# Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

## Question

Find the sum of the first 15 multiples of 8.

**Solution:**

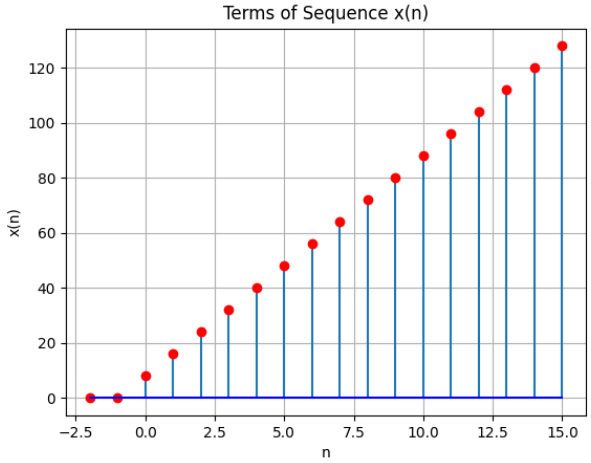


Fig. 1. Plot of x(n) *vs* n

For an AP,

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$\implies X(z) = \frac{8}{1-z^{-1}} + \frac{8z^{-1}}{(1-z^{-1})^2} \tag{2}$$

$$= \frac{8}{(1-z^{-1})^2}, \ |z| > 1 \tag{3}$$

$$y(n) = x(n) * u(n) \tag{4}$$

$$Y(z) = X(z)\,U(z) \tag{5}$$

$$\implies Y(z) = \left(\frac{8}{(1-z^{-1})^2}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{6}$$

$$= \frac{8}{(1-z^{-1})^3}, \ |z| > 1 \tag{7}$$

Using Contour Integration to find the inverse $Z$-transform,

$$\implies y(14) = \frac{1}{2\pi j}\oint_C Y(z)\,z^{13}\,dz \tag{8}$$

$$= \frac{1}{2\pi j}\oint_C \frac{8z^{13}}{(1-z^{-1})^3}\,dz \tag{9}$$

We can observe that there only a repeated pole at z=1,

$$\implies R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{10}$$

$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{8z^{16}}{(z-1)^3}\right) \tag{11}$$

$$= 4\lim_{z\to 1}\frac{d^2}{dz^2}(z^{16}) \tag{12}$$

$$= 960 \tag{13}$$

$$\therefore \boxed{y(14) = 960} \tag{14}$$

| PARAMETER | VALUE | DESCRIPTION |
|---|---|---|
| $x(0)$ | 8 | First term |
| $d$ | 8 | common difference |
| $x(n)$ | $(8+8n)$ | General term of the series |

TABLE I
Parameter Table1