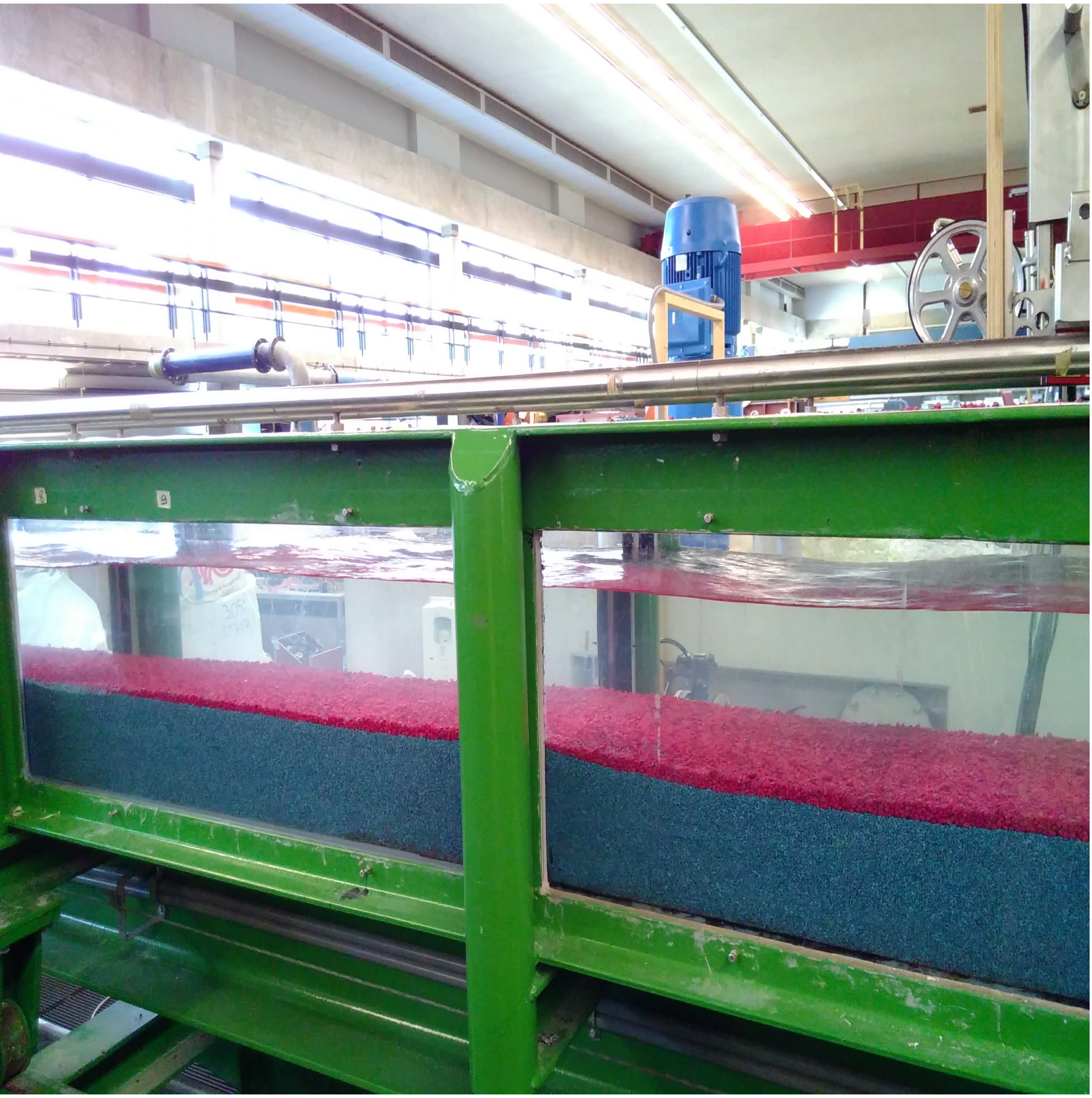
ELV

Technical reference manual



ELV

Technical reference manual

Authors
Victor Chavarrias

Frontpage: Laboratory experiment at the TU Delft modelled using ELV.

Contents

1 Introduction

ELV is a open-source software written in Matlab for modelling one-dimensional river morphodynamic processes ideal for research purposes.

Victor Chavarrias started it on the 23rd of February of 2016. Liselot Arkesteijn has implemented several features as well inestimable support and ideas. Pepijn van Denderen implemented the possibility of modelling several branches. The first backwater solver is thanks to Guglielmo Stecca.

