

# Physics-Informed Neural Network

이동로봇 튜토리얼 #3

박수환

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## 논문 요약

- 2D-INS 물리 법칙을 Loss에 포함한 PINN 기반 순수 관성 내비게이션 프레임워크
- IMU 데이터로 위치·속도·Yaw를 물리 일관성 있게 추정하도록 제약 강화
- 기존 2D-INS-MoRPINet 대비 최대 94% 정확도 향상

## 논문 소개

### MoRPI-PINN: A Physics-Informed Framework for Mobile Robot Pure Inertial Navigation

Arun Kumar Sahoo and Itzik Klein

**Abstract**—A fundamental requirement for full autonomy in mobile robots is accurate navigation even in situations where satellite navigation or cameras are unavailable. In such practical situations, relying only on inertial sensors will result in navigation solution drift due to the sensors' inherent noise and error sources. One way to combat this drift is to periodically maneuver the robot in a snake-like slithering motion to increase the inertial signal to noise ratio allowing the regression of the mobile robot position. In this work, we propose MoRPI-PINN as a physics-informed deep learning framework for accurate inertial-based mobile robot navigation. By embedding physical laws and constraints into the training process, MoRPI-PINN is capable of providing an accurate navigation solution with an error of less than 10 cm. Through experiments, we show accuracy improvements of over 85% compared to other approaches. MoRPI-PINN is a lightweight approach that can be implemented on edge devices and used in any typical mobile robot application.

**Index Terms**—Scientific Machine Learning; Physics-informed Neural Networks; Inertial Navigation System; Mobile Robot; Accelerometer; Gyroscope; Yaw Angle; Dead Reckoning

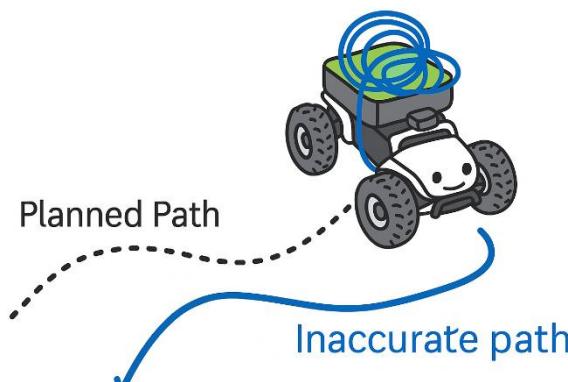
TABLE  
LIST OF ABBREVIATIONS

Abbreviation	Definition
<b>Navigation</b>	
INS	Inertial Navigation System
IMU	Inertial Measurement Unit
GNSS	Global Navigation Satellite System
RTK	Real-Time Kinematic
NED	North-East-Down (Coordinate Frame)
ECEF	Earth-Centered Earth-Fixed
<b>Neural Networks</b>	
PINN	Physics-informed Neural Network
AD	Automatic Differentiation
GT	Ground Truth
<b>Error Metrics</b>	
ATE	Absolute Trajectory Error
MATE	Mean Absolute Trajectory Error
MSE	Mean Squared Error
RMSE	Root Mean Squared Error
NRMSE	Normalized Root Mean Squared Error

#### INTRODUCTION

A. K. S. is supported in part by the Maurice Hatter Foundation. (Corresponding author: Arun Kumar Sahoo)  
The authors are with the Autonomous Navigation and Sensor Fusion Lab (ANSFL), Faculty of Mechanical Engineering Department of Marine Technologies, Chayen School of Marine Sciences, University of Haifa, Haifa 3498838, Israel. (e-mail: asahoo@campus.haifa.ac.il, itzrik@univ.haifa.ac.il)

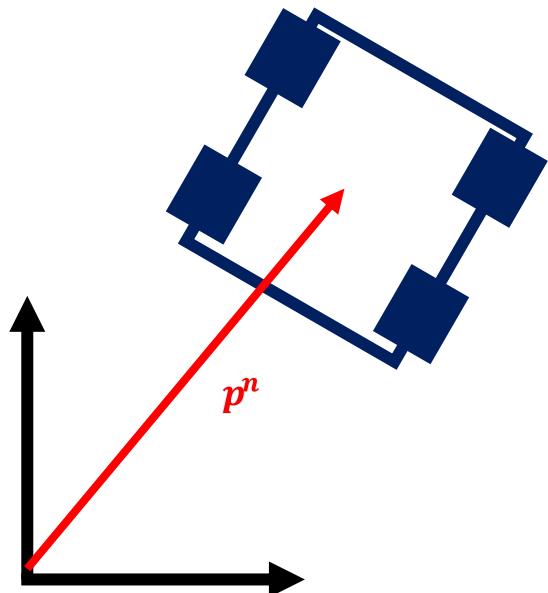
- IMU 단독 내비게이션은 센서 노이즈가 적분 과정에서 증폭되어 급격한 오차 누적 발생
- 직선 주행보다 주기적(Snake-like) 움직임이 높아진 SNR을 제공함을 실험적으로 확인
- 기존 2D-INS학습기반 모델 모두 이러한 동적 환경에서 안정적 추정을 보장하지 못함



## 기본 이론 소개

### 위치 업데이트 수식

$$\begin{aligned}\dot{\mathbf{p}}^n &= \mathbf{v}^n, \\ \dot{\mathbf{v}}^n &= \mathbf{C}_b^n \mathbf{f}_{ib}^b + \mathbf{g}^n, \\ \dot{\mathbf{C}}_b^n &= \mathbf{C}_b^n \boldsymbol{\Omega}_{ib}^b.\end{aligned}$$



