BRIGHT LEARNERS ORGANIZATION AND CONCOUR PREP SCHOOL

CONTINUOUS ASSESSMENT

WEEK 4

ENGINEERING DEPARTMENT

CONTENT

MATHEMATICS 60MARKS

PHYSICS 75MARKS

GOOD LUCK

Mathematics

Section A: Matrices

Question 1. 5mrks

Given the matrix M where M = $\begin{pmatrix} 0 & 2 & -1 \\ 3 & 2 & 2 \\ 3 & -1 & 2 \end{pmatrix}$

find M $^{-1}$ and hence, or otherwise, solve the simultaneous equations

$$2y + z = 5$$
$$3x + y + 2z = 13$$
$$3x - y + 2z = 9$$

T is the transformation with matrix M find

(a) the image of the point $(2,3,-1)$ under T
(b) the point whose image is $(6,-6,12)$

Question 2. 5mrks;

Given the matrix M = $\begin{pmatrix} 1 & 3 & 6 \\ 2 & -1 & 4 \\ -3 & 5 & 0 \end{pmatrix}$

(i) find the image of the line $\frac{x-1}{4} = y + 4 = \frac{z-1}{2}$ under the transformation whose matrix is M find M $^{-1}$ and hence
(ii) Find the point whose image is $(2,3,-1)$

Section B: Functions and Relations

Question 1. 15mrks;

1. The functions f and g are defined by

$$f: x \mapsto \frac{x+2}{x-4}, x \in \mathbb{R}, x \neq 4$$
$$g: x \mapsto x + 5, x \in \mathbb{R}$$

Find
(a) $f^{-1}(x)$, stating its domain
(b) $g^{-1}(x)$
(c) $fg(x)$.

2. A function $f: \mathbb{Z} \to \mathbb{Z}$ is defined by

$$f: x \mapsto \frac{2x}{1+x}$$

(a) show that f is not injective.
(b) Find the range of f.

3. A function f is defined by

$$f: x \mapsto \frac{x}{x-3}, x \in \mathbb{R}, x \neq 3$$

(a) Verify whether f is a bijection.
(b) Find $f^{-1}(x)$ and state its domain.

4. The functions f and g are defined by are defined over the set of real numbers by

$$f: x \mapsto \frac{2}{x^2-4}, g: x \mapsto 2x + 1$$

(a) Determine the domain and range of f

b) Show that g is a bijection.

c) State the three requirements for an equivalence relation on a set S. Given that the relation "is a factor of" is not an equivalence relation on the set $\mathbb{N}$ of integers.

5. A relation R is defined on the set of all points in the plane $(x, y)R(u, v)$ if and only if $v^2 - y^2 = u^2 - x^2$. Show that R is symmetric.

Section C: Sequences and series 25mrks;

Question 1;

$$f(x) = \frac{1}{\sqrt{4+x}}, \quad |x| < 4$$

Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction.
(Total 6 marks).

Question 2;

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)} \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)}$$

a. find the values of B and C and show that $A = 0$. (4marks)
b. Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2. Simplify each term. (6marks).

Question 3;

Find the nth term of the arithmetic sequences defined below:

1. $\frac{2}{5}, \frac{12}{5}, \frac{22}{5}, \frac{32}{5}$, … where $n = 2,3,$ …
2. $\frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3,$ … where $n = 3,4,5,$ …

Find the sum of the first n terms of the arithmetic sequence defined by $T_n = n - 3$ with $n = 1,2,3,4,$ …

Find the sums hence compute the value when $n = 20,24$ and 25:

1. $\sum_{k=1}^{n} (k + 1)$
2. $\sum_{k=1}^{n} (2k + 3)$

Given that $S_n = n^2 + 4n + 4$ with $n = 1,2,3,$ … is the sum of the first n terms of an arithmetic sequence T_n, find the expression of T_n hence or otherwise find the sum from the 11th to the 25th term exclusive.

