Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

QUESTION

Find the sum of the first 15 multiples of 8.

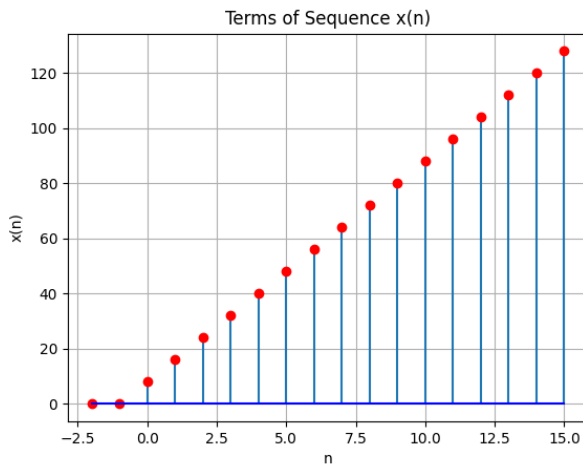**Solution:**

Fig. 1. Plot of x(n) *vs* n

The Z-transform of p(n) is defined as

$$P(Z) = \sum_{n=-\infty}^{\infty} p(n) z^{-n} \tag{1}$$

$$u(n) \xleftrightarrow{Z} U(x) = \frac{1}{(1 - z^{-1})}, \quad |z| > 1 \tag{2}$$

From (1) and (2)

$$U(Z) = \sum_{-\infty}^{\infty} u(n) z^{-n} \tag{3}$$

$$\Rightarrow \frac{d(U(z))}{dz} = -z^{-1} \sum_{n=-\infty}^{\infty} -n u(n) z^{-n} \tag{4}$$

$$\therefore n u(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{5}$$

For an AP,

$$x(n) = [x(0) + nd] u(n) \tag{6}$$

$$x(n) = 8n + 8 \tag{7}$$

$$\Rightarrow X(Z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{8}$$

$$y(n) = x(n) * u(n) \tag{9}$$

$$Y(z) = X(z) U(z) \tag{10}$$

$$Y(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \tag{11}$$

$$n^2 u(n) \xleftrightarrow{Z} \frac{z^{-1} + z^{-2}}{(1 - z^{-1})^3} \tag{12}$$

By performing inverse Z-transform on Y(z)

$$y(n) = x(0)(n+1) u(n) + d\left(\frac{n(n+1)}{2} \right) u(n) \tag{13}$$

$$y(n) = \frac{n+1}{2} (2x(0) + nd) \tag{14}$$

$$y(n) = \frac{15}{2} (16 + 120) \tag{15}$$

$$y(n) = 960 \tag{16}$$

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
d	8	common difference
$y(n)$	960	Sum of n terms
$x(n)$	$(8 + 8n)$	General term of the series
$X(z)$	$8(1 - z^{-1})^{-1} + 8z^{-1}(1 - z^{-1})^{-2}$	Z-transform of x(n)
$u(n)$	$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$	Unit step function
$U(z)$	$(1 - z^{-1})^{-1}$	Z-transform of u(n)

TABLE I
PARAMETER TABLE1