ASSIGNMENT11.15 _ 13Q

EE22BTECH11219 - Sai Sujan Rada

QUESTION:

Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represents (a) travelling wave, (ii) a stationary wave or (iii) none at all:

(a) $y = 2\cos(3x)\sin(10t)$

(b) $y = 2\sqrt{x - vt}$

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

(d) $y = \cos x \sin t + \cos 2x \sin 2t$

SOLUTION:

TRAVELLING WAVE	STATIONARY WAVE
$y(x,t) = A\sin(kx \pm \omega t)$	$y(x,t) = A\sin kx \cos \omega t$

TABLE I
TRAVELLING WAVE vs STATIONARY WAVE

PARAMETERS	DEFINITION
A	Amplitude
ω	Angular Velocity
x	Position
k	Wavenumber

TABLE II
PARAMETER MEANINGS

Let us assume an equation:

$$A(x)\cos(\omega t + \phi(x))$$

CONDITION TO REPRESENT A STATIONARY WAVE:
$A(x)$ should be a function of position x, and it can be expressed as $A(x) = A_0 cos(\omega t + \alpha)$ where A_0 is a constant, k is the wavenumber, x is the position and α is a phase constant.
$\phi(x)$ can be expressed as $\phi(x) = c$ where c is a constant.

CONDITION TO REPRESENT A TRAVELLING WAVE:
$A(x)$ should be a constant, and it can be expressed as $A(x) = A_0$ where A_0 is a constant number.
$\phi(x)$ represents a linear expression in x, and it can be expressed as $\phi(x) = kx + \theta$ where k is the wavenumber and θ is the phaseconstant.

(a) The given equation is: $y = 2\cos(3x)\sin(10t)$

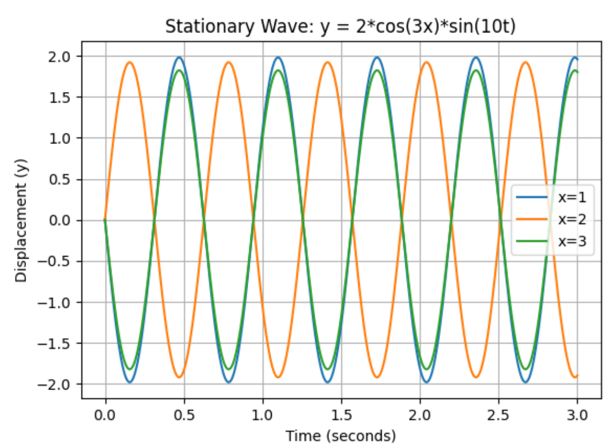

Fig. 1. DIPLACEMENT vs TIME-graph1

We can observe the nodes and antinodes in the graph with fixed spatial pattern and different amplitude peaks at various positions of x maintaining symmetry with axis.This shows that the graph is stationary or a standing wave.

(b) The given equation is: $y = 2\sqrt{x - vt}$

We can observe the graph and conclude that the given equation is not a wave as there is no periodic oscillation and proper wave shape.

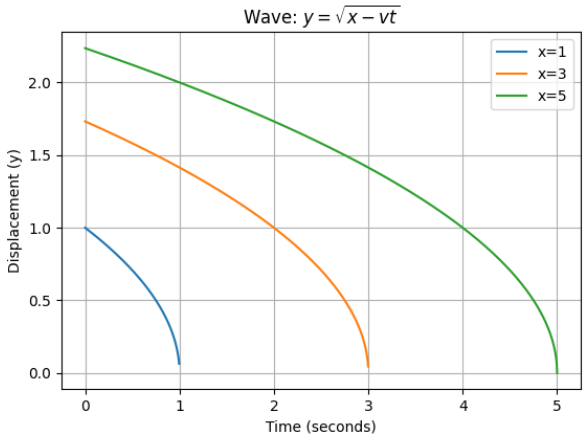

Fig. 2. DIPLACEMENT *vs* TIME-graph2

(c) The given equation is: $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$

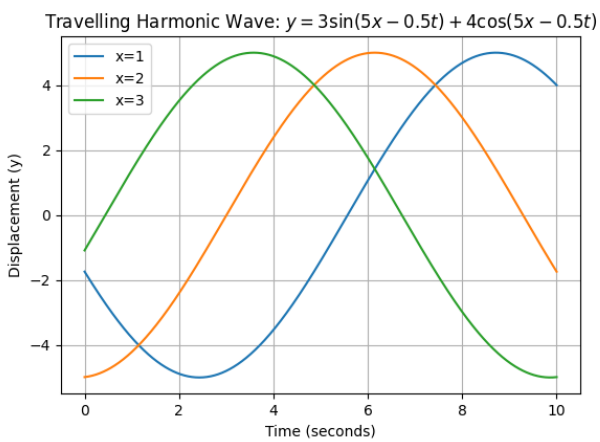

Fig. 3. DIPLACEMENT *vs* TIME-graph3

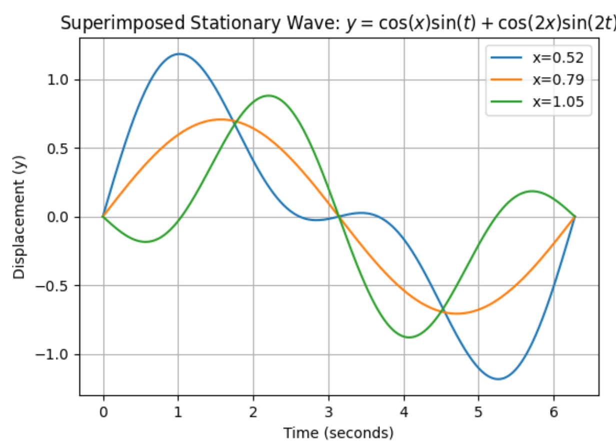

Fig. 4. DIPLACEMENT *vs* TIME-graph3

We can observe the graph having exhibiting periodic oscilltions with equal amplitude and proper sinusoidal wave shape uniformly. Thus, we can conclude that is a travelling wave.

(d) The given equation is: $y = \cos x \sin t + \cos 2x \sin 2t$

We can observe fixed spatial patter but with multiple frequencies. The graph even shows interference patterns having uniformity by which we can say it is a superimposed stationary wave equation.