Section D: MCQs 10mrks;

1. If $\frac{x-4}{x^2-5x-2k} = \frac{2}{x-2} - \frac{2}{x+k}$, then K =
   a) -3 b) -2 c) 2 d) 3
2. If $\frac{ax-1}{(1-x+x^2)(2+x)} = \frac{x}{1-x+x^2} - \frac{1}{2+x}$ then a =
   a) 3 b) -3 c) 2 d) -2
3. If $\frac{3x+4}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$, then A, B
   a) $(7,10)$ b) $(10,7)$ c) $(10, -7)$ d) $(-10,7)$
4. If $\frac{1-x+6x^2}{x-x^3} = \frac{A}{x} + \frac{B}{1-x} + \frac{C}{1+x}$, then A =

a) *1* b) *2* c) *3* d) *4*

5. If $\frac{3x}{(x-a)(x-b)} = \frac{2}{x-a} + \frac{1}{x-b}$ then a : b =
   a) $1:2$ b) $-2:1$ c) $1:3$ d) $3:1$

6. Let a, b, c such that
$$\frac{1}{(1-x)(1-2x)(1-3x)} = \frac{a}{1-x} + \frac{b}{1-2x} + \frac{c}{1-3x}$$
$$\frac{a}{1} + \frac{b}{3} + \frac{c}{5} =$$
   a) $\frac{1}{15}$ b) $\frac{1}{6}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

7. If $\frac{x^2}{(x-2)(x-6)(x-c)} = P(x) + \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)^2}$ then P(x) =
   a) $x-a$ b) $x-a-b$ c) $x-a-b-c$ d) $x+a+b+c$

8. $\frac{x}{(2x-1)(x+2)(x-3)} = A + \frac{B}{2x-1} + \frac{C}{x+2} + \frac{D}{x-3}$ then A =
   a) $\frac{1}{2}$ b) $\frac{1}{50}$ c) $\frac{8}{25}$ d) $\frac{27}{25}$

9. $\frac{3x^2+x+1}{(x-1)^4} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x-1)^4}$ then A + B - C + D =
   a) *0* b) *15* c) *1* d) *10*

10. If $\frac{x^4+24x^2+28}{(x^2+1)^3} = \frac{A}{x^2+1} + \frac{B}{(x^2+1)^2} + \frac{C}{(x^2+1)^3}$ then A + B + C =
    a) *26* b) *27* c) *28* d) *29*

Physics

Section A: current electricity 20mks;

Question 1;

A. Deduce Ohm's law using the concept of drift velocity.

B. Define the term 'drift velocity' of charge carriers in a conductor. Obtain the expression for the current density in terms of relaxation time.

C. Define relaxation time of the free electrons drifting in a conductor. How is it related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material.

(i) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend?
(ii) Why alloys like constantan and manganin are used for making standard resistors?

Question 2;

When 5 V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} m/s. If the electron density in the wire is 8×10^{28} m $^{-3}$, calculate the resistivity of the material of wire.

Section B: Mechanics. 25mks;

Problem 1;

An object A of mass m moving with a velocity of 10ms $^{-1}$ collides with a stationery object B at equal mass m. After collision A moves with a velocity U at an angle of 30 ^{0} to its initial direction and B moves with a velocity V at an angle of 90 ^{0} to the direction U.

i) Calculate the velocities U and V (U = $5\sqrt{3}$ms $^{-1}$, V = 5ms $^{-1}$)
ii) Determine whether the collision was elastic or not.(Kinetic energy before collision = kinetic energy after collision = 50m, hence collision is elastic)

0.45kg 1.5ms $^{-1}$ 8ms $^{-1}$ 0.3kg

0.45kg 0.3kg v

Problem 2

A body of mass 5.0kg is moving with a velocity 2.0ms $^{-1}$ to the right. It collides with a body of mass 3.0 kg moving with a velocity of 2.0ms $^{-1}$ to the left. If the collision is head-on and elastic, determine the velocities of the two bodies after collision.(-1.0 ms $^{-1}$, 3.0ms $^{-1}$)

Problem 3.s

A rectangular block of mass 10 kg is pulled from rest along a smooth inclined plane by a light inelastic string which passes over a light frictionless pulley and carries a mass of 20kg. The inclined plane makes an angle of 30 ^{0} with the horizontal.

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Determine

(i) The acceleration of the block
(ii) The tension in the string
(iii) The K.E of the block when it has moved 2m along the inclined plane.

Section B: MCQs 13mks;

1. What is the new speed of the electron?

$E_k = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2E_k}{m}}$ $v \propto \sqrt{E_k}$

$$v_2 = v\sqrt{\frac{3E}{E}} = \sqrt{3}v$$

A. $\frac{v}{\sqrt{2}}$
B. $\sqrt{3}v$
C. $2v$
D. $4v$

(Total 1 mark)

2. An object of mass m is accelerated from rest to a velocity v by a constant resultant force F.

What is the work done on the object during this acceleration?

