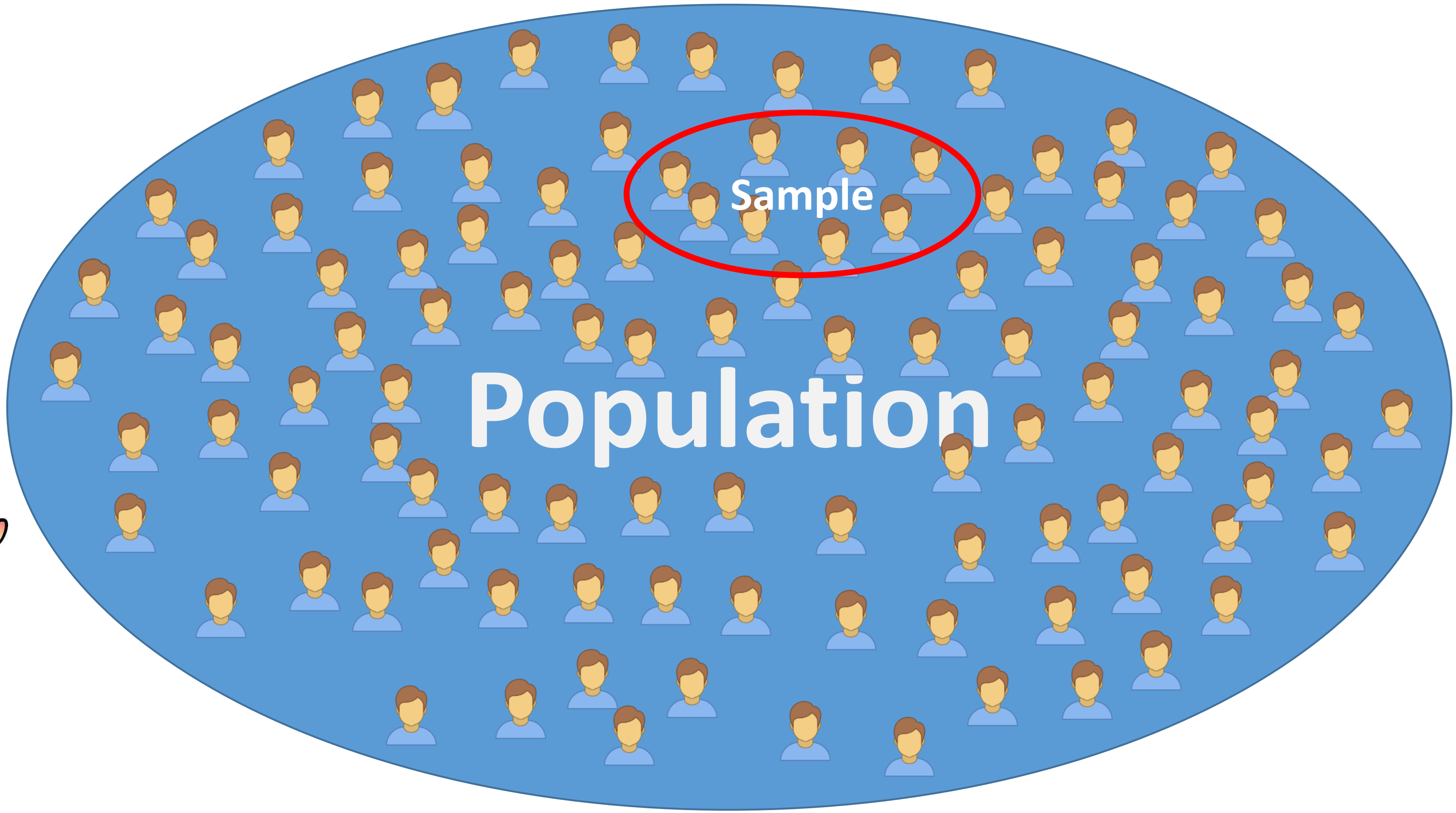
Sampling Theory & Testing of Hypothesis

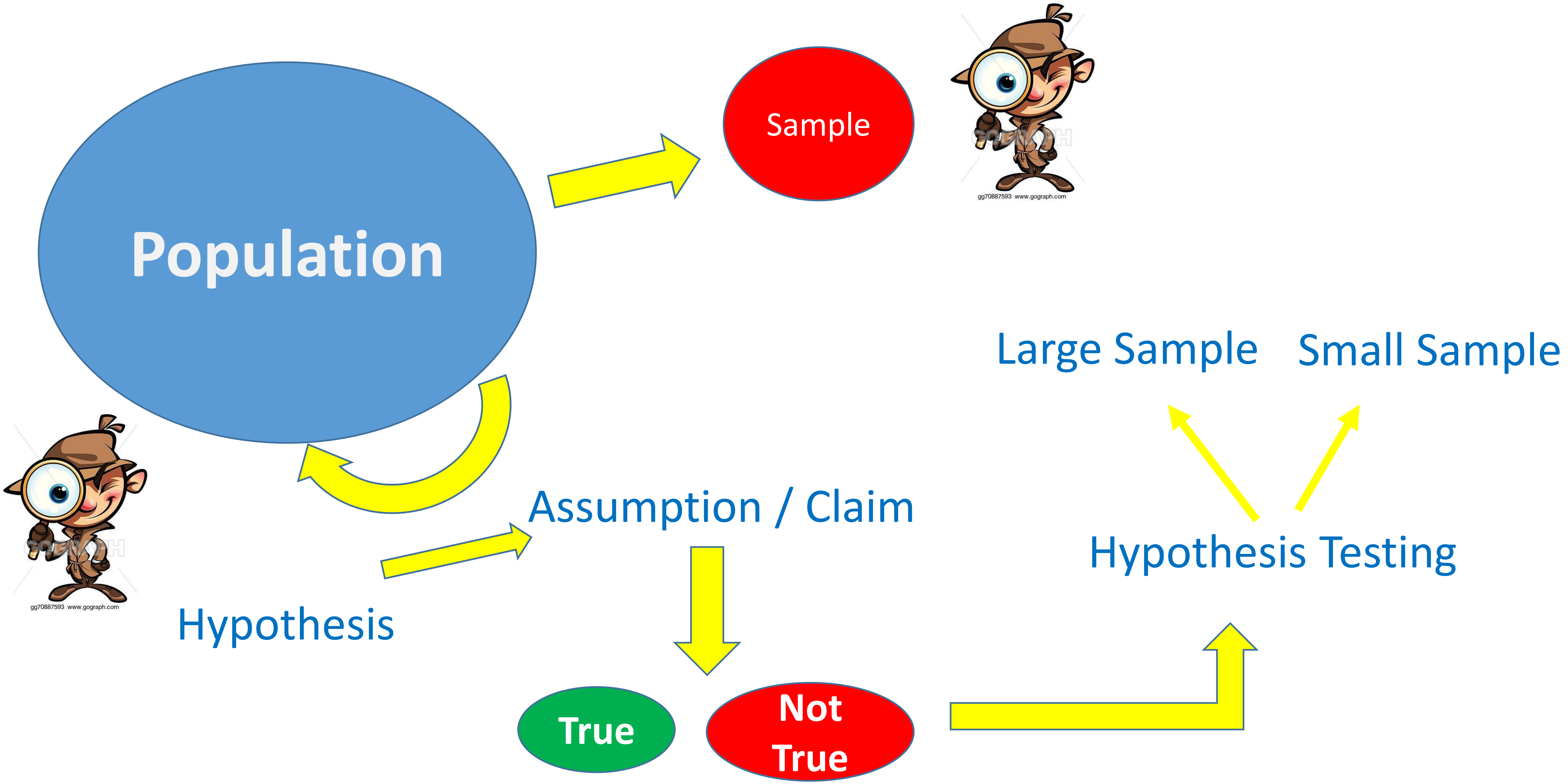


Global Academy of Technology
Growing Ahead Of Time

Dr. Anil P N

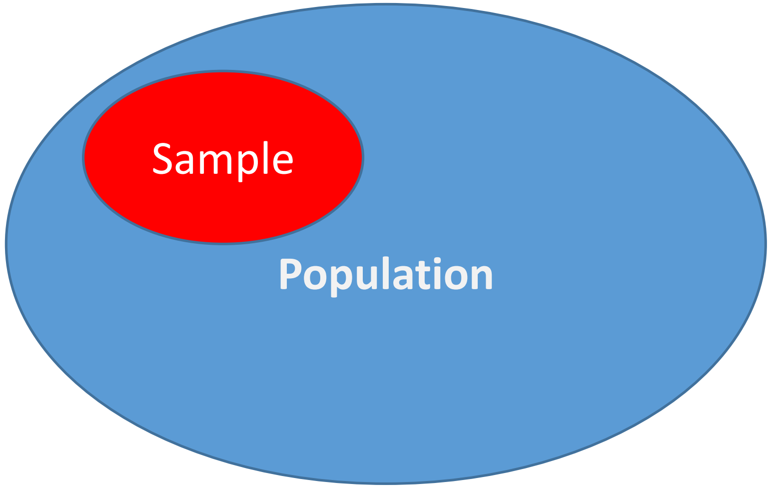

gg70887593 www.gograph.com


Sample
Population


Population
Sample
Hypothesis
Assumption / Claim
True
Not True
Hypothesis Testing
Large Sample
Small Sample

Population and Sample

- Population: Group of individuals or items under investigation or study.
- Sample: A finite subset of a population.
  - Size of Population : N
  - Size of Sample : n



Examples of Population & Sample

- All the people in a country.
- All the items produced by a company.
- All the Students in an University
- All people with ID proofs
- All members of parliament

- People above 60 years in a country.
- All the items produced in Sept 2022.
- All the Students of final year.
- All people with Aadhar card
- All female members of parliament

Parameter and Statistic

Parameter		Statistic
A number that describes the data from a population	Examples : Mean, Standard deviation, Proportion	A number that describes the data from a sample

Notations

μ	Mean	$\bar{x}$
σ	Standard Deviation	S
P	Proportion	p

Sampling Distribution

- Population size : N
- Sample size : n
- Number of samples of size n = ${}^{N}C_{n}$
- Compute a statistic (say Mean) for each sample

