Stochastic Process Assignment Problems

1. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?

2. Suppose that the probability of a dry day following a rainy day is $\frac{1}{3}$ and that the probability of a rainy day following a dry day is $\frac{1}{2}$. Given that May 1 is a dry day, find the probability that (i) May 3 is also a dry day and (ii) May 5 is also a dry day.

3. A housewife buys 3 kinds of cereals, A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. In the long run, how often she buys each of the three cereals?

4. Two boys B_1, B_2 and two girls G_1, G_2 are throwing a ball from one to another. Each boy throws the ball to the other boy with probability $1/2$ and to each girl with probability $1/4$. On the other hand, each girl throws the ball to each boy with probability $1/2$ and never to the other girl. In the long run, how often does each receive the ball?

5. Suppose that on any given sunny day, the next day's weather has a 60% chance of being sunny, a 30% chance of being cloudy, and a 10% chance of being rainy. On any given cloudy day, the next day's weather has a 40% chance of being sunny, a 30% chance of being cloudy, and a 30% chance of being rainy. Lastly, on any given rainy day, there is a 20% chance of being sunny, a 50% chance of being cloudy, and a 30% chance of being rainy the following day. Assuming it's sunny today, what is the probability that it will be sunny two days from now.

6. A gambler's luck follows a pattern. If he wins a game, the probability of his winning the next game is 0.6. However if he loses a game, the probability of his losing the next game is 0.7. There is an even chance that the gambler wins the first game. What is the probability that he wins (i) the second game, (ii) the third game and (iii) in the long run?

7. Each year a man trades his car for a new car. If he has a Chrysler he trades for a Plymouth. If he has a Plymouth he trades it for a Ford. However, if he has a Ford, he is just as likely to trade it in for a Chrysler or a Plymouth. In 2022 he bought his first car which was a Ford.

   (a) Find the probability that he has a (i) 2024 Ford (ii) 2024 Chrysler (iii) 2025 Plymouth.

   (b) In the long run how often will he have a Ford?

8. Define:
   1. Absorbing State
   2. Recurrent State
   3. Transient State
   4. Regular Stochastic Matrix
   5. Irreducible Markov Chain