Work done = energy transferred = $E_k = \frac{1}{2}mv^2$

A. $\frac{Fv}{2}$
B. Fv
C. mv^2
D. $\frac{mv^2}{2}$

(Total 1 mark)

3. Two identical balls, X and Y, are at the same height and a horizontal distance of 25 cm apart. X is projected horizontally with a velocity of 0.10 m s $^{-1}$ towards Y at the same time that Y is released from rest. Both X and Y move freely in the absence of air resistance. What is the distance between the balls 1.0 s later?

A. 0.15 m
B. 0.25 m
C. 2.4 m
D. 4.9 m

4. Two bodies of different masses undergo an elastic collision in the absence of any external force. Which row gives the effect on the total kinetic energy of the masses and the magnitudes of the forces exerted on the masses during the collision?

	Total kinetic energy	Magnitudes of forces

	Total kinetic energy	Magnitudes of forces
A	remains unchanged	same on both masses
B	remains unchanged	greater on the smaller mass
C	decreases	same on both masses
D	decreases	greater on the smaller mass

5. A body of constant mass falls freely due to gravity. The rate of change of momentum of the body is equal to its

A. kinetic energy.
B. mass.
C. gravitational potential energy.
D. weight.

6. A railway truck of mass 2000 kg travelling horizontally at 1.5 m s^{-1} collides with a stationary truck of mass 3000 kg. After the collision they move together. Which row is correct?

	Speed of the trucks immediately after collision / m s^{-1}	Effect of collision on total kinetic energy
A	0.6	no change
B	0.6	decrease
C	1.0	no change
D	1.0	decrease

7. The drag force on a boat is kv^2, where v is the speed and k = *64* kg m $^{-1}$. The boat's engine has a useful power output of 8000 W. What is the maximum speed of the boat?

A. 0.2 m s^{-1}
B. 5 m s^{-1}
C. 11 m s^{-1}
D. 125 m s^{-1}

8. Which row is true for an elastic collision between two objects in an isolated system?

	Kinetic energy	Momentum
A	conserved	conserved
B	not conserved	conserved
C	conserved	not conserved
D	not conserved	not conserved

9. Two ball bearings X and Y are projected from horizontal ground at the same time. X has mass $2m$ and is projected vertically upwards with speed u. Y has mass m and is projected at 30° to the horizontal with speed $2u$. Air resistance is negligible. Which statement is correct?

A. X and Y have the same initial momentum.
B. X and Y reach their maximum heights at different times.
C. The maximum height reached by Y is half that reached by X.
D. X and Y reach the ground at the same time.

10. Which quantity is represented by the area under a force–time graph?

A. average power
B. elastic strain energy stored
C. momentum change
D. work done

11. A Formula 1 racing car uses up its fuel during the race, causing its lap times to decrease. The lap times decrease because

A. the acceleration of the car increases.
B. the drag forces on the car decrease.
C. the maximum speed of the car increases.
D. the tyres become worn, reducing the friction with the road.

12. The question below is about three spheres X, Y and Z. The relative mass and relative diameter of each sphere are given in the table.

	X	Y	Z
relative mass	1	5	1
relative diameter	1	1	5

Each sphere is dropped from rest and accelerates to its terminal speed. What is true about the terminal speeds?

A. The terminal speed of X is greater than that of Y.
B. The terminal speed of X is the same as that of Y.
C. The terminal speed of Y is greater than that of Z.
D. The terminal speed of X is less than that of Z.

13. What is true for an inelastic collision between two isolated objects?

A. Both total momentum and total kinetic energy are conserved.
B. Neither total momentum nor total kinetic energy is conserved.
C. Only total kinetic energy is conserved.
D. Only total momentum is conserved.

Section C: Electromagnetism. 18mks;

Question 1

A long straight wire carries a current of 5 A. A rectangular coil of 100 turns, length 0.2 m, and width 0.1 m is placed 0.05 m away from the wire. The coil is then rotated about its axis (which is parallel to the wire) with an angular velocity of 10 rad/s.

A. Find the magnetic field strength at the center of the coil due to the current-carrying wire.
B. If the coil has a resistance of 10 ohms, find the induced current in the coil.
C. Find the torque acting on the coil due to the magnetic field.

Question 2

A conducting rod of length 1 m and resistance 5 Î© is moving with a speed of 20 m/s in a magnetic field of strength 0.5 T. The rod is connected to a circuit with a resistance of 10 Î© and an inductance of 0.1 H.

A. Find the induced emf in the rod.
B. Find the current in the circuit.
C. Find the force required to maintain the motion of the rod.
D. If the rod is moving in a circular path of radius 2 m, find the magnetic field strength at the center of the circle.