2 Code structure

The code structure is:

1. Initialization (Section 2.1)
2. Time loop
   - 2.1. Flow update (Section 2.2)
   - 2.2. Friction correction (Section 2.3)
   - 2.3. Sediment transport (Section 2.4)
   - 2.4. Interpolation (Section 2.5)
   - 2.5. Struiksma reduction (Section 2.6)
   - 2.6. Particle activity (Section 2.7)
   - 2.7. Nodal point relation (Section 2.8)
   - 2.8. Bed level update (Section 2.9)
   - 2.9. Active layer thickness update (Section 2.10)
   - 2.10. Grain size distribution update (Section 2.11)
   - 2.11. Friction update (Section 2.12)
   - 2.12. Results writing (Section 2.13)
   - 2.13. Time step computation (Section 2.14)
3. Finalization

2.1 Initialization

The space domain x is discretized into N cells of equal length Δx [m]. The equations are solved in a one-dimensional domain of length L [m] extending from $x = x_\mathrm{o}$ until $x = x_\mathrm{f}$ over time T [s] between $t = t_1$ and $t = t_2$.

2.2 Flow update

2.2.1 Physical system of equations

The **?** equations in conservative form read:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \, , \tag{2.1}$$

$$\frac{\partial q}{\partial t} + \frac{\partial q^2/h}{\partial x} + \frac{1}{2} g \frac{\partial h^2}{\partial x} + gh \frac{\partial \eta}{\partial x} = -C_f \frac{q^2}{h^2} \, . \tag{2.2}$$

In solving the flow equations, g and C_f are assumed constant and η constant in time.

At $t = t_1$, $h = H_1(x)$ and $q = Q_1(x)$ for $x_\mathrm{o} \leq x \leq x_\mathrm{f}$.

At $x = x_\mathrm{o}$, $q = Q_\mathrm{o}(x)$ for $t_1 \leq t \leq t_2$.

At $x = x_\mathrm{f}$, $h = H_\mathrm{f}(x)$ for $t_1 \leq t \leq t_2$.

2.2.2 Numerical discretization

The non-linear set of equations ((2.1))-((2.2)) is discretized at the cell centres by means of the θ-box scheme:

$$\frac{\partial f}{\partial t} = \frac{1}{2}\left(\frac{f_{m+1}^{n+1} - f_{m+1}^{n}}{\Delta t} + \frac{f_{m}^{n+1} - f_{m}^{n}}{\Delta t}\right) , \tag{2.3}$$

$$\frac{\partial f}{\partial x} = \theta\left(\frac{f_{m+1}^{n+1} - f_{m}^{n+1}}{\Delta x}\right) + (1-\theta)\left(\frac{f_{m+1}^{n} - f_{m}^{n}}{\Delta x}\right) , \tag{2.4}$$

where $\theta \in [0.5, 1]$ is a parameter, $m \in [1, N]$ is an index indicating cell centre number in increasing order, and $n > 1$ is an index indicating time.

For $\theta = 0.5$ one obtains the Preissmann scheme (**???**).

The first term in Equation ((2.1)) is:

The slope is approximated as:

$$\left.\frac{\partial \eta}{\partial x}\right|_{m} = \frac{\eta_{m+1} - \eta_{m}}{\Delta x} , \tag{2.5}$$

2.2.3 Solution of the algebraic system of equations

The discretized set of equations form a system of algebraic equations:

$$\mathbf{AQ} = \mathbf{B} . \tag{2.6}$$

Vector:

$$\mathbf{Q} = [h_1, h_2, \cdots, h_{N-1}, h_N, q_1, q_2, \cdots, q_{N-1}, q_N]^{\intercal} , \tag{2.7}$$

is the $2N$x1 vector of unknowns.

Matrix:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} , \tag{2.8}$$

is the $2N$x$2N$ matrix containing the implicit terms, which is subdivided into 4 NxN submatrices contain.

Vector:

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} , \tag{2.9}$$

is the $2N$x1 vector of explicit terms.

The system is solved using the Newton–Raphson method until $r < \epsilon$:

$$\mathbf{Q}^{j+1} = \mathbf{Q}^{j} - (\mathbf{J_0}^{j})^{-1}\left(\mathbf{A}^{j}\mathbf{Q}^{j} - \mathbf{B}^{j}\right) , \tag{2.10}$$

where j is the iteration index. The operation to the right of equation ((2.10)) is done using function `mldivide` applied to arguments $\mathbf{J_0}^j$ and $\mathbf{A}^j\mathbf{Q}^j - \mathbf{B}^j$. The residual r is computed as:

$$r = \max \left| \left(\mathbf{A}^j \mathbf{Q}^j - \mathbf{B}^j \right) \right| \; . \tag{2.11}$$

2.3 Friction correction

2.4 Sediment transport

2.5 Interpolation

Some functions require information at cell edges ($m - 1/2$ and $m + 1/2$). This is done by linear interpolation. Extrapolation is applied at the first and last cells.

