Assignment 10.5.3_13Q

EE23BTECH11219 - Rada Sai Sujan

Question

Find the sum of the first 15 multiples of 8.
Solution:

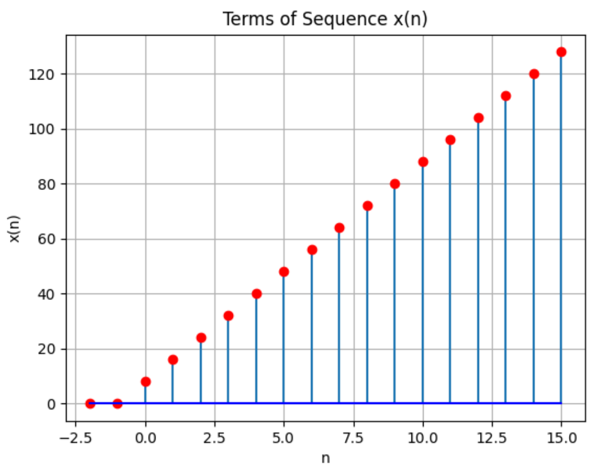

Fig. 1. Plot of x(n) *vs* n

For an AP,

$$x(n) = [x(0) + nd]u(n) \tag{1}$$
$$x(n) = 8n + 8, \quad \forall n \geq 0 \tag{2}$$
$$\Rightarrow X(Z) = \frac{8}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2}, |z| > 1 \tag{3}$$
$$y(n) = x(n) * u(n) \tag{4}$$
$$Y(z) = X(z)\,U(z) \tag{5}$$
$$Y(z) = \left(\frac{8}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{6}$$
$$= \frac{8}{(1 - z^{-1})^2} + \frac{8z^{-1}}{(1 - z^{-1})^3} \tag{7}$$
$$= \frac{8}{(1 - z^{-1})^3}, \ |z| > 1 \tag{8}$$

By using Contour Integration to find the inverse Z-transform,

$$y(n) = \frac{1}{2\pi j}\oint_C Y(Z)\, Z^{n-1}\, dz \tag{9}$$
$$= \frac{1}{2\pi j}\oint_C \frac{8Z^{n-1}}{(1 - z^{-1})^3}\, dz \tag{10}$$
$$= \sum_i R_i \tag{11}$$

We can observe that there only a repeated pole at z=1,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{12}$$
$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{8Z^{n+2}}{(z-1)^3}\right) \tag{13}$$
$$= 4\lim_{z\to 1}\frac{d^2}{dz^2}(Z^{n+2}) \tag{14}$$
$$= 4\,(n+1)\,(n+2)\lim_{z\to 1} z^n \tag{15}$$

$$\Rightarrow y(n) = \sum_i R_i \tag{16}$$
$$= 4\,(n+1)\,(n+2)\,(1)^n \tag{17}$$

$$\therefore y(14) = 960 \tag{18}$$

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
d	8	common difference
$y(n)$	960	Sum of n+1 terms
$x(n)$	$(8+8n)$	General term of the series
$X(z)$	$8(1-z^{-1})^{-1}+$ $8z^{-1}(1-z^{-1})^{-2}$	Z-transform of x(n)
$u(n)$	$u(n)=\begin{cases}1 & \text{if } n\geq 0\\ 0 & \text{if } n<0.\end{cases}$	Unit step function
$U(z)$	$(1-z^{-1})^{-1}$	Z-transform of u(n)
$\sum_i R_i$	$4\,(n+1)\,(n+2)\,(1)^n$	Sum of residues evaluated at poles

TABLE I
PARAMETER TABLE1