How to Compute Integer Quotient and Remainder

Introduction

Suppose there are two integers called “dividend” and “divisor”. We are looking for two new integers, which we’ll call “quotient” and “remainder” such that the following equation is satisfied.

dividend = divisor x quotient + remainder

The problem is that the equation above is one equation with two unknowns. Therefore, there may exist multiple solutions. We want rules that will give use a unique correct solution to any such quotient-remainder problem.

Example: dividend = -14
divisor = 3
then both equations are correct:
-14 = 3*(-4) + (-2)
-14 = 3*(-5) + (+1)
but which solution is universally implemented by programming languages?

The two rules

If the dividend and the divisor have the same sign then the quotient is positive.
If the signs are opposites then the quotient is negative.

The sign of the remainder is always the same as the sign of the dividend.

Algorithm for manually computing quotient and remainder

1. Start with two integers, say -21 and -4. We wish to divide -21 by -4.
2. Divide the absolute value of the dividend by the absolute value of the divisor: 21/4 = 5 with remainder 1.
3. Apply the first rule to the quotient. The two original integers have the same sign, therefore the preliminary quotient 5 remains.
4. Apply the second rule to the remainder. The dividend -21 has sign “minus”. Therefore, the remainder is -1.

Let’s verify: -21 = (-4)*5 + (-1). The equation is true.

Therefore, -21 /(-4) yields quotient 5 and remainder -1.

What about zero? Example

Suppose the dividend is +15 and the divisor is -3. Then 15 divided 3 is 5 with remainder 0. Since the dividend and the divisor have opposite signs the computed 5 becomes a negative 5. Let's verify: 15 = (-3)*(-5) + 0, which is correct.

Therefore 15 divided by -3 yields quotient -5 and remainder 0.

The information in this document was verified by a C++ program.
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