### IMU 센서 데이터

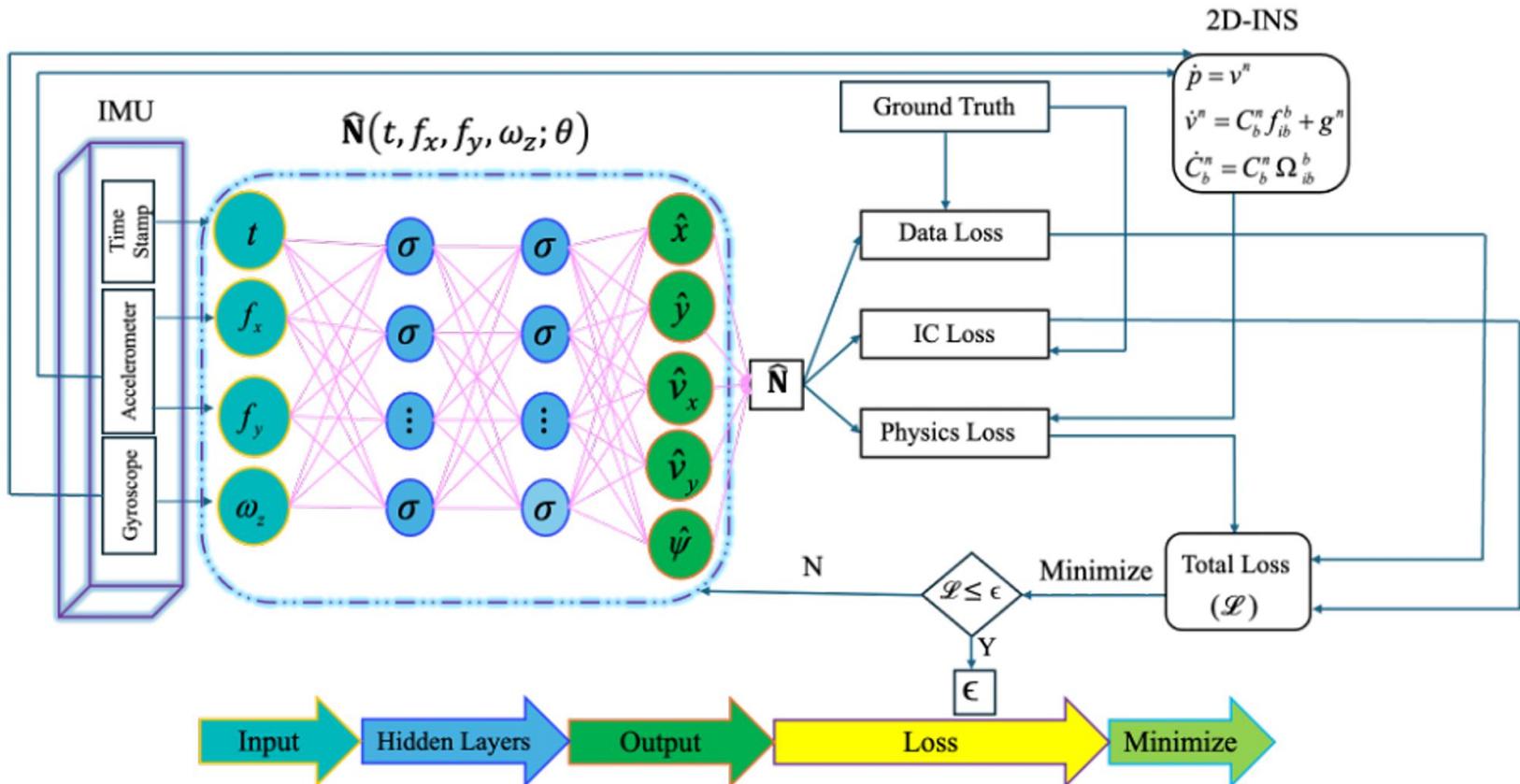
$$\mathbf{f}_{ib}^b = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}^T \quad \boldsymbol{\omega}_{ib}^b = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}^T$$

### 회전관련

$$\boldsymbol{\Omega}_{ib}^b = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad \mathbf{C}_b^n(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

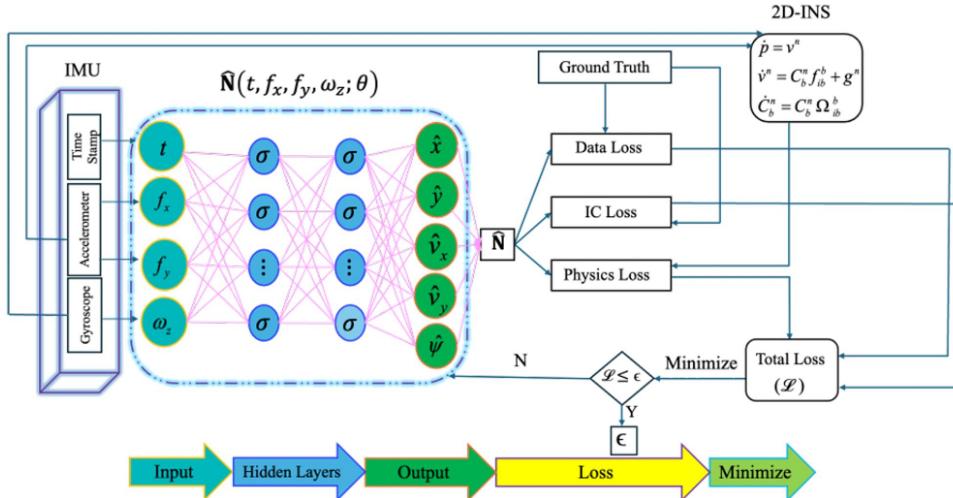
# Paper #1 Navigation

## 핵심 Framework



# Paper #1 Navigation

## 핵심 Framework



### Data Loss

$$\mathcal{L}_{\text{data}} = \frac{1}{N_{\text{data}}} \sum_{i=1}^{N_{\text{data}}} \left( \|\mathbf{p}_i - \hat{\mathbf{p}}_i\|^2 + \|\mathbf{v}_i - \hat{\mathbf{v}}_i\|^2 \right)$$

$$\mathcal{L}_{\text{init}} = \frac{1}{N_{\text{ic}}} \sum_{i=1}^{N_{\text{ic}}} \|\hat{\mathbf{p}}_i - \mathbf{p}_0\|^2$$

