Sensor applications



Human Activities Classification Using Biaxial Seismic Sensors

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Abstract—In this letter, we propose a method for passive human activity classification exploiting ground vibrations observed by a biaxial geophone. The solution is grounded on the idea that some activities can be better analyzed by the horizontal channel (bicycle and car) and others by the vertical one (walk and run). Thus, the following two solutions are proposed: first, joint processing of the vertical and horizontal data by a single classifier and, second, cascade processing by two classifiers that analyze the two channels separately. Numerical results based on real data show that while a parametric method such as a support vector machine performs well in both cases, a nonparametric method such as the k-nearest neighbors reaches a higher accuracy in cascade processing. Besides, the results are compared with those obtained using a monoaxial geophone only.

Index Terms—Sensor applications, classification, geophone, human activity, k-nearest neighbors (k-NN), principal component analysis (PCA), support vector machine (SVM).

I. INTRODUCTION

The problem of identifying and classifying the presence of a target in a particular environment with low-cost sensors remains a key issue for outdoor security applications [1]–[4]. The variability of the ground and environment characteristics (i.e., weather conditions, humidity, temperature, and wind speed) makes the target detection more complicated than a controlled indoor environment.

In the literature, many works propose to use networks of geophones (which present weak dependencies from the environment) to capture the ground vibration to detect the presence of persons in indoor scenarios [5] or to classify several vehicles with different weights in a well defined outdoor area [6]. In [7], a method for detecting intruders and predicting their activities outdoor using a seismic sensor is presented. Similarly, in [8], the objective is to detect and classify different targets (e.g., humans, vehicles, and animals led by a human) using seismic and passive infrared sensors. Other solutions exploit cooperative sensors (i.e., microphones and geophones) and data fusion techniques to improve vehicle classification accuracy and estimate their velocity [9], [10]. In [11], a system architecture for the classification of moving objects using both scalar and multimedia sensors is proposed.

In this letter, we aim to distinguish between four different human activities, ride a bike, drive a car, walk, run, and investigate the possibility of doing it using only a biaxial geophone (i.e., with two channels, horizontal, and vertical). In particular, the intuition behind this letter is that forces involved in the activities based on sliding contact with the ground solicit mainly horizontal ground vibrations, while activities characterized by footsteps are responsible for vertical vibrations. The consequences of this consideration on the solution proposed are explained more in-depth in Section III. The presented solution is based on the dimensionality reduction of frequency domain features of the data, based on principal component analysis (PCA) to ensure good classification performance at low computational cost [12]. Then, two different classification methods, using two distinct classifiers for each one, are presented.

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The main contributions of this letter are as follows.

- We propose a human activities classification system that uses only a biaxial geophone.
- 2) We propose the power spectral density (PSD) as a sensitive feature for the activity identification and PCA to remap the data in an advantageous feature space.
- We propose two different classification methods: classification made with a single classifier or using a cascade of two classifiers.
- 4) For both methods, we compare two distinct classifiers, i.e., support vector machine (SVM) and *k*-nearest neighbors (*k*-NN), and we measure the impact of vertical and horizontal components on classification performance.

Moreover, we present a comparison with conventional algorithms that exploit cross-correlation, named template matching, and an alternative classification method based on linear discriminant analysis (LDA).

Throughout this letter, capital boldface letters denote matrices, lowercase bold letters denote vectors, $(\cdot)^T$ stands for transposition, and $||\cdot||_2$ is the ℓ_2 -norm operator. The rest of this letter is organized as follows. In Section II, the system model and the signal processing chain are described. Section III presents the solution based on the two different classification methods. In Section IV, extensive numerical results based on real waveforms captured in an outdoor scenario are given. Finally, Section V concludes this letter.

II. SYSTEM MODEL

As depicted in Fig. 1, a passive geophone with two channels (vertical and horizontal) captures ground vibrations in an outdoor environment. The acquired data are then processed to classify between four human activities: ride a bike, drive a car, walk, and run.