2.6 Struiksma reduction

2.7 Particle activity

2.8 Nodal point relation

2.9 Bed level update

2.9.1 Physical system of equations

Mass conservation of sediment is solved in its flux form (Section **??**):

$$\frac{\partial \eta}{\partial t} + \frac{M_\mathrm{f}}{c_\mathrm{b} \beta} \frac{\partial q_\mathrm{b}}{\partial x} = 0 \; , \tag{2.12}$$

or in entrainment-deposition form (Section 2.9.3):

$$\frac{\partial \eta}{\partial t} = \frac{M_\mathrm{f}}{c_\mathrm{b}} \left(D - E \right) = 0 \; . \tag{2.13}$$

2.9.2 Numerical discretization of the flux form

The flux version (Equation (2.12)) can be solved using a FTBS (Forward in Time and Backward in Space) scheme (Section **??**) or the Borsboom scheme (Section **??**).

2.9.2.1 FTBS

$$\eta_m^{n+1} = \eta_m^n - \frac{M_\mathrm{f} \Delta t}{c_\mathrm{b} \beta_m \Delta x} \left({q_\mathrm{b}}_{m-1}^{\,n} - {q_\mathrm{b}}_{m}^{\,n} \right) \tag{2.14}$$

2.9.2.2 Borsboom scheme

The bed elevation at the cell centre of the first and last two cells is found applying FTBS (Section 2.9.2.1).

The remaining cell centre bed elevation is updated as:

$$\eta_m^{n+1} = \eta_m^n - \frac{M_\mathrm{f} \Delta t}{c_\mathrm{b} \beta_m \Delta x} \left(\varphi_{m-1/2}^n - \varphi_{m+1/2}^n \right) \; , \tag{2.15}$$

$$\varphi_{m-1/2} = q_{\mathrm{b}\,m-1/2} + \frac{1}{2} c_{m-1/2} \left[\left(1 - \sigma_{\mathrm{b}\,m-1/2} \right) \Phi_{m-1} - 1 \right] \left(\eta_m - \eta_{m-1} \right) \ , \tag{2.16}$$

$$\sigma_{\mathrm{b}\,m-1/2} = c_{m-1/2} \frac{\Delta t}{\Delta x} \ , \tag{2.17}$$

$$\sigma_{\mathrm{b}\,m-1/2} = c_{m-1/2} \frac{\Delta t}{\Delta x} \ , \tag{2.18}$$

ϕ_m is a flux limiter (Appendix B) based on r_m:

$$r_m = \frac{\eta_m - \eta_{m-1}}{\eta_{m+1} - \eta_m + \epsilon} \ , \tag{2.19}$$

where $\epsilon = 1 \times 10^{-10}$ is a tolerance. c is the bed celerity (Appendix C)

2.9.3 Numerical discretization of the entrainment-deposition form

2.10 Active layer thickness update

2.11 Grain size distribution update

2.12 Friction update

2.13 Results writing

2.14 Time step computation

2.15 Finalization

A Variables

Table 1 Variables in ELV

Symbol	Unit	Meaning
c	m/s	bed celerity
c^*	m/s	non-dimensional bed celerity
c_b	-	1-porosity
C_f	-	non-dimensional friction coefficient
D	m/s	Deposition rate
E	m/s	Entrainment rate
Fr	-	Froude number
g	m/s^2	acceleration due to gravity
h	m	flow depth
m	-	index of the cell centre
n	-	index of time
M_f	-	morphodynamic acceleration factor
q	m^2/s	discharge per unit width
q_b	m^2/s	sediment discharge per unit width
t	s	time coordinate
x	m	streamwise coordinate
α	-	preconditioning variable for volume fraction content
β	-	preconditioning variable for bed elevation
Δt	s	time step
Δx	m	space step
Φ	-	flux limiter
η	m	bed elevation
σ_b	-	CFL number of the bed

B Flux limiters

A Koren

$$\Phi = \max\left(0, \min\left(2, \min\left(2 + \frac{r}{3}, 2r\right)\right)\right) \tag{B.1}$$

C Celerities

The bed celerity is computed as:

$$c = \frac{M_{\mathrm{f}} u}{c_{\mathrm{b}} \beta} c^* \; , MorFac * u. * celerities.lb./(1 - porosity)./beta \tag{C.1}$$

where the non-dimensional bed celerity $c*$ [-] is computed as:

$$c^* = \frac{\psi}{1 - \mathrm{Fr}^2} \; , \tag{C.2}$$

$$\psi = \frac{\partial q_{\mathrm{b}}}{\partial u} \; , \tag{C.3}$$

where the derivative is computed with finite differences.