

## Grammar - Types

$$\begin{array}{ll}
 \text{(Base types)} & A, B ::= \text{unit} \mid A_1 \oplus \dots \oplus A_n \mid A_1 \otimes \dots \otimes A_n \mid \\
 & \mu X.A \mid (A_1, \dots, A_n) \mid X \\
 \text{(Isos)} & T ::= A \leftrightarrow B \mid T_1 \rightarrow T_2 \mid X
 \end{array}$$

## Grammar - Terms

$$\begin{array}{ll}
 \text{(Values)} & v ::= () \mid x \mid c \mid c v \mid (v_1, \dots, v_n) \\
 \text{(Expressions)} & e ::= v \mid \text{let } v_1 = v_2 \text{ in } e \mid \text{let } v_1 = \omega v_2 \text{ in } e \\
 \text{(Isos)} & \omega ::= (\text{case } v_1 \leftrightarrow e_1 \mid \dots \mid v_n \leftrightarrow e_n) \\
 & \quad \text{fix } \phi.\omega \mid \text{fun } \phi \rightarrow \omega \mid \phi \mid c \mid \text{invert } \omega \mid \omega_1 \omega_2 \\
 \text{(Terms)} & t ::= () \mid x \mid (t_1, \dots, t_n) \mid \omega t \mid \text{let } v = t_1 \text{ in } t_2 \mid \text{iso } x = \omega \text{ in } t
 \end{array}$$

## Typing Rules - Terms

$$\frac{}{\Psi; \emptyset \vdash () : \text{unit}} \quad \frac{\Psi; x : A \vdash x : A}{\Psi; \Delta \vdash (t_1, \dots, t_n) : A_1 \otimes \dots \otimes A_n} \quad \frac{\Psi; \Delta_1 \vdash t_1 : A_1 \quad \dots \quad \Psi; \Delta_n \vdash t_n : A_n}{\Psi; \Delta \vdash t : A}$$

$$\frac{\Psi \vdash_\omega \omega : A \leftrightarrow B \quad \Psi; \Delta \vdash t : A}{\Psi; \Delta \vdash \omega t : B}$$

$$\frac{\Psi; \Delta_1 \vdash t_1 : A_1 \otimes \dots \otimes A_n \quad \Psi; \Delta_2 \vdash x_1 : A_1, \dots, x_n : A_n \vdash t_2 : B}{\Psi; \Delta_1, \Delta_2 \vdash \text{let } (x_1, \dots, x_n) = t_1 \text{ in } t_2 : B}$$

## Typing Rules - Isos

$$\frac{}{\Psi; \phi : T \vdash_\omega \phi : T} \quad \frac{\Psi \vdash_\omega \omega_1 : T_1 \quad \Psi \vdash_\omega \omega_2 : T_1 \rightarrow T_2}{\Psi \vdash_\omega \omega_2 \omega_1 : T_2} \quad \frac{\Psi, \phi : T_1 \vdash_\omega \omega : T_2}{\Psi \vdash_\omega \lambda \phi. \omega : T_1 \rightarrow T_2}$$

$$\frac{\Psi; \Delta_1 \vdash v_1 : A \quad \dots \quad \Psi; \Delta_n \vdash v_n : A \quad \forall i \neq j, v_i \perp v_j}{\Psi; \Delta_1 \vdash e_1 : B \quad \dots \quad \Psi; \Delta_n \vdash e_n : B \quad \forall i \neq j, e_i \perp e_j}$$

$$\frac{}{\Psi \vdash_\omega \{v_1 \leftrightarrow e_1 \mid \dots \mid v_n \leftrightarrow e_n\} : A \leftrightarrow B}$$