Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

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QUESTION

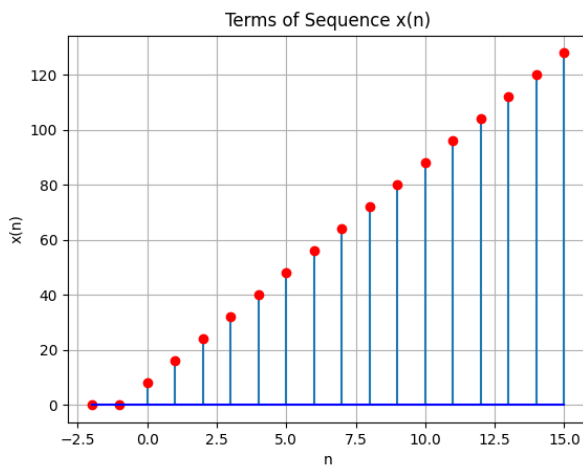Find the sum of the first 15 multiples of 8.
Solution:

Fig. 1. Plot of x(n) *vs* n

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \tag{1}$$

$$\Rightarrow X(z) = \frac{8}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

$$= \frac{8}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z) U(z) \tag{5}$$

$$\Rightarrow Y(z) = \left(\frac{8}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \tag{6}$$

$$= \frac{8}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{7}$$

Using Contour Integration to find the inverse *Z*-

transform,

$$\Rightarrow y(14) = \frac{1}{2\pi j} \oint_C Y(z) \, z^{13} \, dz \tag{8}$$

$$= \frac{1}{2\pi j} \oint_C \frac{8z^{13}}{(1 - z^{-1})^3} \, dz \tag{9}$$

$$= \sum_i R_i \tag{10}$$

We can observe that there only a repeated pole at z=1,

$$\Rightarrow \sum_i R_i = R \tag{11}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 \frac{8z^{16}}{(z - 1)^3} \right) \tag{12}$$

$$= 4 \lim_{z \to 1} \frac{d^2}{dz^2} (z^{16}) \tag{13}$$

$$= 960 \tag{14}$$

$$\therefore \boxed{y(14) = 960} \tag{15}$$

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
d	8	common difference
$x(n)$	$(8 + 8n)$	General term of the series

TABLE I
PARAMETER TABLE1