$$\overline{x} = \{\overline{x}_1, \overline{x}_2, \overline{x}_3, \cdots\cdots\cdots\cdots\}$$

Population Size : N = 100

Sample Size: n = 5

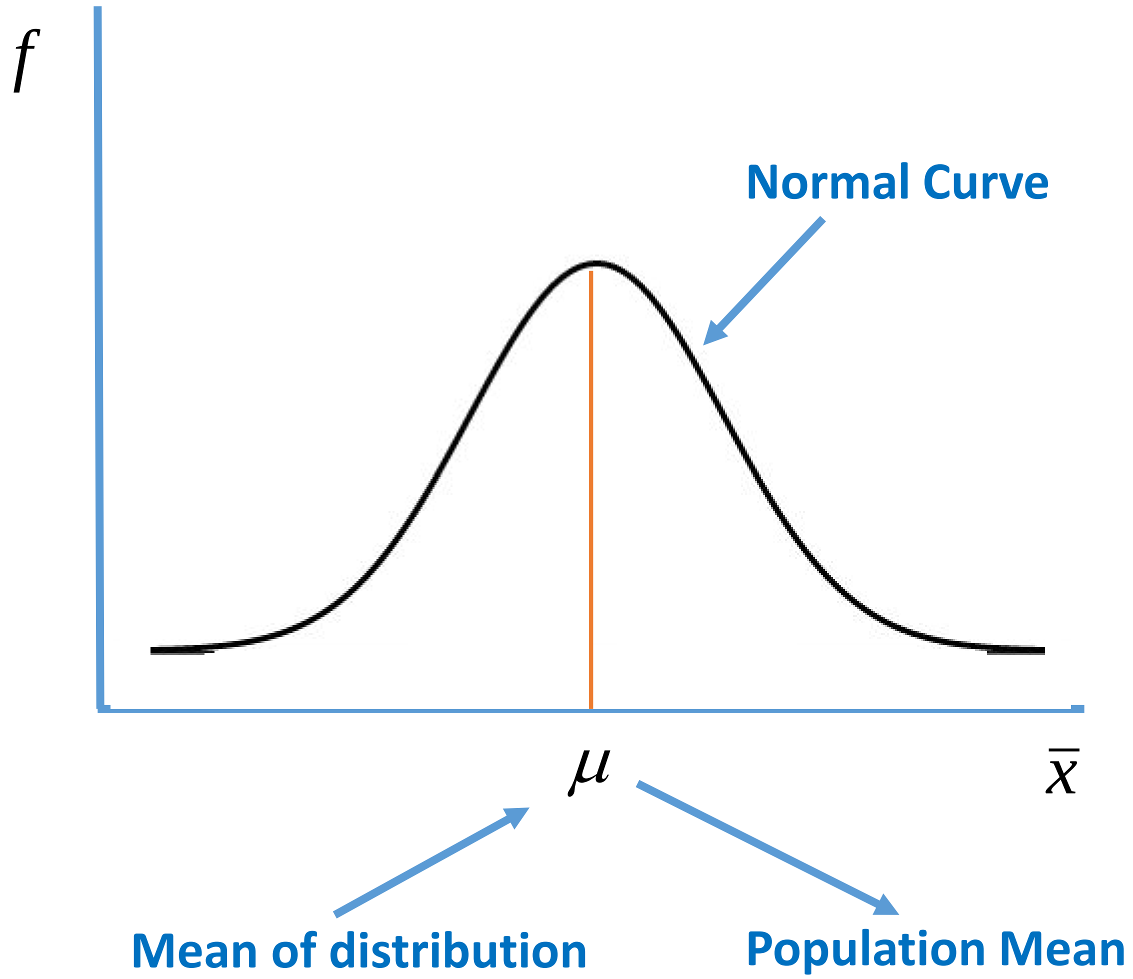
Number of samples = 7,52,87,520

Sampling Distribution

$\overline{x}$	f
$\overline{x}_1$	f_1
$\overline{x}_2$	f_2
$\overline{x}_3$	f_3
.	.
.	.
.	.

Sampling Distribution

$\bar{x}$	f
$\bar{x}_1$	f_1
$\bar{x}_2$	f_2
$\bar{x}_3$	f_3
.	.
.	.
.	.



Standard deviation of sampling distribution (Standard Error) :

$$S.E. = \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Testing of Hypothesis

Dr. Anil P N

Testing of Hypothesis

- Hypothesis: Assumptions about the data (population)
  - Average weight of newborn babies in India is more than 6.5 pounds.
  - Average life of electric bulbs produced by XYZ company is 1600 hours.
  - Average life of a human being in India is less than 65 years
  - Children in India watch on an average of 10 hours of TV per week.
- Hypothesis testing: Testing validity of hypothesis using sample data.
  - Interpretation or drawing conclusions about the Population using Sample data.

Null Hypothesis and Alternate Hypothesis

- Null Hypothesis(H_0): Statement about a population parameter, such as the population mean, that is assumed to be true.
- Alternate Hypothesis (H_1) : Statement that directly contradicts a null hypothesis.