### Physics Loss

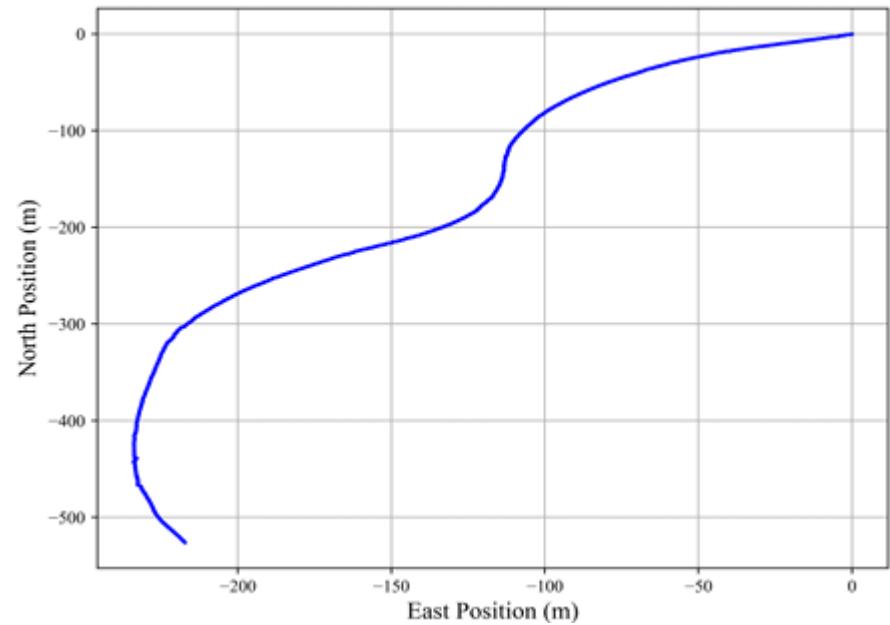
$$\mathcal{L}_{\text{phys}} = \frac{1}{N_{\text{phys}}} \sum_{i=1}^{N_{\text{phys}}} \left( \left\| \frac{d}{dt} \begin{bmatrix} \hat{x}(t_i) \\ \hat{y}(t_i) \end{bmatrix} - \begin{bmatrix} \hat{v}_x(t_i) \\ \hat{v}_y(t_i) \end{bmatrix} \right\|^2 + \left\| \frac{d}{dt} \begin{bmatrix} \hat{v}_x(t_i) \\ \hat{v}_y(t_i) \end{bmatrix} - \left( \mathbf{C}_b^n(\hat{\psi}(t_i)) \begin{bmatrix} f_x(t_i) \\ f_y(t_i) \end{bmatrix} + \begin{bmatrix} g_x \\ g_y \end{bmatrix} \right) \right\|^2 + \left( \frac{d\hat{\psi}(t_i)}{dt} - \omega_z(t_i) \right)^2 \right)$$

**Total Loss**  $\mathcal{L}_{\text{total}} = \lambda_{\text{data}} \mathcal{L}_{\text{data}} + \lambda_{\text{init}} \mathcal{L}_{\text{init}} + \lambda_{\text{phys}} \mathcal{L}_{\text{phys}}$

# Paper #1 Navigation

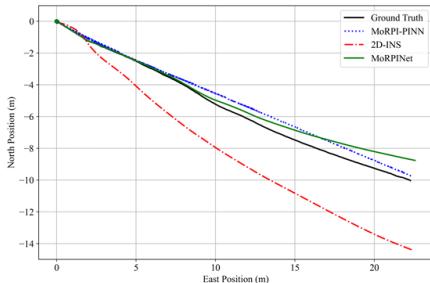
## Train Data

Specification	Gyroscope	Accelerometer
Bias	$10^\circ/\text{h}$	$0.03 \text{ mg}$
Noise Density	$0.007^\circ/\text{s}/\sqrt{\text{Hz}}$	$120 \mu\text{g}/\sqrt{\text{Hz}}$

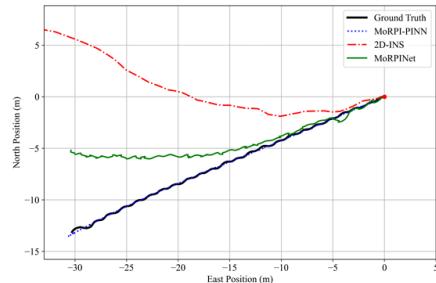


# Paper #1 Navigation

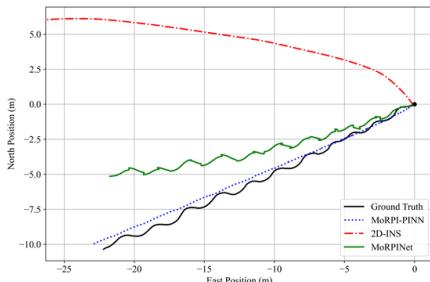
## Experiment



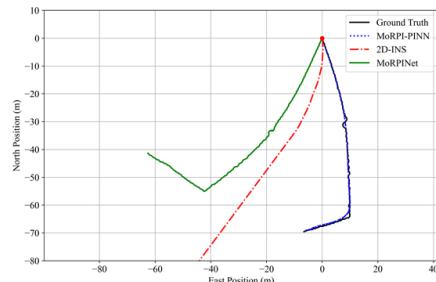
(a) Trajectory 1



(b) Trajectory 2

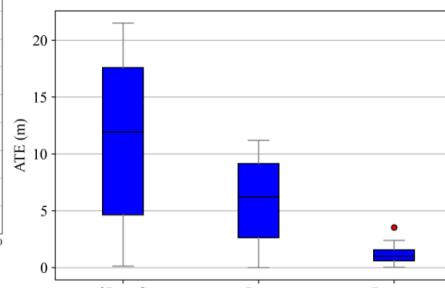


(c) Trajectory 3

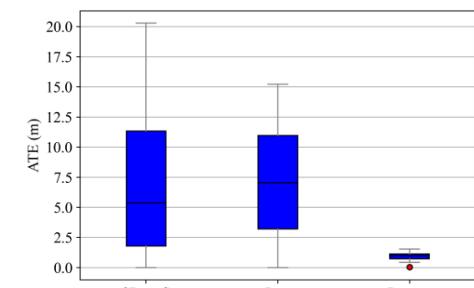


(d) Trajectory 4

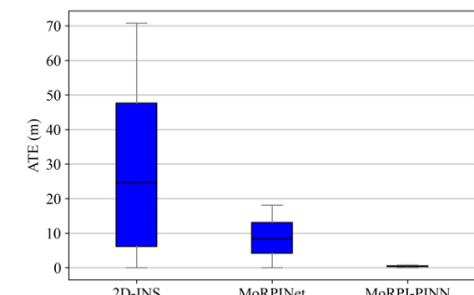
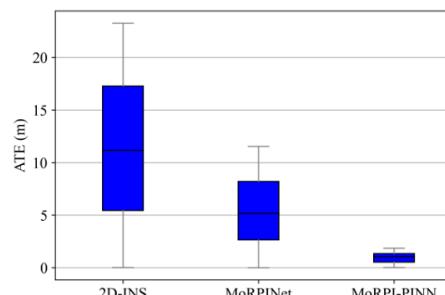
$$\text{ATE}(\mathbf{p}_i, \hat{\mathbf{p}}_i) = \sqrt{\frac{1}{N} \sum_{i=1}^N \|\mathbf{p}_i - \hat{\mathbf{p}}_i\|^2}$$



