# Assignment 11.9.5_1Q

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## Question

Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an $A.P.$, is equal to twice the $m^{th}$ terms.
**Solution:**

| PARAMETER | VALUE | DESCRIPTION |
|---|---|---|
| $x(0)$ | $x(0)$ | First term |
| $d$ | 8 | common difference |
| $x(n)$ | $[x(0)+nd]u(n)$ | General term of the series |

TABLE I
Parameter Table1

For an $AP$,

$$X(z) = \frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2}, \quad |z| > 1 \tag{1}$$

Using Contour Integration to find the inverse $Z$-transform,

$$x(n) = \frac{1}{2\pi j}\oint_C X(z)\, z^{n-1}\, dz \tag{2}$$

$$= \frac{1}{2\pi j}\oint_C \left(\frac{x(0)\, z^{n-2}}{1-z^{-1}} + \frac{dz^{n-1}}{(1-z^{-1})^2}\right) dz \tag{3}$$

$$= R_1 + R_2 \tag{4}$$

We can observe a non-repeated pole at $z = 1$,

$$R_1 = (z-1)\left(\frac{x(0)\, z}{z-1}\right)\bigg|_{z=1} \tag{5}$$

$$= x(0) \tag{6}$$

Similarly, we can observe that a pole is repeated 2 times at $z = 1$ and thus $m = 2$,

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{7}$$

$$\implies R_2 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}(z-1)^2\left(\frac{dz^n}{(z-1)^2}\right) \tag{8}$$

$$= \lim_{z\to 1}\frac{d}{dz}\left(dz^n\right) \tag{9}$$

$$= nd \tag{10}$$

$$\therefore x(n) = [x(0) + nd]u(n) \tag{11}$$

Now, calculating the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms,

$$x(m+n) + x(m-n) = [x(0) + (m+n)\, d] + [x(0) + (m - \tag{12}$$

$$= 2[x(0) + md] \tag{13}$$

$$\implies x(m+n) + x(m-n) = 2x(m) \tag{14}$$