Claim : Average weight of newborn children in India is more than 6.5 pounds.

- H_0 : $\mu = 6.5$
- H_1 : $\mu > 6.5$

Null Hypothesis and Alternate Hypothesis

Claim : Average life of electric bulbs produced by XYZ company is less than 1600 hours.

- $H_0 : \mu = 1600$
- $H_1 : \mu < 1600$

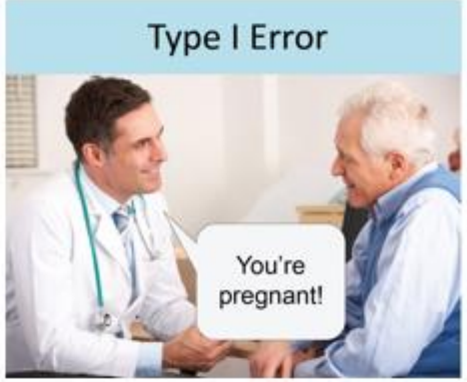
Claim : Children in India watch on an average of 10 hours of TV per week.

- $H_0 : \mu = 10$
- $H_1 : \mu \neq 10$

Errors in Testing of Hypothesis

- Error of rejecting null hypothesis when it is true is called Type I error.

- Error of accepting null hypothesis when it is not true is called Type II error.



H_0 : Not Pregnant	Null hypothesis is TRUE	Null hypothesis is FALSE
Reject null hypothesis	Type I Error (False positive)	Correct outcome! (True positive)
Fail to reject null hypothesis	Correct outcome! (True negative)	Type II Error (False negative)

Level of Significance

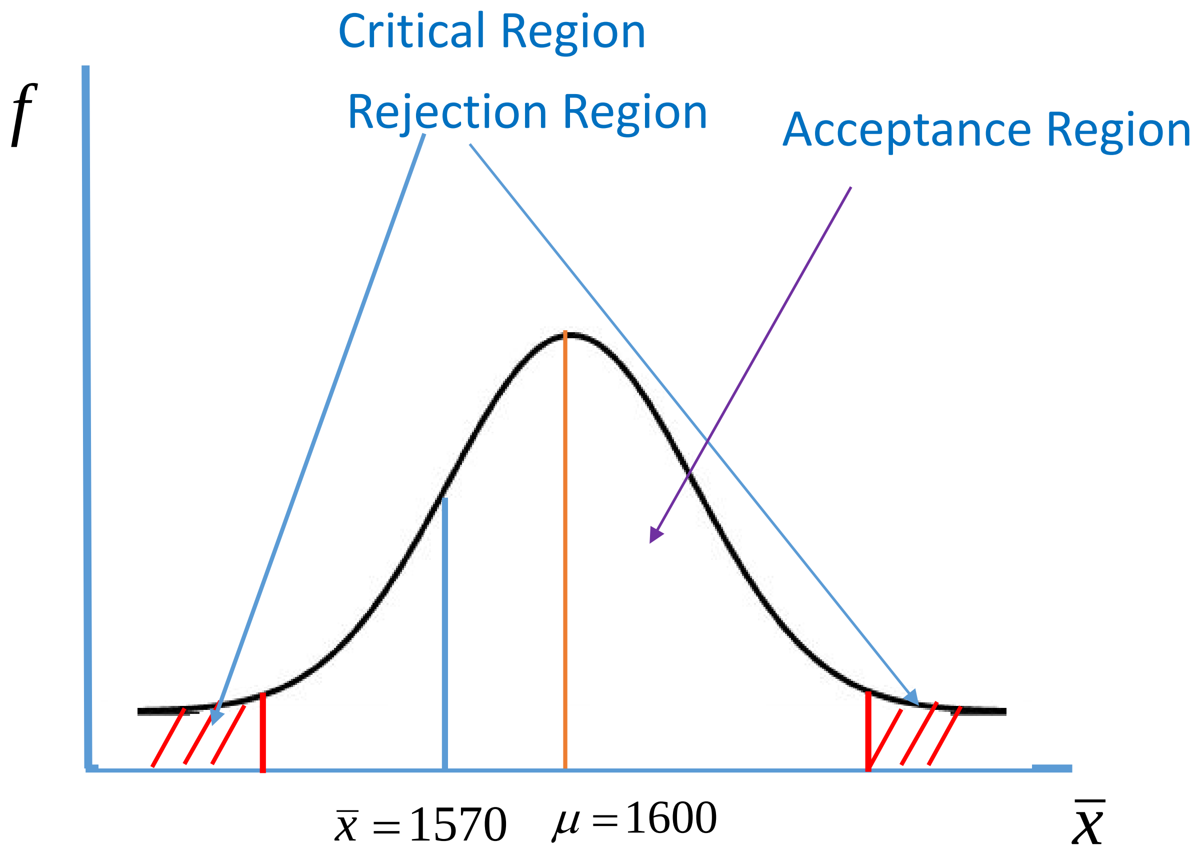
Claim : Average life of electric bulbs produced by XYZ company is 1600 hours.

- $H_0: \mu = 1600$
- $H_1: \mu \neq 1600$

Suppose Sample Mean is $\bar{x} = 1570$

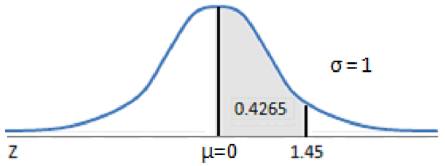
Level of Significance $\alpha = 5\%$

Probability of rejecting the null hypothesis when it is true



Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

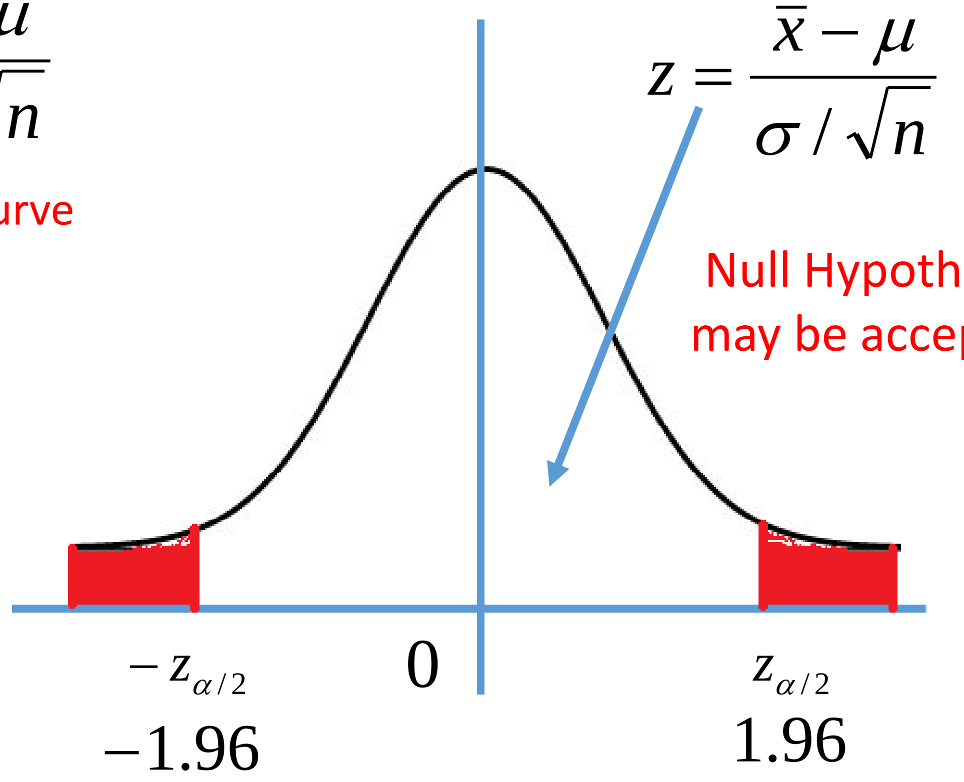
gion

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Standard Normal Curve

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Null Hypothesis may be accepted