At first, the analog signals produced by the horizontal and vertical channels of the geophone, $x_h(t)$ and $x_v(t)$, pass through an analog-to-digital converter (ADC) with sampling frequency f_s and resolution N_{bit} . The output of the conversion consists in two time series $\mathbf{x}_h = \{x_h(k/f_s)\}_{k=0}^{K-1} \text{ and } \mathbf{x}_v = \{x_v(k/f_s)\}_{k=0}^{K-1} \text{ of } K \text{ samples. Subsequently, the time series are first split in } N_w \text{ row vectors, obtained through a partially overlapped sliding window of length } W \text{ samples, with } W \leq K, \text{ and a}$

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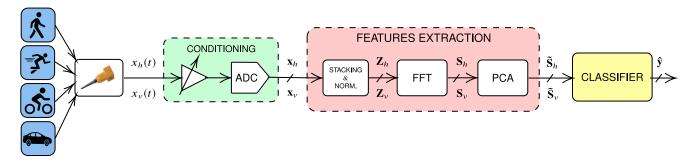


Fig. 1. Illustration of the processing data chain to extract features from the geophone signals and perform classification.

sliding step $\Delta_{\rm w}$, and then rearranged into the matrices $\mathbf{Z}_{\rm h}$ and $\mathbf{Z}_{\rm v}$, of size $N_{\rm w} \times W$, by stacking them. From now on, since the processing stages apply to \mathbf{Z}_h and \mathbf{Z}_v separately, we indicate both with \mathbf{Z} for the sake of conciseness. In this phase, the samples of each observation window are normalized column-wise such that the results are zero mean row vectors with $\max_i |z_{n,i}| = 1, n = 1, \dots, N_w$. Finally, the data are labeled by activity for the classification during the training phase.

A. PSD Estimation

For the identification of relevant features for human activity classification, we analyze the signal in the frequency domain [13]. In particular, we estimate the PSD of the acquired samples using the Welch overlapping segment averaging for each row of Z, with Hann window and 50% overlap. The number of points, D, of the fast Fourier transform (FFT) regulates the tradeoff between the frequency resolution and the PSD estimation accuracy [14]. The estimated PSDs $\{\mathbf{s}_i\}_{i=1}^{N_{\mathrm{w}}}$ are then organized in the matrix **S** of size $N_{\mathrm{w}} \times D$, generically used at this stage to indicate both S_h and S_v . Afterward, PCA is applied to S to reduce the dimensionality of the data and extract the features. From now on, we split $N_{\rm w}$ into two subsets $N_{\rm o}$ and $N_{\rm t}$, for the training and the test phases, respectively.

B. Principal Component Analysis

PCA distills the essential information from the dataset, which is then represented as a set of new orthogonal variables called principal components obtained from a linear combination of the original data [15]. For the calculation of the principal components, we consider only the N_0 points of the training subset. After centering the matrix S by subtracting its column-wise sample mean, we evaluate the sample covariance matrix $\Sigma = \frac{1}{N_0} \mathbf{S}^T \mathbf{S}$. Then, Σ is factorized by eigenvalue decomposition $\Sigma = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^T$, where $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues, ordered from the largest to the smallest, and Q is the matrix of eigenvectors [16].

In order to perform dimensionality reduction, we only keep the first D_h or D_v eigenvalues of Λ and the corresponding eigenvectors $\tilde{\mathbf{Q}}$ (i.e., selected columns of \mathbf{Q}). The projections $\hat{\mathbf{S}}$ of the observations in the components subspace through the new projection matrix $\tilde{\mathbf{Q}}$ are $\tilde{\mathbf{S}} =$ $\hat{\mathbf{SQ}}$, where $\hat{\mathbf{S}}$ is an $N_{\rm w} \times D_{\rm h}$ or $N_{\rm w} \times D_{\rm v}$ matrix, and $D_{\rm h}$, $D_{\rm v} \leq D$ are the number of principal components considered for $\tilde{\mathbf{S}}_h$ and $\tilde{\mathbf{S}}_v$. Note that, while the principal components are calculated solely over the training points $N_{\rm o}$, all the $N_{\rm w}$ points are projected in the components subspace. These two matrices represent the selected features used to train and test the classifiers described in Section III.

III. CLASSIFICATION TECHNIQUES

After dimensionality reduction, the features $\tilde{\mathbf{S}}_h$, $\tilde{\mathbf{S}}_v$ are used by a classifier to determine the type of human activity. In particular, two classification methods are proposed.

