

ASSIGNMENT11.15 _ 13Q

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QUESTION:

Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represents (a) travelling wave, (ii) a stationary wave or (iii) none at all:

- (a) $y = 2 \cos(3x) \sin(10t)$
- (b) $y = 2 \sqrt{x - vt}$
- (c) $y = 3 \sin(5x - 0.5t) + 4 \cos(5x - 0.5t)$
- (d) $y = \cos x \sin t + \cos 2x \sin 2t$

SOLUTION:

TRAVELLING WAVE:

Travelling waves are the kind of waves where the position of the particles shifts as the wave advances. The wave's maximum and minimum amplitudes pass across the medium, and its constituent particles oscillate in accordance with the wave's progression. Progressive waves are another name for moving waves. A travelling wave is a wave that goes through a medium while being disturbed. It moves energy from one location to another.

The general equation of a travelling wave is,
 $y(x, t) = A \sin(kx \pm \omega t)$

Here, the amplitude of the wave is A , it's angular velocity is ω and it's position is x and it's wavenumber is k .

STATIONARY WAVE:

The wave formed due to the superposition of two identical waves traveling with the same speed in opposite directions is called a stationary wave or standing wave. In stationary waves the resultant harmonic disturbance of the particles doesn't travel in any direction and there is no transport of energy.

The general equation of a stationary wave is,

$$y(x, t) = A \sin kx \cos \omega t$$

Here, the amplitude of the wave is A , it's angular velocity is ω and it's position is x and it's wavenumber is k .

- (a) The given equation is: $y = 2 \cos(3x) \sin(10t)$

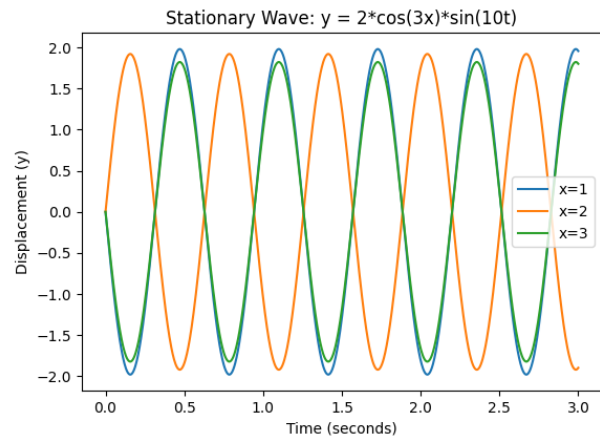
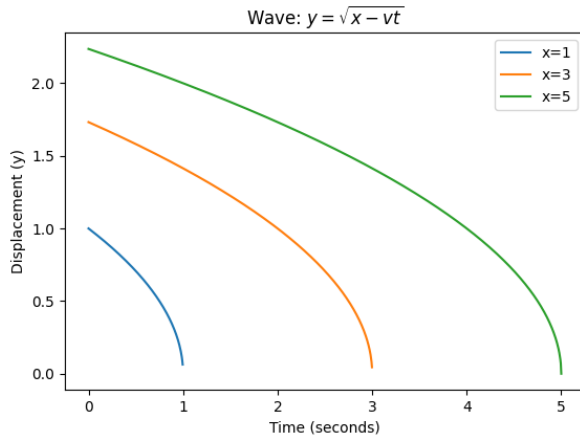
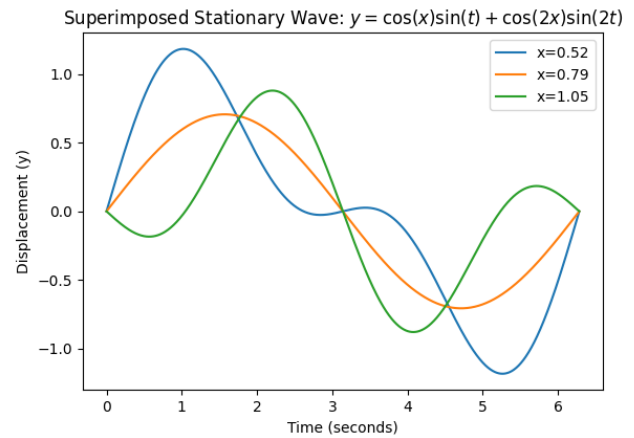


Fig. 0. DIPLACEMENT vs TIME-graph1

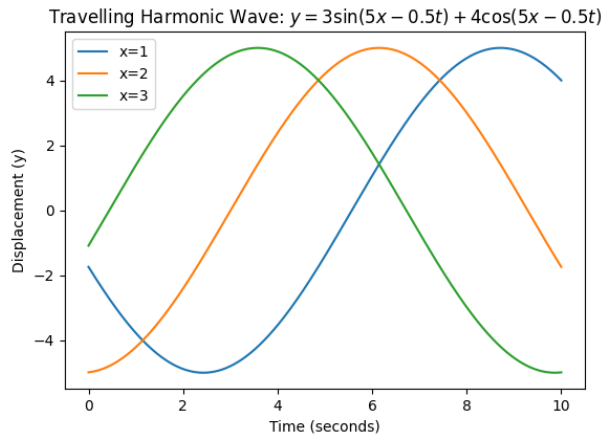
As the given equation is similar to the standard standing wave equation, so the given wave is a stationary wave. Similarly, we can observe the nodes and antinodes in the graph with fixed spatial pattern and different amplitude peaks at various positions of x maintaining symmetry with axis. This shows that the graph is stationary or a standing wave.

- (b) The given equation is: $y = 2 \sqrt{x - vt}$

The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave. Similarly, we can observe the graph and conclude that the given equation is not a wave as there is no periodic oscillation and proper wave shape.

Fig. 0. DIPLACEMENT ν s TIME-graph2Fig. 0. DIPLACEMENT ν s TIME-graph3

(c) The given equation is: $y = 3 \sin(5x - 0.5t) + 4 \cos(5x - 0.5t)$

Fig. 0. DIPLACEMENT ν s TIME-graph3

As the given equation is similar to the standard harmonic wave equation, so the provided wave is a travelling harmonic wave. Similarly, we can observe the graph having exhibiting periodic oscillations with equal amplitude and proper sinusoidal wave shape uniformly. Thus, we can conclude that is a travelling wave.

(d) The given equation is: $y = \cos x \sin t + \cos 2x \sin 2t$

The given equation represents a stationary wave because the harmonic terms kx and ωt appear separately in the equation. This equation actually represents the superposition of two stationary waves. Similarly, we can observe fixed spatial pattern but with multiple frequencies. The graph even shows interference patterns having uniformity by which we can say it is a superimposed stationary wave equation.