(a)



(b)



## 논문 요약

- PINN 기반 마찰 모델링으로 고비선형(static/dynamic friction)까지 정밀하게 추정
- UKF에 PINN 마찰을 직접 측정치로 입력, 모델 불확실성 외란 상황에서도 견고한 토크 추정
- 실험에서 RNEA 대비 토크 추종·에너지 효율·외란 대응 능력 크게 향상

## 논문 소개

IEEE ROBOTICS AND AUTOMATION LETTERS, VOL. 10, NO. 6, JUNE 2025

5705

### Physics-Informed Neural Networks With Unscented Kalman Filter for Sensorless Joint Torque Estimation in Humanoid Robots

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**Abstract**—This paper presents a novel framework for whole-body torque control of humanoid robots without joint torque sensors, designed for environments with electric motors and high-gain haptic actuators. The approach uses Physics-Informed Neural Networks (PINNs) for friction modeling and Unscented Kalman Filtering (UKF) for joint torque estimation, within a real-time torque control loop. The PINNs learn the complex static and dynamic friction from joint and motor velocity readings, capturing effects of motor acceleration without joint movement. The UKF utilizes PINNs for friction estimation and directly optimizes them, improving torque estimation robustness. Experimental validation on the ergo humanoid robot demonstrates improved torque tracking performance, enhanced stability, and superior disturbance rejection compared to the state-of-the-art Recursive Newton-Euler Algorithm (RNEA), using a dynamic modeling approach. The framework is evaluated across various friction conditions, across robots with similar hardware but different friction characteristics, without prior knowledge. A comparative analysis with position control highlights the accuracy of the proposed friction control approach. The results establish the method as a scalable and practical solution for sensorless torque control in humanoid robots, enabling torque tracking, adaptability, and stability in dynamic environments.

**Index Terms**—Humanoid robot systems, calibration and identification, humanoid and bipedal locomotion.

#### I. INTRODUCTION

THE ongoing evolution of humanoid robotics is expanding the boundaries of functionality, safety, and integration into human environments. Humanoid robots are designed for tasks

- 다수의 휴머노이드 로봇은 관절 토크 센서가 없거나, 비용무게내구성 문제로 장착이 불가능
- 토크를 추정하려면 마찰, 관성, 비선형성까지 포함해야 하는데 기존 모델링만으로는 정확하지 않음



This paper presents a novel framework for whole-body torque control of humanoid robots without joint torque sensors, designed for environments with electric motors and high-gain haptic actuators. The approach uses Physics-Informed Neural Networks (PINNs) for friction modeling and Unscented Kalman Filtering (UKF) for joint torque estimation, within a real-time torque control loop. The PINNs learn the complex static and dynamic friction from joint and motor velocity readings, capturing effects of motor acceleration without joint movement. The UKF utilizes PINNs for friction estimation and directly optimizes them, improving torque estimation robustness. Experimental validation on the ergo humanoid robot demonstrates improved torque tracking performance, enhanced stability, and superior disturbance rejection compared to the state-of-the-art Recursive Newton-Euler Algorithm (RNEA), using a dynamic modeling approach. The framework is evaluated across various friction conditions, across robots with similar hardware but different friction characteristics, without prior knowledge. A comparative analysis with position control highlights the accuracy of the proposed friction control approach. The results establish the method as a scalable and practical solution for sensorless torque control in humanoid robots, enabling torque tracking, adaptability, and stability in dynamic environments.

**Index Terms**—Humanoid robot systems, calibration and identification, humanoid and bipedal locomotion.

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## 기존 방법의 한계

### Model Based Method (RNEA)

- 마찰 모델 자체가 단순함 → 실제 마찰 재현 불가
  - RNEA는 마찰을 Coulomb + viscous처럼 단순화하여 실제 복잡한 마찰을 재현하지 못함
  - 기어박스트랜스미션의 비선형 마찰(static, Stribed, stick-slip 등)을 RNEA만으로는 표현 불가
- 모델 불일치→ 토크 추정 오차 증가
- 환경마모 변화에 유연하지 않음
  - 마찰은 온도·마모·기어박스 상태에 따라 계속 변하지만 RNEA는 고정 파라미터만 사용

### Neural Network

- 데이터 의존적 → 일반화 성능 약함
  - 훈련 데이터 분포 바깥(새 동작새 속도새 환경)에서는 성능 급격히 저하
- 물리 기반 제약 부재 → 비물리적 추정 가능
- 환경 변화(온도·마모 등)에 적응 어려움
  - 마찰 특성이 계속 변하지만 NN은 고정된 데이터 기반 모델이라 외란마모 변화에 취약

**PINN은 물리식 + 학습을 결합해 두 방식의 단점을 동시에 극복**

## 기본 이론 정리

로봇 동역학 식

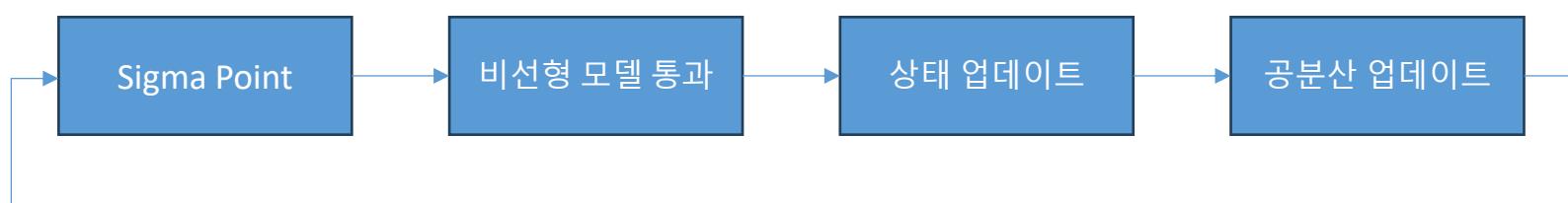
$$M(q) \begin{bmatrix} {}^B\alpha_{A,B}^g \\ \ddot{s} \end{bmatrix} + C(q, {}^B\nu_{A,B}) = B\tau + \sum_k J_k^\top(q) f_{\text{ext}}^k$$

