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Question 1:-

Let $(x_1, x_2, x_3 .. x_n)$ be a random sample of size taken from a normal population with parameters, mean $= \theta_1$ and variance $= \theta_2$. Find the maximum likelihood. Estimates of these of parameters.

Given that $x_1, x_2 .... x_n$ is a random sample from a normal distribution with mean $= \theta_1$ and variance θ_2 the likelihood function is

$$L(\theta_1, \theta_2 | x_1, x_2 ... x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Taking log on Both sides

$$\ln L(\theta_1, \theta_2 | x_1, x_2 .... x_n) = -n/_2 \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

To find MLE, we will differentiate the log. likelihood with respect to θ_1 and θ_2, set derivative equal to zero.

For θ_1

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2 .... x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)$$

setting this equal to zero

$$\frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0 \Rightarrow \sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0.$$

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

So MLE for θ_1 is the sample mean, For θ_2

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2 | x_1 x_2 \cdots x_n) = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Setting this equal to zero.

$$\frac{-n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\frac{n}{2\hat{\theta}_2} - \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

So, MLE for θ_2 is the sample variance

Question 2:

Let $X_1, X_2 \dots x_n$ be random sample from $B(m, \theta)$ distribution, where $\theta \in \Theta = (0,1)$ is unknown and m is known +ve integer, compute values of θ using the MLE.

Answer:-

To find the MLE of θ for random sample $X_1, X_2 \dots x_n$ from a Bernoulli Distribution with parameter θ and a known m. The likelihood for this scenario is.

$$L(\theta \mid x_1 x_2 \dots x_n) = \prod_{i=1}^{n} P(x_i = x_i \mid \theta)$$

Since x_i follows a Bernoulli distribution

$$P\left(X_i = \frac{x_i}{\theta}\right)$$

$$= \theta^{x_i}(1-\theta)^{m-x_i} \text{ for each } i$$

Taking log on Both sides.

$$\ln(L\theta \mid x_1, x_2 \dots x_n) = \sum_{i=1}^{n} \ln\left(\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n}\left(x_i \ln\theta + (m-x_i)\ln(1-\theta)\right)$$

Now differentiate with respect to θ and set to zero.

$$\frac{d}{d\theta}\left(\ln L(\theta \mid x_1, x_2 \dots x_n)\right) = 0$$

$$\sum_{i=1}^{n}\left(x_i/\theta - \frac{m-x_i}{1-\theta}\right) = 0.$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = n.m - \frac{\sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \theta = \sum_{i=1}^{n} \frac{x_i}{n.m}$$

So max likelihood estimate for θ.

$$\boxed{\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n.m}}$$