- 1) Single classifier. For each observation window, the data of the two channels are jointly processed. The $N_{\rm w} \times (D_{\rm h} + D_{\rm v})$ matrix $\mathbf{\tilde{S}}_{hv}$ is built concatenating the matrices $\mathbf{\tilde{S}}_h$ and $\mathbf{\tilde{S}}_v$ as $\mathbf{\tilde{S}}_{hv}=$ $[\tilde{\mathbf{S}}_h \ \tilde{\mathbf{S}}_v]$. The monoaxial (i.e., horizontal or vertical) geophone configuration is obtained setting $D_v = 0$ or $D_h = 0$, respectively.
- 2) Cascade classifier. The classification is carried out in two steps. First, a three-class classifier uses the data acquired from the horizontal channel $\tilde{\mathbf{S}}_{h}$ to classify between bike, car, and footsteps (i.e., run and walk are treated as if they are the same class). Then, if the first classifier does not choose for the bike or car classes, a second one uses the $\tilde{\mathbf{S}}_v$ features to discriminate between run and walk. The rationale behind this approach is that based on the dominant forces during the interaction between the target and the ground, all the activities based on sliding contact (i.e., car and bike) tend to stimulate horizontal ground vibrations. In contrast, activities characterized by footsteps tend to excite vertical vibrations.

Hereafter, the two classifiers used in this work are briefly described. Besides, ordinary cross-correlation-based classification method used both in time and frequency domain is reviewed. For the sake of clarity, y is the vector of actual classification labels of length $N_{\rm w}$, $\hat{\mathbf{y}}$ is the vector of classification labels estimated by the algorithms, $\tilde{\mathbf{s}}_n$ is the *n*th row of $\tilde{\mathbf{S}}_{h}$ (either $\tilde{\mathbf{S}}_{v}$ or $\tilde{\mathbf{S}}_{hv}$), and \mathcal{S} is the feature space of dimension D (either $D_{\rm h}$ or $D_{\rm v}$). Thus, each point, both for training and test, is represented by the pair $(\tilde{\mathbf{s}}_n, y_n)$.

A. Support Vector Machine

The SVM constructs a set of hyperplanes in high-dimensional space that can be used for tasks like classification or regression [17]. Hence, it is a parametric learning algorithm whose error function includes a regularization term as follows:

$$g(\tilde{\mathbf{w}}) = \sum_{n=1}^{N_{o}} \ln\left(1 + e^{-y_{n}(\tilde{\mathbf{s}}_{n}^{T}\tilde{\mathbf{w}})}\right) + \lambda ||\tilde{\mathbf{w}}||_{2}^{2}$$

where $\tilde{\mathbf{w}}$ are the weights of the parametrical model, and λ is the regularization parameter [17], [18]. The hyperplane that performs a good separation is the one that maximizes its distance from the nearest training points of each class, and it is identified by the set of weights $\tilde{\mathbf{w}}$ that minimizes the error function. Given a test point $(\tilde{\mathbf{s}}_m, y_m)$, the estimated label \hat{y}_m is given by $\hat{y}_m = \tilde{\mathbf{s}}_m \tilde{\mathbf{w}}^T$.

B. k-Nearest Neighbors

In k-NN a set of N_0 pairs $\{(\tilde{\mathbf{s}}_n, y_n)\}_{n=1}^{N_0}$ is given as training set, where $\mathbf{\tilde{s}}_n$ takes values in the feature space S upon which is defined the metric $d(\mathbf{\tilde{s}}_n, \mathbf{\tilde{s}}_m)$: the Euclidean distance in this work. Given a test point $(\mathbf{\tilde{s}}_m, y_m)$, the estimate of y_m is given by the nearest neighbor training point with respect to the test point as

$$\hat{y}_m = \left\{ y_k : \tilde{\mathbf{s}}_k = \arg\min_{\tilde{\mathbf{s}}_m} d(\tilde{\mathbf{s}}_n, \tilde{\mathbf{s}}_m) \right\}. \tag{1}$$

With (1) we can assign $\tilde{\mathbf{s}}_m$ to the same class of the nearest $\tilde{\mathbf{s}}_n$. If the number of training points is large enough, it makes sense to use the majority rule of the nearest k neighbors, instead of the single nearest neighbors [19]. In the case of binary classification, k is chosen to be odd to avoid that a point is assigned to two different classes.