모터 동역학 식

$$k_t I_m = J_m \ddot{\theta} + \frac{1}{R} \tau_F + \frac{1}{R} \tau$$

## UKF

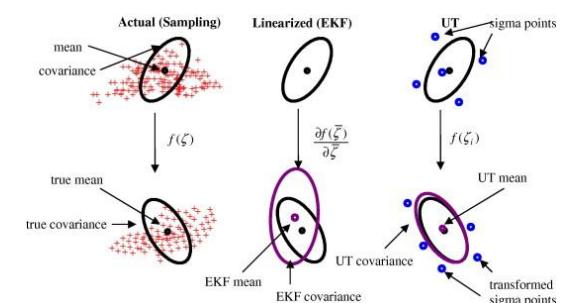
- 시그마 포인트 기반 비선형 칼만 필터
- 기존 EKF에서 비선형 함수를 선형화하는 과정을 생략한 상태로 샘플링을 통한 근사화 전략을 사용
- 동역학마찰 불확실성에 강한 확률적 상태 추정 제공



$$x_k = [\dot{s}^\top \quad \hat{\tau}_m^\top \quad \hat{\tau}_F^\top \quad \hat{f}_{FT}^\top \quad \hat{f}_{ext}^\top \quad \hat{\alpha}_{acc}^\top \quad \hat{\omega}_{gyro}^\top]_k^\top$$

$$y_k = [\dot{s}^\top \quad I_m^\top \quad \tau_F^\top \quad f_{FT}^\top \quad \alpha_{acc}^\top \quad \omega_{gyro}^\top]_k^\top$$

[2] I. Sorrentino et al, "Physics-informed neural networks with unscented Kalman filter for sensorless joint torque estimation in humanoid robots," IEEE Robotics and Automation Letters, vol. 10, no. 6, pp. 5705–5712, 2025



## PINN

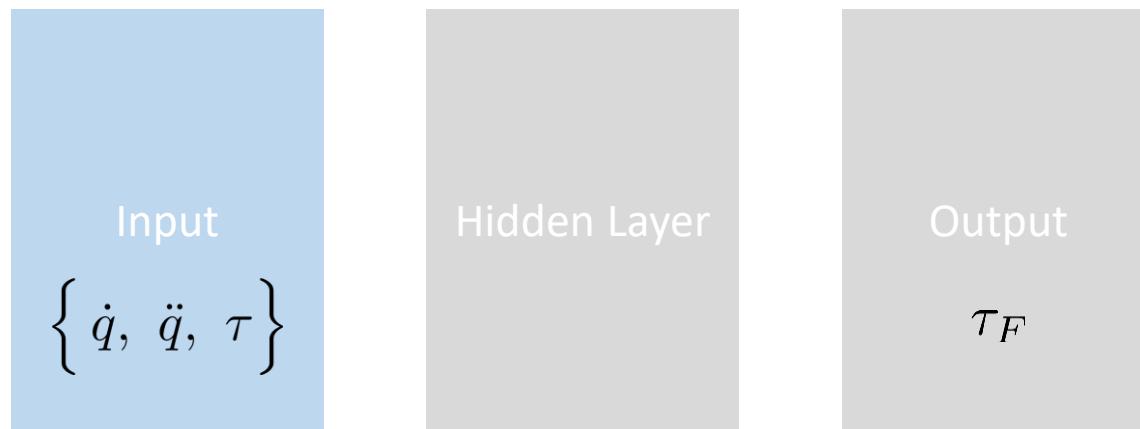
- 조인트 속도값의 Noise, 모터 속도를 차분으로 진행해 부정확성을 방지하기 위해 1틱이 아닌 Buffer 형태로 입력
- 마찰토크로 Stribeck–Coulumb–Viscous 마찰모델 기반으로 마찰토크를 만들도록 제약 (정확하지만 구조가 복잡해 모델 기반으로는 추정이 어려움)

### Stribeck–Coulumb–Viscous

$$\tau_F(\dot{q}) = \tau_c \operatorname{sgn}(\dot{q}) + (\tau_s - \tau_c) \exp\left(-\left|\frac{\dot{q}}{v_s}\right|^\alpha\right) + b \dot{q}$$

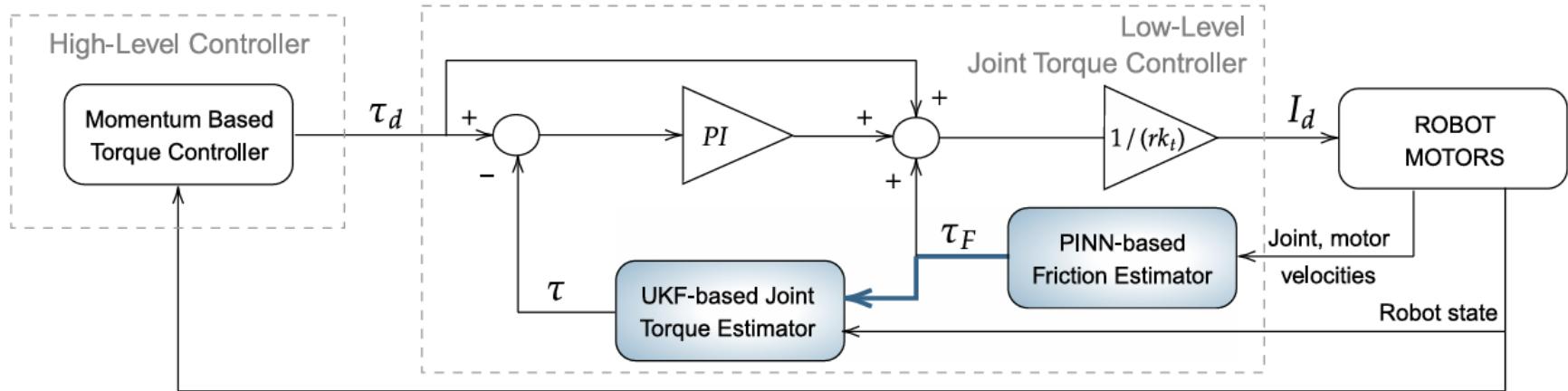
### Total Loss

$$\mathcal{L} = \mathcal{L}_{\text{data}} + \mathcal{L}_{\text{physics}} = (1-\lambda) \frac{1}{N} \sum_{i=1}^N (\tau_{F,\text{pred}} - \tau_{F,\text{true}})^2 + \lambda \frac{1}{N} \sum_{i=1}^N (\tau_{F,\text{pred}} - \tau_{F,\text{physics}})^2.$$