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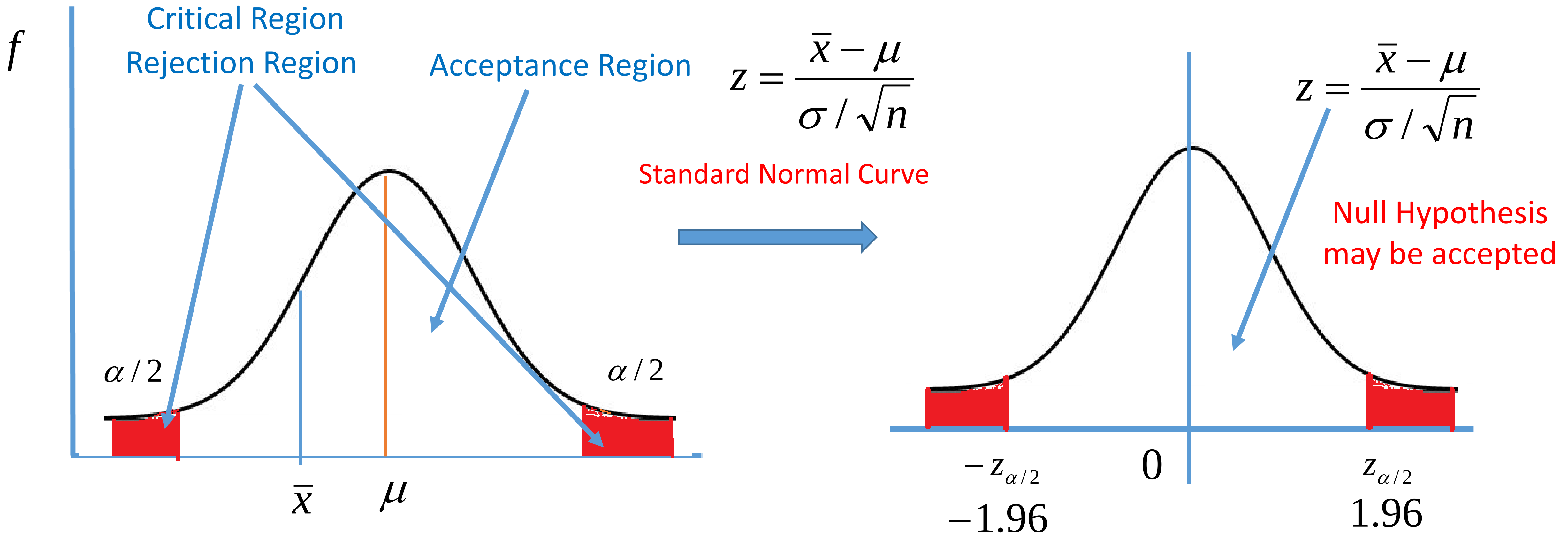
$$< z_{\alpha/2}) = 0.95 \longrightarrow p(0 < z < z_{\alpha/2}) = 0.475$$

$$\longrightarrow z_{\alpha/2} = 1.96$$

Level of Significance



Acceptance Region $p(-z_{\alpha/2} < z < z_{\alpha/2}) = 0.95 \Rightarrow p(0 < z < z_{\alpha/2}) = 0.475$

$\Rightarrow z_{\alpha/2} = 1.96$

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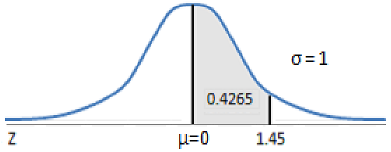

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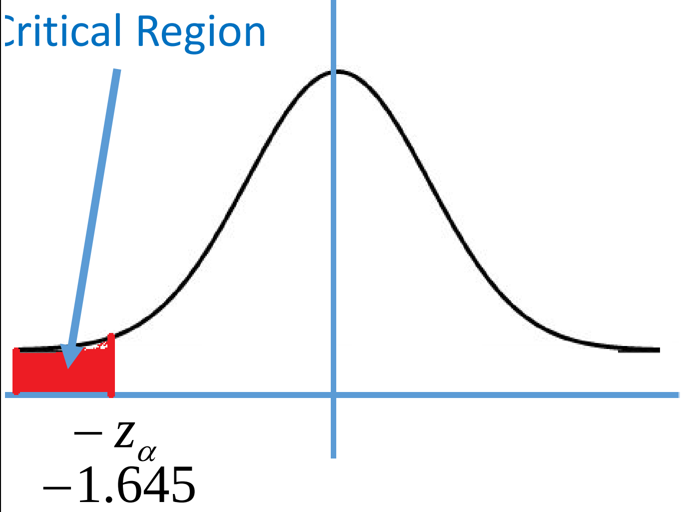
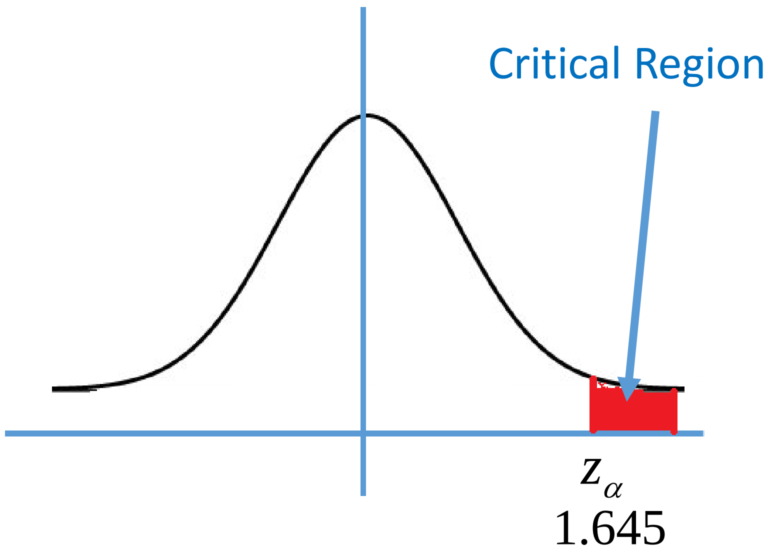
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000



