

Assignment 10.5.3 _ 13Q

EE23BTECH11219 - Rada Sai Sujan

QUESTION

Find the sum of the first 15 multiples of 8.

Solution:

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
d	8	common difference
$x(n)$	$[8 + 8n]u(n)$	General term of the series

TABLE I
PARAMETER TABLE I

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{8}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$= \frac{8}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (5)$$

$$Y(z) = \left(\frac{8}{(1 - z^{-1})^2} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (6)$$

$$= \frac{8}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (7)$$

Using Contour Integration to find the inverse Z-transform,

$$y(14) = \frac{1}{2\pi j} \oint_C Y(z) z^{13} dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \frac{8z^{13}}{(1 - z^{-1})^3} dz \quad (9)$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (10)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{8z^{16}}{(z-1)^3} \right) \quad (11)$$

$$= 4 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{16}) \quad (12)$$

$$= 960 \quad (13)$$

$$\therefore \boxed{y(14) = 960} \quad (14)$$

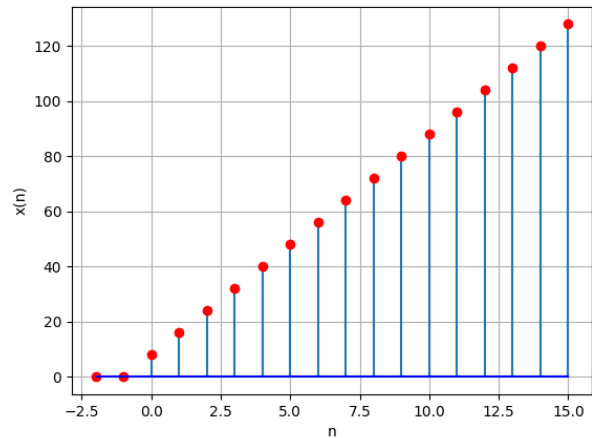


Fig. 1. Plot of $x(n)$ vs n