# Paper #2 Friction Estimation

## Framework



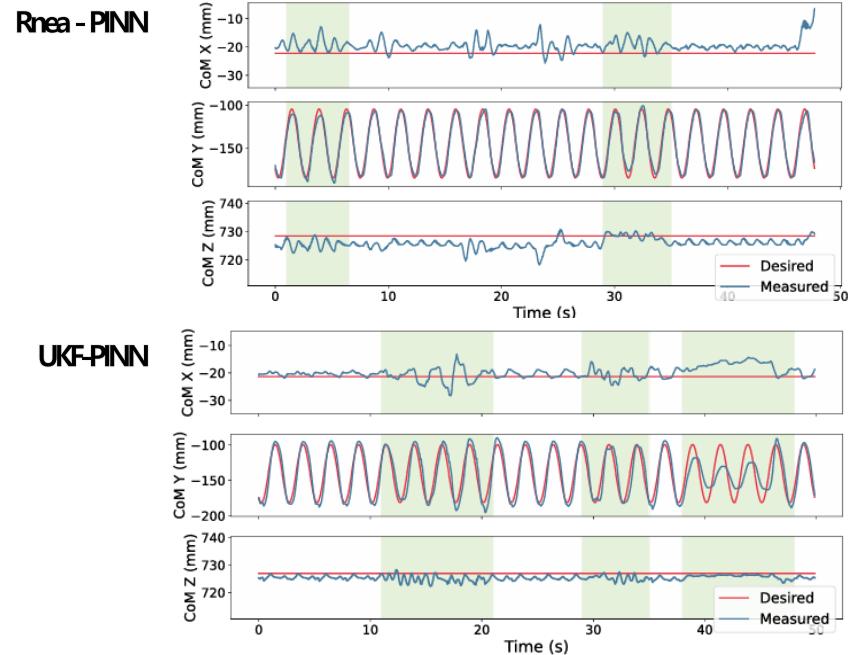
# Paper #2 Friction Estimation

## Experiment #1 Without Disturbances

Control Architecture	Mean Error (mm)	Max Error (mm)
Feedforward	[1.2, 13.1, 1.9]	[3.1, 24.2, 4.6]
RNEA No Compensation	[0.8, 6.5, 0.7]	[2.3, 15.2, 2]
UKF No Compensation	[3.2, 11.5, 6.4]	[7.2, 20.6, 7.5]
Feedforward PINN	[2.8, 9.4, 1.1]	[4.4, 25.4, 2.5]
RNEA-PINN	[2.8, 3.8, 3.8]	[8.2, 18.1, 6.7]
UKF-PINN	[2.9, 8.1, 0.6]	[4.5, 17.5, 1.3]

Control Architecture	MSE	RMSE	MAE
RNEA No Compensation	19.2	4.37	2.82
UKF No Compensation	4.13	2.03	1.45
RNEA-PINN	49.43	7.03	4.12
UKF-PINN	1.18	1.08	0.7

## Experiment #2 With Disturbances



[2] I. Sorrentino et al, "Physics-informed neural networks with unscented Kalman filter for sensorless joint torque estimation in humanoid robots," IEEE Robotics and Automation Letters, vol. 10, no. 6, pp. 5705–5712, 2025

# Paper #3 Modeling Estimation

## 논문 요약

- OPI-PINNPC 구조를 제안하여 Payload가 달라져도 동역학을 안정적으로 예측할 수 있도록 했음
- 실시간 Payload 질량위치 정보를 추정하여 PINN의 physics-loss에 직접 입력함으로써 모델 불확실성을 크게 감소시켰음
- PINN을 NMPC의 수치적분 대체 모델로 사용하여 예측 속도·계산 효율성을 동시에 확보했음

## 논문 소개

IEEE ROBOTICS AND AUTOMATION LETTERS, VOL. 10, NO. 11, NOVEMBER 2025

11275

### PINN-Based Predictive Control Combined With Unknown Payload Identification for Robots With Prismatic Quasi-Direct-Drives

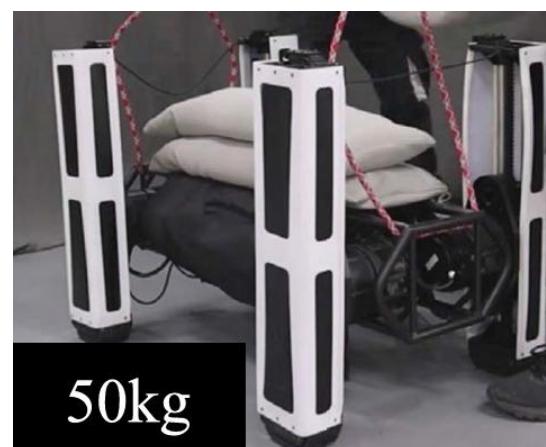
Haojin Li , Haotang Chen, Yikang Chai, Hang Zhao , Ye Zhao , Yu Han , and Jianwen Luo 

**Abstract**—This study introduces a unified control framework that addresses the challenges of payload robots with Quasi-Direct-Drives under unknown payloads, named as online payload identification-based predictive control (OPI-PINNPC). By integrating online payload identification with physics-informed neural networks (PINNs), our approach enables real-time payload identification while maintaining the network's loss function, ensuring physical consistency while adapting to changing load conditions. The physics-constrained neural representation is used to build a model predictive controller, enabling real-time optimization despite the complex dynamics of robots with Quasi-Direct-Drives. Experimental results demonstrate that OPI-PINNPC achieves significant improvement in position and orientation tracking accuracy across diverse payload conditions, with substantially faster convergence compared to traditional methods. This research also provides an adaptive solution for maintaining tracking performance under variable payload conditions without sacrificing computational efficiency.

**Index Terms**—Physics-informed neural network, nonlinear model predictive control, quadraped locomotion, identification.

**L. INTRODUCTION** Online Model Predictive Control (NMPC) has emerged as a powerful and versatile control strategy,

- QDD 로봇은 Payload 변화에 민감해 기존 모델만으로는 정확한 제어가 어려웠음
- Payload 포함 동역학은 비선형 요소가 많아 기존 방식만으로는 정확한 추정이 어려웠음
- 실시간 Payload 추정 + PINN + NMPC 결합으로 안정적 예측제어를 구현했음



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**N. ONLINEAR Model Predictive Control (NMPC)** has

emerged as a powerful and versatile control strategy,

particularly in the realm of robots, version [1]–[3].