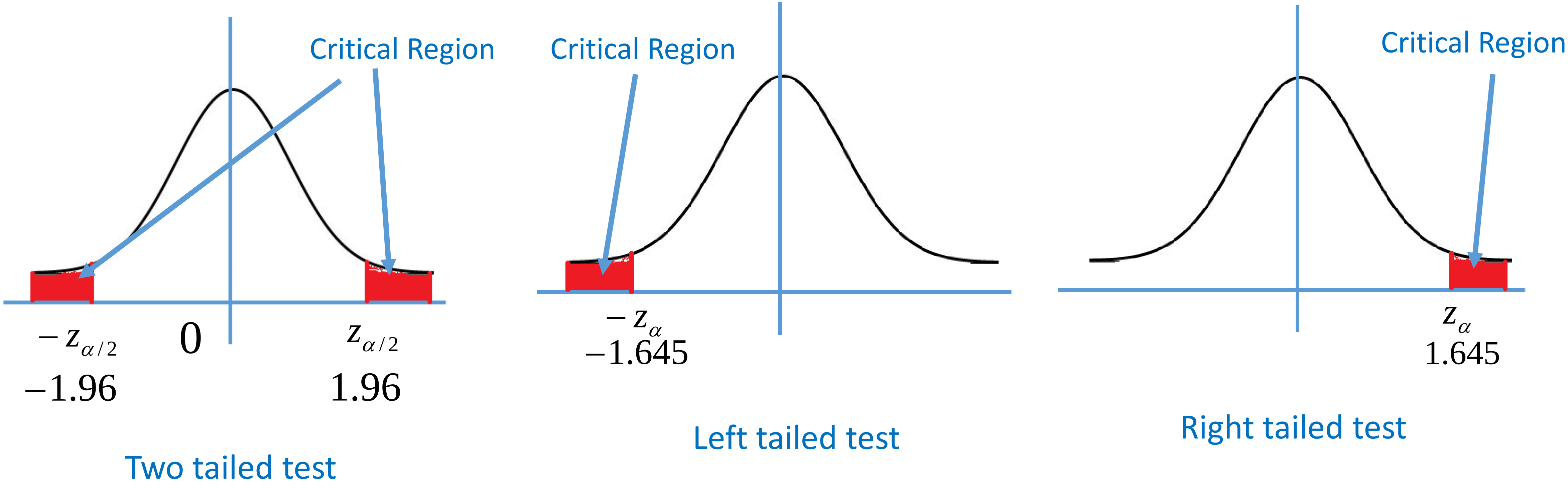
Left tailed test



Right tailed test

f Significance $\alpha = 5\% = 0.05$

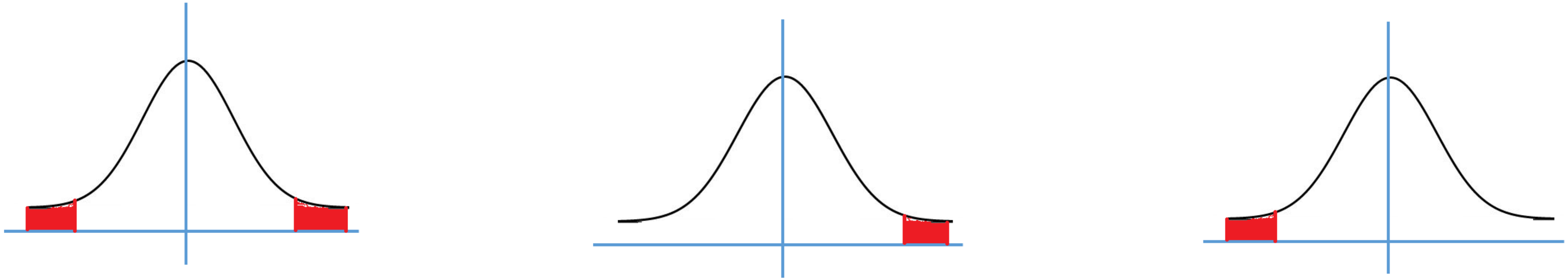
Level of Significance



Level of Significance $\alpha = 5\% = 0.05$

Level of Significance

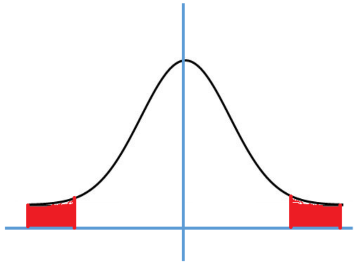
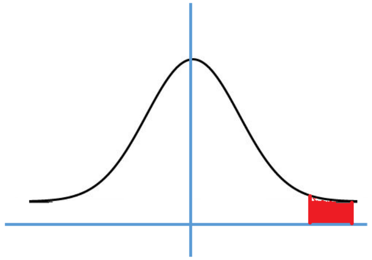
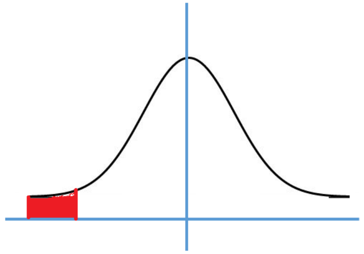
Nature of Test	Level of Significance		
	$\alpha = 1\% = 0.01$	$\alpha = 5\% = 0.05$	$\alpha = 10\% = 0.1$
Two tailed test	2.58	1.96	1.645
Right tailed test	2.33	1.645	1.28
Left tailed test	-2.33	-1.645	-1.28



Procedure for Testing of Hypothesis (Large Sample)

Step 1: Setup Null hypothesis H_0 $E(t) = K$

Step 2: Setup Alternate hypothesis H_1 and decide nature of test

$E(t) \neq K$	$E(t) > K$	$E(t) < K$	Step 3: Choose level of significance
			
-2.58 2.58	2.33	-2.33	$\alpha = 1\%$
-1.96 1.96	1.645	-1.645	$\alpha = 5\%$
-1.645 1.645	1.28	-1.28	$\alpha = 10\%$

Step 4:
Compute test statistic

$$z = \frac{t - E(t)}{S.E.(t)}$$

Conclusion

$z \in$ Critical Region

Reject null hypothesis

$z \notin$ Critical Region

Accept null hypothesis

Procedure for Testing of Hypothesis (Large Sample)

Test statistic $$z = \frac{t - E(t)}{S.E.(t)}$$

Test for Mean $$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Test for Proportion $$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}, \quad Q = 1 - P$$

Test for Equality of Mean $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Test for Equality of Proportions

$$z = \frac{p_1 - p_2}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}}$$

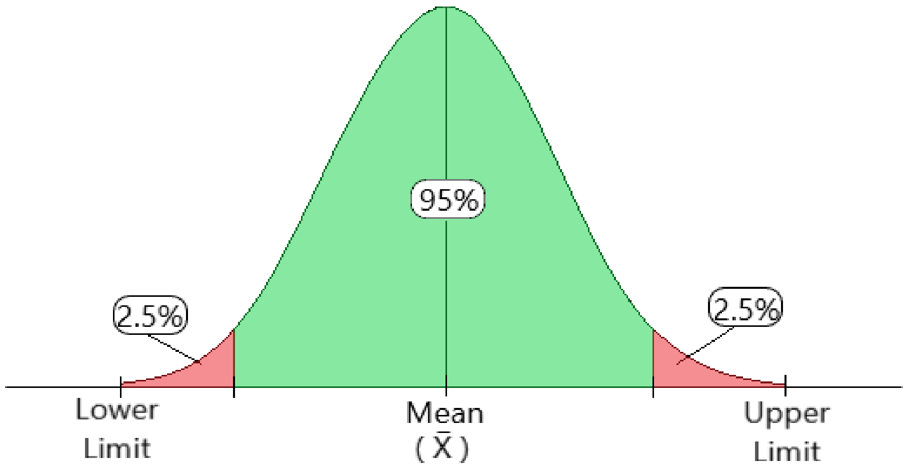
$$z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

Confidence Interval or limits

- Confidence interval is an interval within which the parameter is expected to lie.

	Confidence Interval
99% Confidence interval	$t \pm 2.58SE(t)$
95% Confidence interval	$t \pm 1.96SE(t)$
90% Confidence interval	$t \pm 1.645SE(t)$



Confidence Interval or limits

Parameter	**95% Confidence Interval**	**99% Confidence Interval**	**90% Confidence Interval**
Mean	$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm 1.645 \frac{\sigma}{\sqrt{n}}$
Proportion	$p \pm 1.96 \sqrt{\frac{pq}{n}}$	$p \pm 2.58 \sqrt{\frac{pq}{n}}$	$p \pm 1.645 \sqrt{\frac{pq}{n}}$

Example

A random sample of 400 items is found to have a mean 82 and standard deviation 18. Find 95% confidence limits for the mean of population from which sample is drawn.

