Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

Question

Find the sum of the first 15 multiples of 8.

Solution:

$$8 + 16 + 24 + \ldots + 120 \tag{1}$$

Sum of n terms of an AP is given by

$$S = \frac{n}{2}\left(2x\left(0\right) + \left(n-1\right)d\right) \tag{2}$$

Now,

$$S = \frac{15}{2}\left(2\left(8\right) + \left(15-1\right)\left(8\right)\right) \tag{3}$$

$$\boxed{S = 960} \tag{4}$$

General term $x\left(n\right)$ can be given by

PARAMETER	VALUE	DESCRIPTION
x(0)	8	First term
n	15	Number of terms
d	8	common difference
S	960	Sum of n terms

TABLE I
Parameter Table1

$$x\left(n\right) = \left(8 + 8n\right) \times u\left(n\right) \tag{5}$$

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

Applying Z transform for $u(n)$,

$$U(z) = \sum_{n=-\infty}^{\infty} z^{-n}u(n) \tag{6}$$

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{7}$$

$$= \left(1 - z^{-1}\right)^{-1};\ ROC = |z| > 1 \tag{8}$$

$$\frac{d(U(z))}{dz} = \sum_{n=0}^{\infty} -nz^{-n-1} \tag{9}$$

$$= -z^{-2}(1 - z^{-1})^{-2};\ ROC : |z| > 1 \tag{10}$$

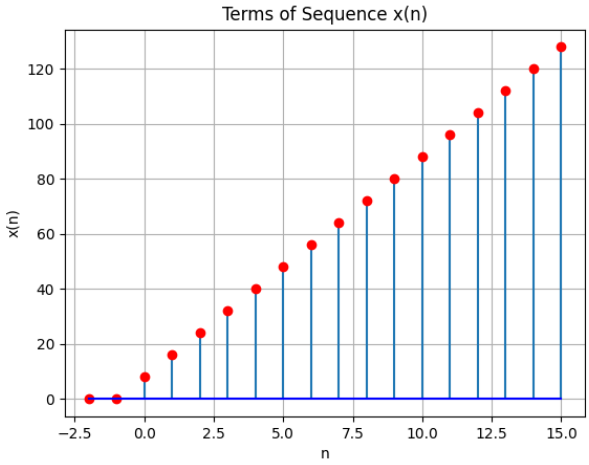

Fig. 1. Plot of x(n) *vs* n

Now, Applying Z transform for $x(n)$,

$$X\left(z\right) = \sum_{n=-\infty}^{\infty} x\left(n\right)z^{-n} \tag{11}$$

(12)

$$\boxed{x\left(n\right) = \left(8 + 8n\right) \times u\left(n\right)} \tag{13}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 8\left(n+1\right) \cdot u\left(n\right)z^{-n} \tag{14}$$

$$= \sum_{n=0}^{\infty} 8\left(n+1\right)z^{-n} \tag{15}$$

$$= 8U(n) + 8\left(-z\frac{d(U(z))}{dz}\right) \tag{16}$$

$$= 8(1 - z^{-1})^{-1} + 8z^{-1}(1 - z^{-1})^{-2};\ ROC : |z| > 1 \tag{17}$$

PARAMETER	VALUE	DESCRIPTION
$x(n)$	$(8 + 8n)$	General term of the series
$X(z)$	$8(1 - z^{-1})^{-1} + 8z^{-1}(1 - z^{-1})^{-2}$	Z-transform of x(n)
$u(n)$		Unit step function
$U(z)$	$(1 - z^{-1})^{-1}$	Z-transform of u(n)

TABLE II
PARAMETER TABLE2