Robotic systems, such as robotic manipulators or quadruped robots, with their complex dynamics and multiple degrees of freedom, present unique challenges for motion control. Traditional model-based methods often struggle to meet these demands due to the highly nonlinear and coupled dynamics [4]–[6].

NMPC addresses these challenges by leveraging a predictive model of the system to optimize control inputs over a finite horizon [8], [9]. Unlike linear control approaches [10], NMPC fully embraces the nonlinearity inherent in system dynamics, allowing it to handle complex tasks like walking, running, and jumping. It also considers the future states and constraints of the robot, enabling it to anticipate and react to changes in terrain disturbances, or desired trajectories in real-time [12]. This predictive capability is crucial for tasks that require high precision, robustness, and adaptability. As a result, the robot can adapt rapidly to maintain stability and achieve its objectives [13]. Compared with analytical methods [14], [15], NMPC incorporates constraints like joint limits, friction, and contact forces into optimization, ensuring control inputs are both optimal and safe for robot's hardware [16], [17].

NMPC has been successfully applied to various applications and operational scenarios [18], [19]. Recent advances in computing and optimization have made NMPC practical for real-time control.

However, accurately modeling nonlinear dynamics in NMPC remains challenging, especially in the presence of disturbances, especially with the many parameters of the locomotion model under varying payloads [20], [21]. Therefore,

it is common in traditional methods to incorporate parameter identification modules to handle model uncertainties. For example, the Hong Kong-Suzhou-Kalman Filter (HSKF) in [22] effectively adapts to dynamic changes in inertial parameters.

[23] proposes an estimation method for the center of mass and full momentum of humanoid robots for controlling the movement of humanoid robots.

As robots systems have grown increasingly complex, traditional machine learning models and purely physics based approaches face distinct limitations: the former typically sacrifice physical interpretability and consistency, while the latter struggle to adapt to complex environments and uncertainties [24], [25].

[26] proposed Physics-informed Neural Networks (PINNs) has emerged as a groundbreaking methodology in robot modeling and control,

effectively bridging the gap between conventional physics-based techniques and contemporary machine learning [27],

[28], [29].

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## 사전지식

### Payload 포함 동역학

$$\begin{cases} m\ddot{r} = \sum F_{ci} - \sum m_i g - m_p g, \\ I\dot{\theta} = \sum r_{ci} \times F_{ci} + \sum r_i \times (m_i g) + r_p \times (m_p g). \end{cases}$$

- 해당 부분을 PINN으로 학습하여 최적화 반영이 쉽게 함
- Numerical integration 없이 빠른 예측 가능
- 비선형 동역학을 더 정확하게 반영

### Payload 포함 MPC

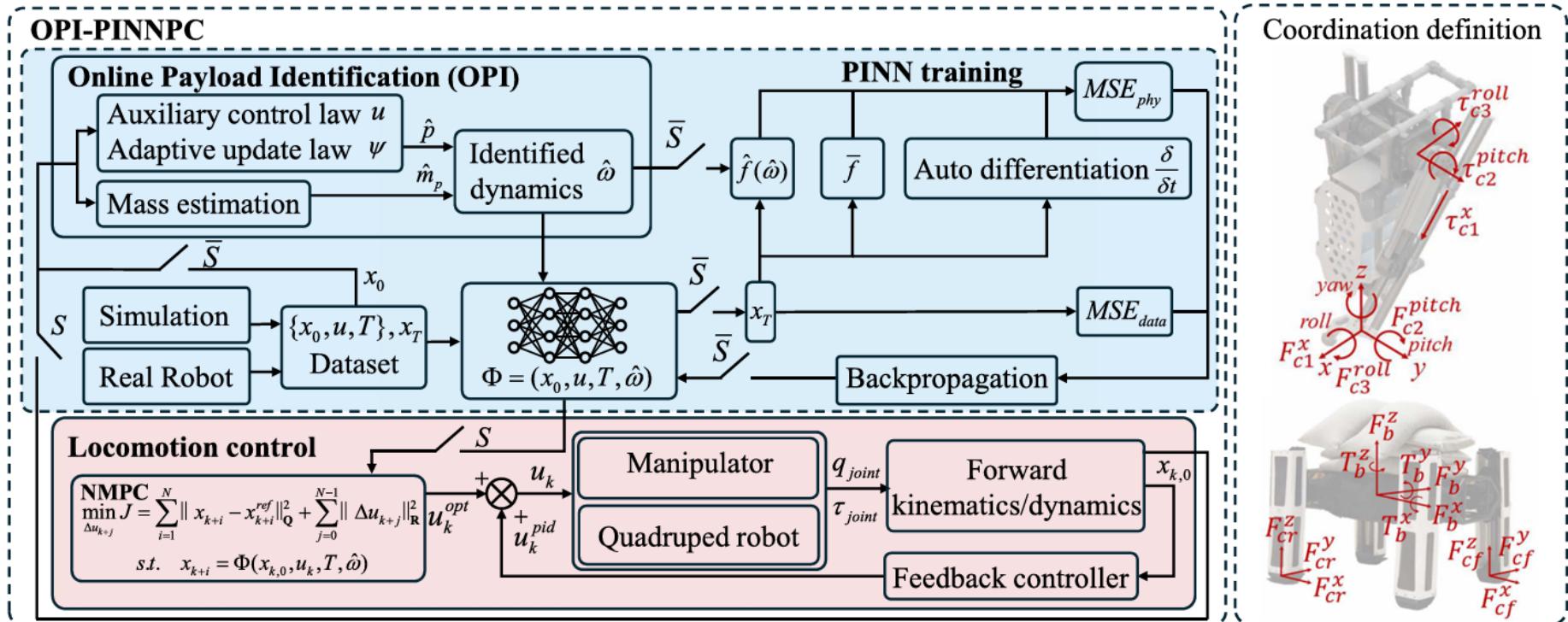
$$\min_{\Delta u_{k+j}} \left( \sum_{i=1}^N \|x_{k+i} - x_{k+i}^{\text{ref}}\|_Q^2 + \sum_{j=0}^{N-1} \|\Delta u_{k+j}\|_R^2 \right)$$

$$\left. \begin{array}{l} x_{k+i+1} = \Phi(x_{k+i}, u_{k+i}, T, \hat{\omega}), \\ u_{k+i+1} = u_k + \sum_{j=0}^i \Delta u_{k+j}, \\ u_{\min} \leq u_{k+i} \leq u_{\max}, \\ \Delta u_{\min} \leq \Delta u_{k+i} \leq \Delta u_{\max}, \\ x_{\min} \leq x_{k+i} \leq x_{\max}, \\ \forall i = 0, \dots, N-1. \end{array} \right\}$$