Solution:

Given $\bar{x} = 82, \sigma = 18 \, and \, n = 400$

95% confidence limits for population mean are given by

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 82 \pm 1.96 \frac{18}{\sqrt{400}} = 82 \pm 1.764$$

Hence Confidence interval is given by $(80.236, \quad 83.764)$

Testing of Hypothesis

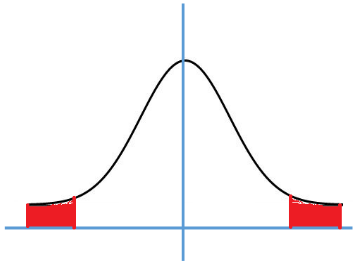
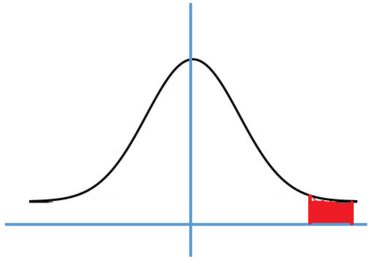
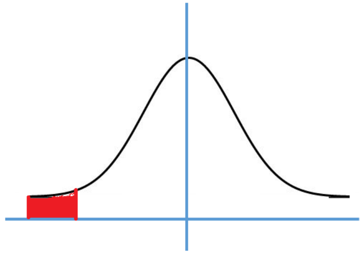
Examples

Dr. Anil P N

Procedure for Testing of Hypothesis (Large Sample)

Step 1: Setup Null hypothesis H_0 $E(t)=K$

Step 2: Setup Alternate hypothesis H_1 and decide nature of test

$E(t) \neq K$		$E(t) > K$	$E(t) < K$	Step 3: Choose level of significance
				
-2.58	2.58	2.33	-2.33	$\alpha = 1\%$
-1.96	1.96	1.645	-1.645	$\alpha = 5\%$
-1.645	1.645	1.28	-1.28	$\alpha = 10\%$

Step 4:
Compute test statistic

$$z = \frac{t - E(t)}{S.E.(t)}$$

Conclusion

$z \in$ Critical Region

Reject null hypothesis

$z \notin$ Critical Region

Accept null hypothesis

Procedure for Testing of Hypothesis (Large Sample)

Test statistic $$z=\frac{t-E(t)}{S.E.(t)}$$

Test for Mean $$z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

Test for Proportion $$z=\frac{p-P}{\sqrt{\frac{PQ}{n}}},\ Q=1-P$$

Test for Equality of Mean $$z=\frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1{}^2}{n_1}+\frac{\sigma_2{}^2}{n_2}}}$$

Test for Equality of Proportions

$$z=\frac{p_1-p_2}{\sqrt{\frac{P_1Q_1}{n_1}+\frac{P_2Q_2}{n_2}}}$$

$$z=\frac{p_1-p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$P=\frac{n_1p_1+n_2p_2}{n_1+n_2}$$

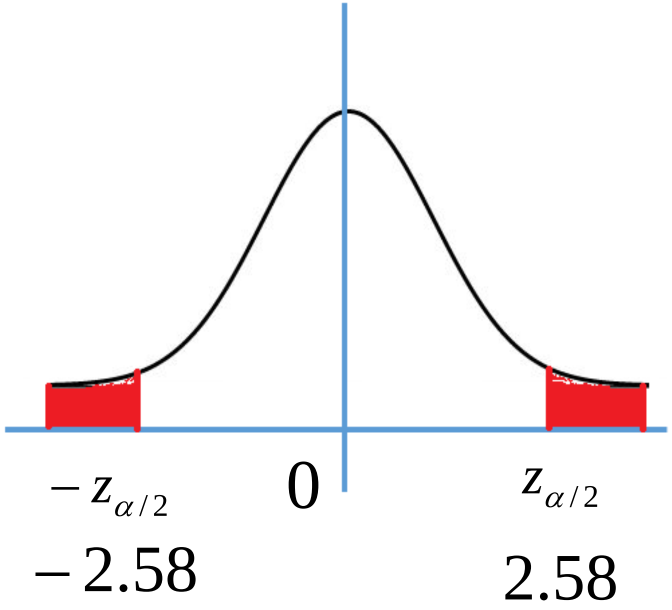
Example 1

The mean life of sample of 100 bulbs produced by a company is computed to be 1570 hours with population standard deviation 120 hours. If μ is the mean life of all bulbs produced by the company, test the hypothesis μ = 1600 hours, using level of significance α=0.01.

Solution:

H_0 : Population mean μ = 1600 hours

H_1 : Population mean μ ≠ 1600 hours (Two tailed)



Test Statistic:

$$z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}=\frac{1570-1600}{120/\sqrt{100}}=-2.5\notin \text{Critical Region}$$

Hence H_0 may be accepted

Thus mean life of bulbs produced by the company is 1600 hours

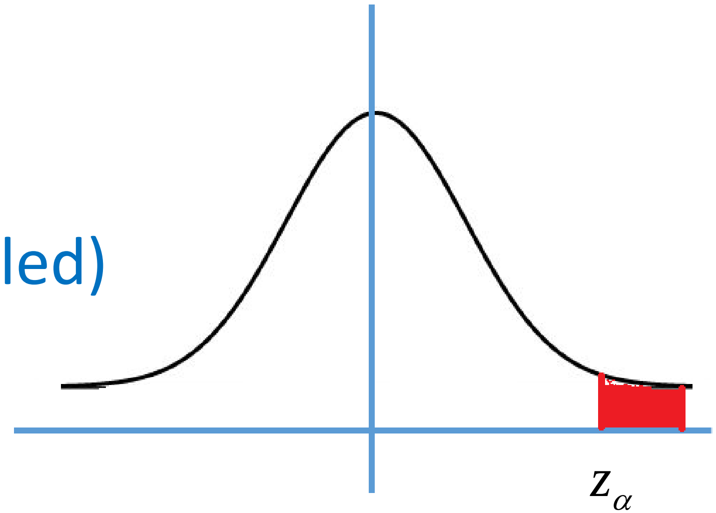
Example 2

A random sample of 100 recorded deaths in past year showed an average life span of 71.8 years. Assuming population standard deviation 8.9 years, does the data indicate that average span today is greater than 70 years. Use 0.05 level of significance.

Solution:

H_0 : Average life span is 70 years : $\mu = 70$

H_1 : Average life span is more than 70 years: $\mu > 70$ (Right tailed)



Test Statistic:

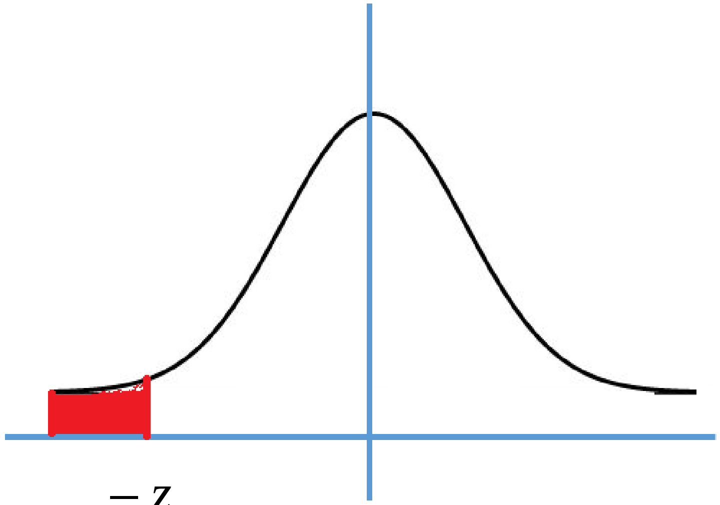
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{71.8 - 70}{8.9 / \sqrt{100}} = 2.022 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus average life span may be greater than 70 years.