C. Template Matching

Classification methods that use cross-correlation as a similarity metric, often called *template matching*, are extensively used in signal processing [20], [21]. Similarity can be searched both in time and frequency domain. In the frequency domain case, at first the estimated PSDs, $\{s_j\}_{j=1}^{N_w}$, of both horizontal and vertical channel, are normalized to have zero mean and unitary standard deviation. Then, given a set of N_0 pairs $\{(s_n, y_n)\}_{n=1}^{N_0}$ as training set, and a test point (s_m, y_m) , we define the vector \mathbf{r} such that its nth element is

$$r_n = \max\{\operatorname{corr}(\mathbf{s}_n, \mathbf{s}_m)\} \quad n = 1, \dots, N_o$$
 (2)

where $corr(\mathbf{s}_n, \mathbf{s}_m)$ is the cross-correlation vector, of length 2D-1, between the sequences \mathbf{s}_n and \mathbf{s}_m [20], [21]. Carrying out the same operations for both the horizontal and the vertical channel, we obtain the vectors \mathbf{r}_h and \mathbf{r}_v , respectively, which are then concatenated to obtain $\mathbf{r}_{hv} = [\mathbf{r}_h \ \mathbf{r}_v]$. The estimated label \hat{y}_m , is, thus

$$\hat{y}_m = \left\{ y_{(k-1) \bmod N_0 + 1} : k = \arg \max_n (\mathbf{r}_{\text{hv}}) \right\}.$$
 (3)

Similar operations are performed in the time domain case.

IV. NUMERICAL RESULTS

In this section, we present several tests to compare the performance of the classifiers in different settings. For the measurements, a biaxial two-channel geophone, with natural frequency of 4.5 Hz, frequency bandwidth of 0.2 - 240 Hz, and sensitivity of 78 V/m/s has been used. The geophone has been placed in a flowerbed next to a car park. All the activities have been performed within a distance of 6 m from the geophone. The seismic sensor was interfaced with the PC using an ADC with sampling frequency $f_{\rm s}=400\,{\rm Hz}$ and resolution $N_{\rm bit}=14\,{\rm b}$. Furthermore, for each channel, an LM 386 amplifier has been adopted to increase the signal-to-quantization noise ratio.

Each activity has been repeated several times for a whole period equal to $T_{\rm a}=20\,{\rm min}$, using different vehicles and involving people with various height and gait. In particular, six people took part in the run and walk measurements, while two bikes and a car, driven by two persons, were used. Then, the acquisition period has been split into several observation windows of duration $T_{\rm ob}=W/f_s$ ranging between 5 and 30 s, depending on the test, and using a sliding step of $\Delta_{\rm w}=5\,{\rm s}$; the PSDs was computed with a D=64 points FFT.

To properly study the classifiers' performance, the acquisitions have been randomly split in *training* and *test* sets as follows.

- 1) Training and validation: 60% of all PSDs of each activity i.e., are randomly chosen to calculate the projection matrix of PCA, $\tilde{\mathbf{Q}}$. Then, the projected points are used to train the classifiers. To set the hyperparameters of SVM and k-NN, 10% of these points are randomly chosen and then used to perform cross-validation. The resulting optimal values are k=3 and $\lambda=0.1$.
- 2) *Test*: the other 40% of PSDs are used to test the performance of the algorithms. In this case, the projection in the principal

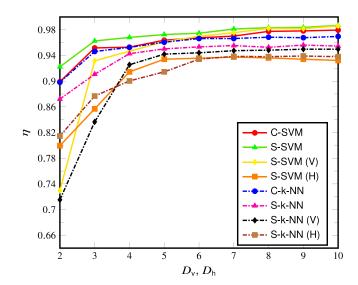


Fig. 2. Accuracy varying PCA components of the vertical and horizontal channels with $T_{\rm ob}=20\,{\rm s}$.

components subspace through PCA is made using the projection matrix computed during the training phase.