# Paper #3 Modeling Estimation

## Framework

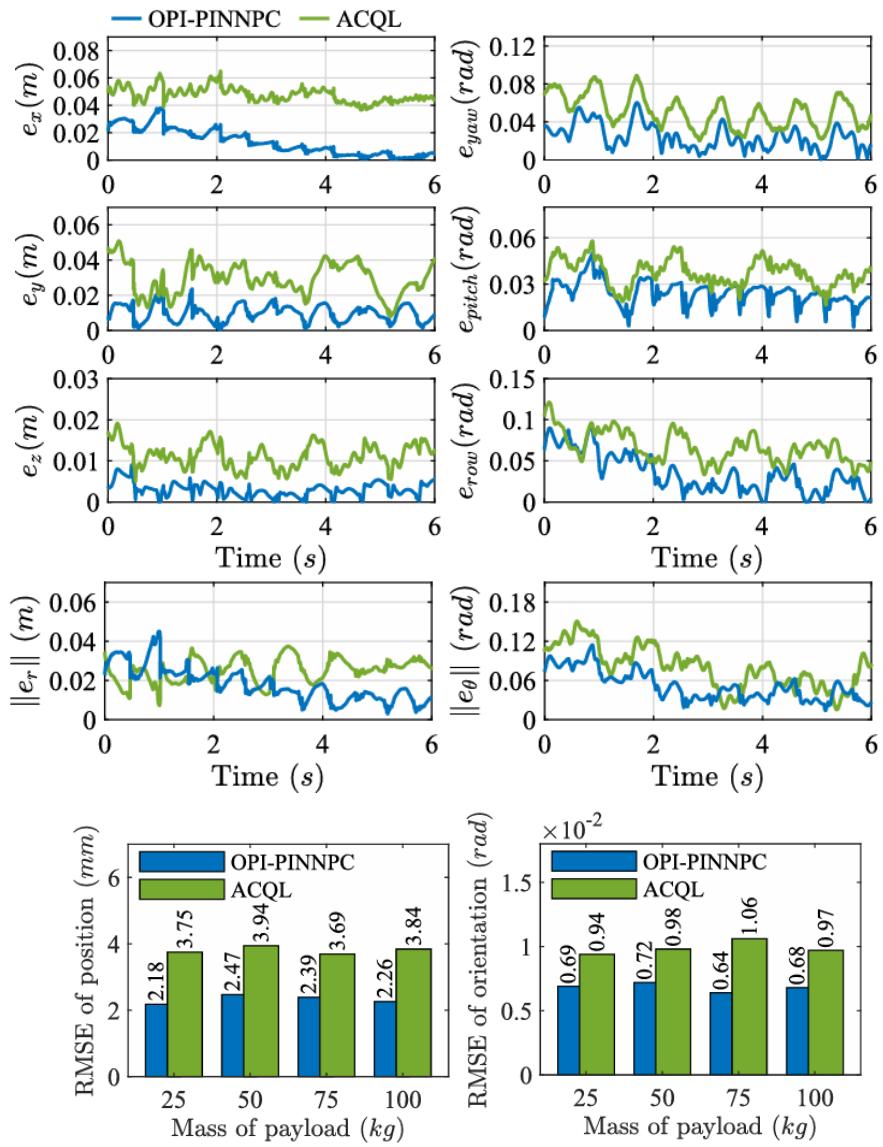
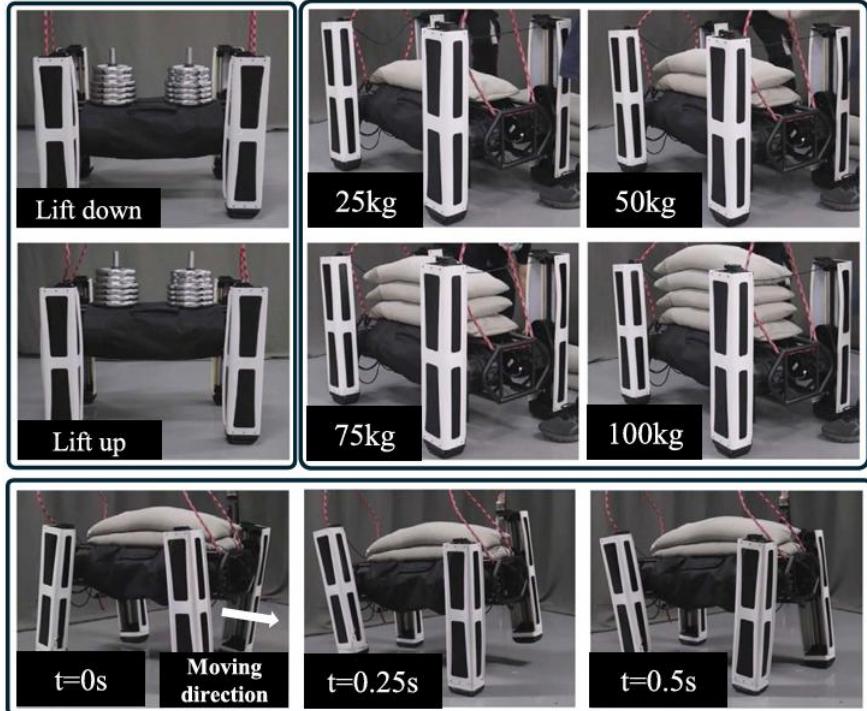


$$MSE_{data} = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \|\Phi(x_k, u_k, T, \hat{\omega}) - x_{k+1}\|^2.$$

$$MSE_{phy} = \frac{1}{N_{phy}} \sum_{i=1}^{N_{phy}} \|\dot{\Phi}(x_k, u_k, T, \hat{\omega}) - \tilde{f}(x_k, u_k, \hat{\omega})\|^2.$$

# Paper #3 Modeling Estimation

## Experiment



# 감사합니다

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