Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

Question

Find the sum of the first 15 multiples of 8.

Solution:

$$8 + 16 + 24 + \ldots + 120 \tag{1}$$

Sum of n terms of an AP is given by

$$S = \frac{n}{2}\left(2x\left(0\right) + \left(n-1\right)d\right) \tag{2}$$

Now,

$$S = \frac{15}{2}\left(2\left(8\right) + \left(15-1\right)\left(8\right)\right) \tag{3}$$

$$\boxed{S = 960} \tag{4}$$

General term $x\left(n\right)$ can be given by

PARAMETER	VALUE	DESCRIPTION
x(0)	8	First term
n	15	Number of terms
d	8	common difference
S	960	Sum of n terms

TABLE I
Parameter Table1

$$x\left(n\right) = \left(8 + 8n\right) \times u\left(n\right) \tag{5}$$

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

$$u\left(n\right) ZU\left(x\right) \tag{6}$$

$$U(z) = \sum_{n=-\infty}^{\infty} z^{-n} u(n) \tag{7}$$

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{8}$$

$$= z(z-1)^{-1}\,;\; ROC = |z| > 1 \tag{9}$$

$$\frac{d(U(z))}{dz} = \sum_{n=0}^{\infty} -nz^{-n-1} \tag{10}$$

$$= -(z-1)^{-2}\,;\; ROC : |z| > 1 \tag{11}$$

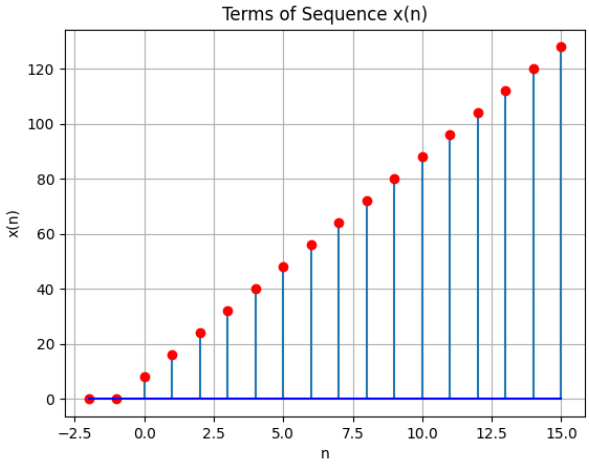

Fig. 1. Plot of x(n) *vs* n

Now,

$$x\left(n\right) ZX\left(x\right) \tag{12}$$

$$X\left(z\right) = \sum_{n=-\infty}^{\infty} x\left(n\right) z^{-n} \tag{13}$$

$$\tag{14}$$

using (5),

$$X(z) = \sum_{n=-\infty}^{\infty} 8\left(n+1\right) \cdot u\left(n\right) z^{-n} \tag{15}$$

$$= \sum_{n=0}^{\infty} 8\left(n+1\right) z^{-n} \tag{16}$$

$$= 8U(n) + 8\left(-z\frac{d(U(z))}{dz}\right) \tag{17}$$

$$= 8z(z-1)^{-1} + 8z(z-1)^{-2}\,;\; ROC : |z| > 1 \tag{18}$$

PARAMETER	VALUE	DESCRIPTION
x(n)	$(8 + 8n)$	General term of the series
X(n)	$8z(z-1)^{-1} + 8z(z-1)^{-2}$	Z-transform of x(n)
u(n)		Unit step function
U(n)	$z(z-1)^{-1}$	Z-transform of u(n)

TABLE II
PARAMETER TABLE2