The same ratio of training and test points has also been used for the template matching based approaches and the LDA. However, no cross-validation is required in these cases. As a figure of merit, to evaluate the performance of the classifiers, we consider the *accuracy*, defined as

$$\eta = \frac{\text{number of points correctly classified}}{\text{number of total points}}.$$
 (4)

A. Accuracy Versus Number of PCA Components

For both the horizontal and the vertical channel, the number of selected components $D_{\rm h}$ and $D_{\rm v}$ have been varied and the effect on the accuracy of the algorithms has been studied. In this test, the observation window has been set to W=8000 samples, corresponding to an observation time $T_{\rm ob}=20\,{\rm s}$.

As shown in Fig. 2, the single SVM (S-SVM) classifier provides always the best performance, for each value of D_h and D_v ; in particular, with only $D_h = D_v = 3$ components, it reaches an accuracy greater than 96%, while to achieve the same accuracy with the cascade SVM (C-SVM) configuration, it is necessary to use a higher number of PCA components $(D_h, D_v \ge 5)$. On the contrary, the single k-NN (S-k-NN) converges to an accuracy roughly equal to 93%, much lower than that of the cascade k-NN (C-k-NN), which stands at almost 97%. These results prove that, in the case of k-NN and for this experiment setup, it is better to adopt the cascade solution and use the samples coming from the channels separately. With regard to the monoaxial scenario, we can see that for the SVM, the performance in case we use only the samples of the vertical channel is comparable with those of C-SVM and S-SVM solutions, when $D_v = D_h = 10$. On the contrary, if we use only the horizontal channel, the accuracy is always worse. Differently, the performance of k-NN is always better when using a biaxial geophone.

B. Accuracy Versus Observation Duration

In this test, the observation window duration varies between 5 and 30 s. The selected principal components are $D_{\rm h}=D_{\rm v}=5$. As shown in Fig. 3, here too, the performance of the S-k-NN is worse than the cascade solution. On the contrary, the accuracy of the S-SVM is always

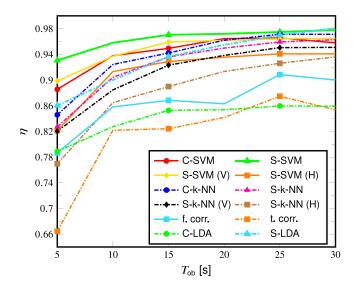


Fig. 3. Accuracy varying the observation window duration $T_{\rm ob}$. For PCA-based classifiers, $D_{\rm h}=D_{\rm v}=5$.

better than the cascade solution, with a higher gap when $T_{\rm ob} = 5\,\rm s$. Moreover, we can notice that for $T_{\rm ob} > 20\,\rm s$, the accuracy of the C-k-NN and of the S-SVM are comparable, while that of the C-SVM decreases. The results for the monoaxial geophone confirm what has been experimented in Section IV-A.

The performance of template matching based approaches, both in time and frequency domain, are included in Fig. 3 for comparison. As can be noted, the traditional techniques which do not exploit data structure are outperformed by the proposed solutions. In addition, the accuracy of LDA, in both cascade (C-LDA) and single (S-LDA) configurations, are also included [22]. While for very long (> 30 s) observation windows, the S-LDA classifier reaches the same performance as the proposed solution based on PCA, followed by SVM, for short observations some of the proposed solutions [C-SVM, S-SVM, and S-SVM (V)] outperform LDA. Note that, C-LDA classification always exhibit a poor performance almost equivalent to template matching.

V. CONCLUSION

In this letter, we proposed a passive human activity classification method exploiting the ground vibrations observed by a two-channel geophone. Data collected by the two channels are processed by a PCA-based dimensionality reduction to extract significant features in the frequency domain. The classification step is performed by either a single classifier or a cascade of two classifiers. The analysis considered the most important parameters (observation window and the number of PCA components) and setups (joint processing or cascade processing) to provide a complete set of results which may assist the system designer. Based on performance assessment on real waveforms, the extensive numerical results show that the number of selected PCA components and the observation window duration strongly impact the performance of the algorithms. For example, for large observation windows, the best solution is represented by the S-SVM classifier that jointly processes both vertical and horizontal data. However, as

complexity may have a role in designing autonomous low-power devices, if the k-NN classifier is preferred, the cascade solution performs the best.

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