

## Grammar - Types

(Base types)  $A, B ::= \mathbf{unit} \mid A_1 \oplus \dots \oplus A_n \mid A_1 \otimes \dots \otimes A_n \mid$   
 $\mu X. A \mid (A_1, \dots, A_n) X \mid X$   
 (Isos)  $T ::= A \leftrightarrow B \mid T_1 \rightarrow T_2 \mid X$

## Grammar - Terms

(Values)  $v ::= () \mid x \mid c \mid c v \mid (v_1, \dots, v_n)$   
 (Expressions)  $e ::= v \mid \mathbf{let} \ v_1 = v_2 \ \mathbf{in} \ e \mid \mathbf{let} \ v_1 = \omega \ v_2 \ \mathbf{in} \ e$   
 (Isos)  $\omega ::= (\mathbf{case} \ v_1 \leftrightarrow e_1 \mid \dots \mid v_n \leftrightarrow e_n)$   
 $\mathbf{fix} \ \phi. \omega \mid \mathbf{fun} \ \phi \rightarrow \omega \mid \phi \mid c \mid \mathbf{invert} \ \omega \mid \omega_1 \ \omega_2$   
 (Terms)  $t ::= () \mid x \mid (t_1, \dots, t_n) \mid \omega \ t \mid \mathbf{let} \ v = t_1 \ \mathbf{in} \ t_2 \mid \mathbf{iso} \ x = \omega \ \mathbf{in} \ t$

## Typing Rules - Terms

$$\frac{}{\Psi; \emptyset \vdash () : \mathbf{unit}} \quad \frac{}{\Psi; x : A \vdash x : A} \quad \frac{\Psi; \Delta_1 \vdash t_1 : A_1 \quad \dots \quad \Psi; \Delta_n \vdash t_n : A_n}{\Psi; \Delta \vdash (t_1, \dots, t_n) : A_1 \otimes \dots \otimes A_n}$$

$$\frac{\Psi \vdash_\omega \omega : A \leftrightarrow B \quad \Psi; \Delta \vdash t : A}{\Psi; \Delta \vdash \omega \ t : B}$$

$$\frac{\Psi; \Delta_1 \vdash t_1 : A_1 \otimes \dots \otimes A_n \quad \Psi; \Delta_2 \vdash x_1 : A_1, \dots, x_n : A_n \vdash t_2 : B}{\Psi; \Delta_1, \Delta_2 \vdash \mathbf{let} \ (x_1, \dots, x_n) = t_1 \ \mathbf{in} \ t_2 : B}$$

## Typing Rules - Isos

$$\frac{}{\Psi; \phi : T \vdash_\omega \phi : T} \quad \frac{\Psi \vdash_\omega \omega_1 : T_1 \quad \Psi \vdash_\omega \omega_2 : T_1 \rightarrow T_2}{\Psi \vdash_\omega \omega_2 \ \omega_1 : T_2} \quad \frac{\Psi, \phi : T_1 \vdash_\omega \omega : T_2}{\Psi \vdash_\omega \lambda \phi. \omega : T_1 \rightarrow T_2}$$

$$\frac{\Psi; \Delta_1 \vdash v_1 : A \quad \dots \quad \Psi; \Delta_n \vdash v_n : A \quad \forall i \neq j, v_i \perp v_j \quad \Psi; \Delta_1 \vdash e_1 : B \quad \dots \quad \Psi; \Delta_n \vdash e_n : B \quad \forall i \neq j, e_i \perp e_j}{\Psi \vdash_\omega \{v_1 \leftrightarrow e_1 \mid \dots \mid v_n \leftrightarrow e_n\} : A \leftrightarrow B}$$