Example 3

The standard deviation of marks scored by candidates in CET is 10.5 and the mean marks scored by 64 randomly selected candidates is 38. Can we conclude at 1% level of significance that the average CET marks is less than 40?



Solution:

H_0 : Average CET marks is 40: $\mu = 40$

H_1 : Average CET marks is less than 40: $\mu < 40$ (Left tailed)

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{38 - 40}{10.5 / \sqrt{64}} = -1.5238 \notin \text{Critical Region}$$

Hence H_0 may be accepted   Thus average marks scored in CET is 40.

Example 4

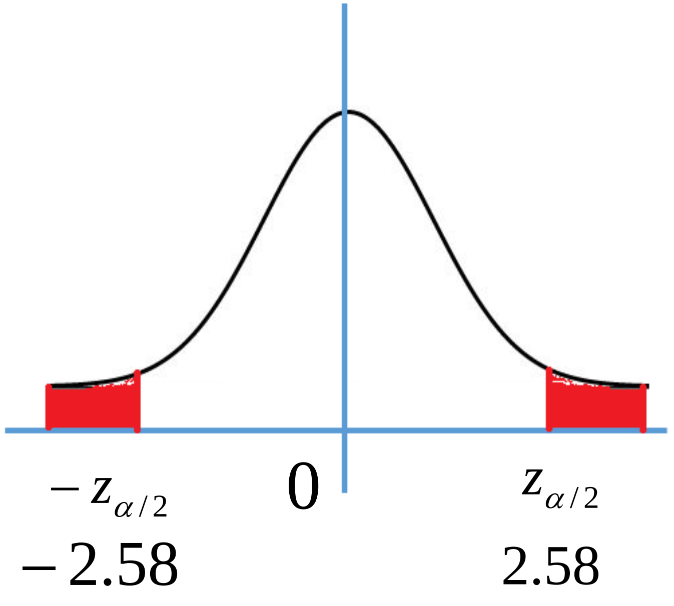
A die was thrown 1200 times & number 6 was obtained 236 times. Can the die is considered as fair at 0.01 level of significance

Solution:

H_0 : Die is fair (Un biased) – Population Proportion P = 1/6

H_1 : Die is not fair (Biased) P ≠ 1/6 (Two tailed)

P = Sample Proportion = 236/1200 = 0.1967



Test Statistic:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.1967 - 1/6}{\sqrt{\frac{1/6 \times 5/6}{1200}}} = 2.78 \quad \in \text{Critical Region}$$

Hence H_0 is rejected

Thus, Die is not fair

Example 5

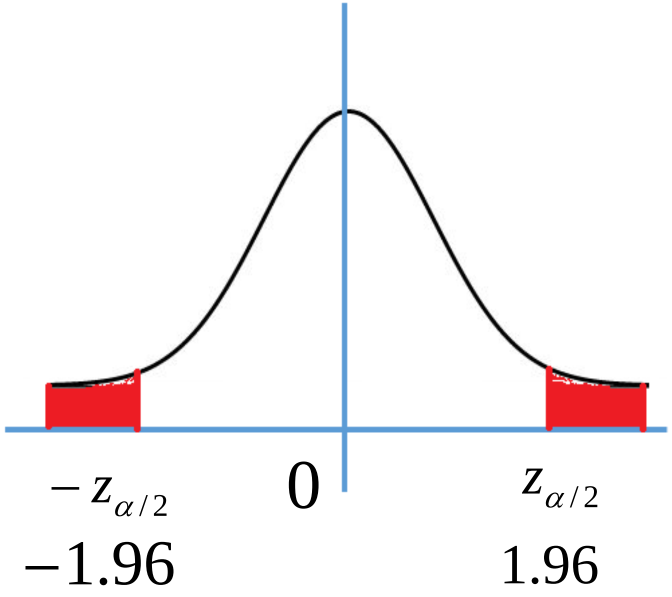
A die was thrown 9000 times & a throw of 5 or 6 was obtained 3240 times. Do the data indicates that die is unbiased at 5% level of significance

Solution:

H_0 : Die is unbiased $P = 1/6 + 1/6 = 1/3$

H_1 : Die is biased $P \neq 1/3$ (Two tailed)

p = Sample Proportion = 3240/9000 = 0.36



Test Statistic:

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.36 - 1/3}{\sqrt{\frac{1/3 \times 2/3}{9000}}} = 5.37 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus, Die is biased

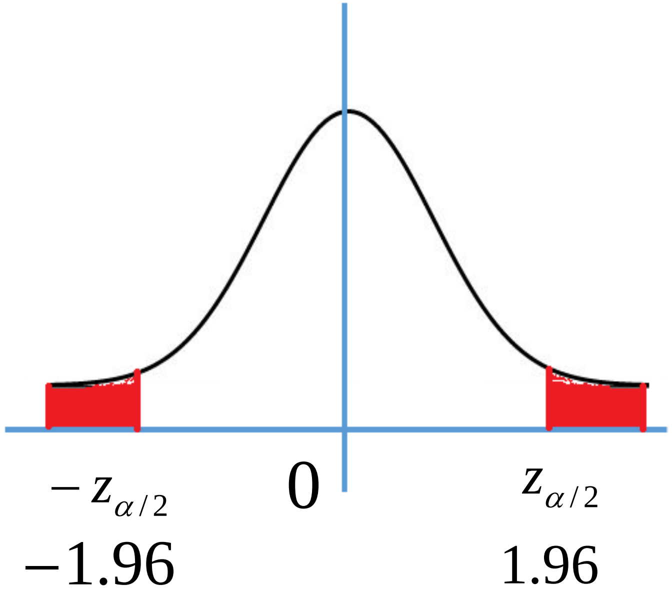
Example 6

A sample of 100 bulbs produced by a company A showed a mean life of 1190 hours & SD of 90hrs. Also a sample of 75 bulbs, produced by a company B showed a mean life of 1230 hours & a SD of 120 hrs. Is there a difference between the mean life of the bulbs produced by two companies at 5% level of significance.

Solution:

H_0 : There is no significant difference $\mu_1 = \mu_2$

H_1 : There is significant difference $\mu_1 \neq \mu_2$ (Two tailed)



Test Statistic:

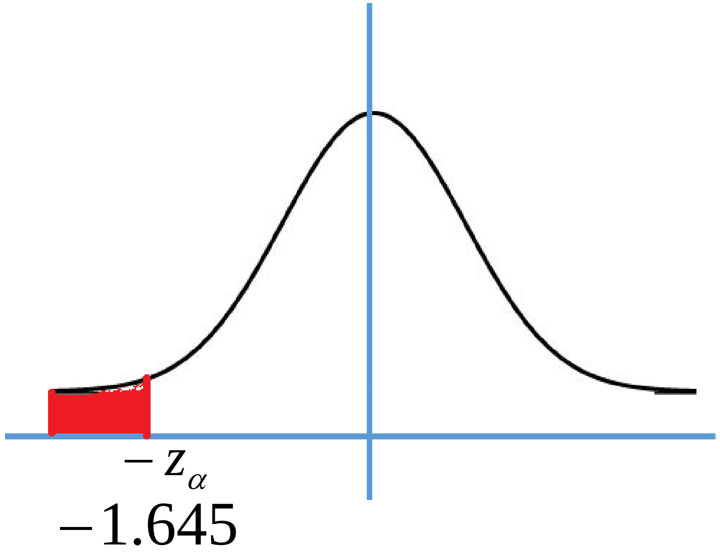
$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1190 - 1230}{\sqrt{\frac{90^2}{100} + \frac{120^2}{75}}} = -2.42 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus, there is significant difference between the mean life of the bulbs produced by two companies

Example 7

In the past the average length of an outgoing telephone call from a business office has been 143 seconds. A manager wishes to check whether that average has decreased after the introduction of policy changes. A sample of 110 telephone calls produced a mean of 133 seconds, with a standard deviation of 35 seconds. Perform the relevant test at 5% level of significance.



Solution:

H_0 : Average length of call: $\mu = 143$

H_1 : Average length of call has decreased: $\mu < 143$ (Left tailed)

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{133 - 143}{35 / \sqrt{110}} = -2.996 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus average length of call has decreased after the introduction of policy changes

Example 8

The mean household income in a region served by a chain of clothing stores is Rs. 38,750. In a sample of 50 customers taken at various stores the mean income of the customers was Rs. 41,505 with standard deviation Rs. 7,652. Test at the 1% level of significance the null hypothesis that the mean household income of customers of the chain is Rs. 38,750 against that alternative that it is different from Rs. 38,750.

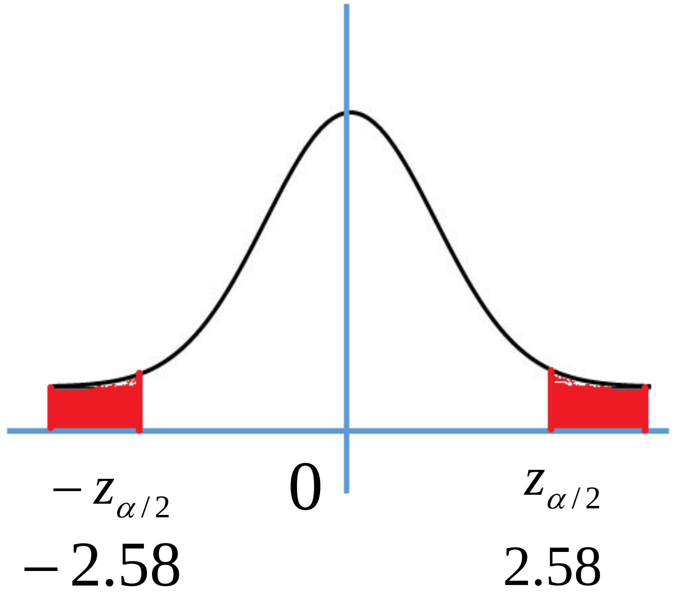
Solution:

H_0 : Mean household income $\mu = 38750$

H_1 : Mean household income $\mu \neq 38750$ (Two tailed)

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{41505 - 38750}{7652 / \sqrt{50}} = 2.546 \notin \text{Critical Region}$$



Hence H_0 may be accepted

Thus mean household income of customers of the chain is Rs. 38,750

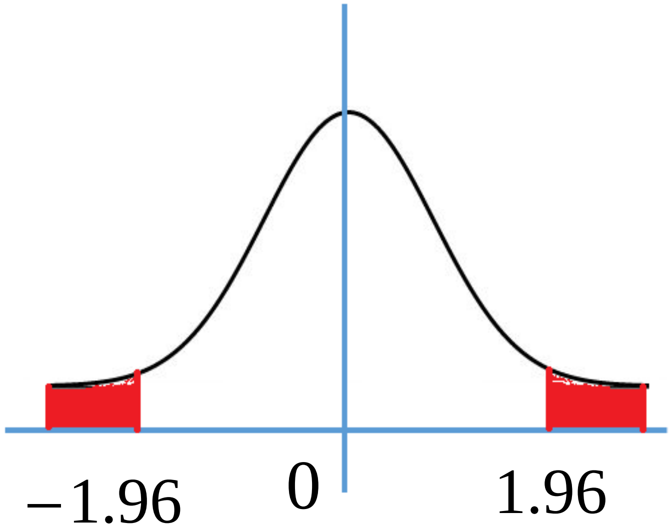
Example 9

A Sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40650 kms with standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000kms (use 0.05 level of significance)

Solution:

H_0 : Population mean μ = 40,000kms

H_1 : Population mean μ ≠ 40,000kms (Two tailed)



Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{40650 - 40000}{3260 / \sqrt{100}} = 1.9938 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus mean life of tyres is not 40,000kms

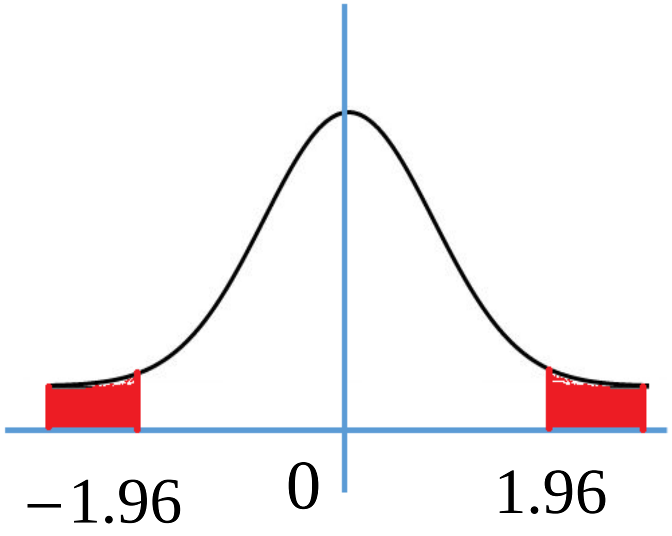
Example 10

A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160cm. Can it be reasonably regarded that in the population the mean height is 165cm and the standard deviation is 10cm at 5% level of significance.

Solution:

H_0 : Population mean $\mu = 165$cm

H_1 : Population mean $\mu \neq 165$cms (Two tailed)



Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{160 - 165}{10 / \sqrt{100}} = -5.00 \in \text{Critical Region}$$

Hence H_0 is rejected

Thus mean height is not 165cms

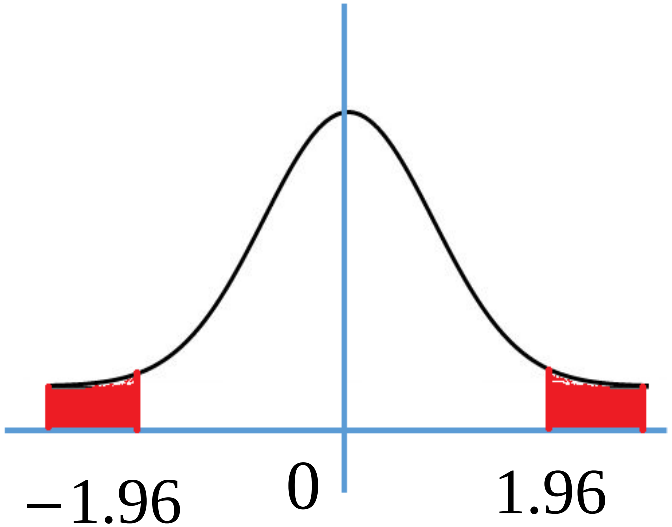
Example 11

A stenographer claims that she can type at the rate of 120 words per minute. Can we reject her claim on the basis of 100 trails in which she demonstrates a mean of 116 words with a standard deviation of 15 words? Use 5% level of significance.

Solution:

H_0 : Population mean $\mu = 120$

H_1 : Population mean $\mu \neq 120$ (Two tailed)



Test Statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{116 - 120}{15 / \sqrt{100}} = -2.667 \in \text{Critical Region}$$

Hence H_0 is rejected

We reject her claim that she can type at the rate of 120 words per minute.