GATE: EE - 47.2022

EE22BTECH11219 - Rada Sai Sujan

Question

Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

The output of the system is

(a) $2\sin(10\pi t) + 5\cos(15\pi t)$
(b) $2\sin(10\pi t) + 4\cos(45\pi t)$
(c) $7\sin(42\pi t) + 4\cos(45\pi t)$
(d) $5\sin(15\pi t) + 7\cos(42\pi t)$

Solution:

Given,

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t) \tag{1}$$

$$= \frac{\sin 50\pi t}{\pi t} - \frac{\sin 30\pi t}{\pi t} \tag{2}$$

$$= h_1(t) - h_2(t) \tag{3}$$

where,

$$h_1(t) = \frac{\sin 50\pi t}{\pi t} \tag{4}$$

$$h_2(t) = \frac{\sin 30\pi t}{\pi t} \tag{5}$$

Taking Fourier transform of $h(t)$

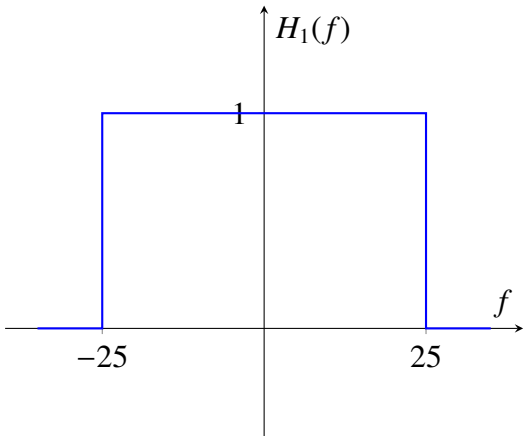
$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H_1(f) - H_2(f) \tag{6}$$

where,

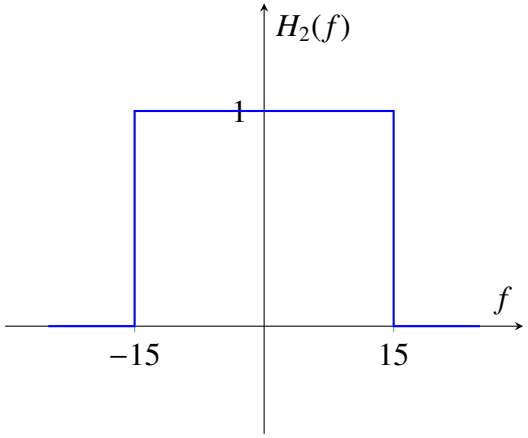
$$h_1(t) \overset{\mathcal{F}}{\longleftrightarrow} H_1(f) \tag{7}$$

$$h_2(t) \overset{\mathcal{F}}{\longleftrightarrow} H_2(f) \tag{8}$$

Plotting $H_1(f)$ and $H_2(f)$ we get,

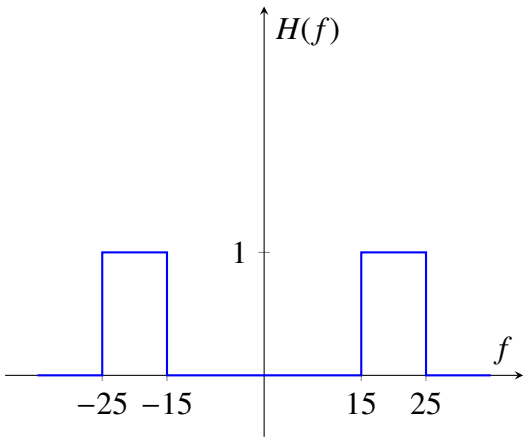


(9)



(10)

Plotting $H(f)$ from (9) and (10)



(11)

Therefore, the given system is a Bandpass filter with passband:

$$15 \leq |f| \leq 25 \tag{12}$$

Veryfying Table I with (12), only f_3 and f_4 will be passed through the system.

Frequency components of input	Value
f_1	$\frac{10\pi}{2\pi} = 5Hz$
f_2	$\frac{15\pi}{2\pi} = 7.5Hz$
f_3	$\frac{42\pi}{2\pi} = 21Hz$
f_4	$\frac{45\pi}{2\pi} = 22.5Hz$

TABLE I
FREQUENCY COMPONENTS

$$\therefore y(t) = 7\sin(42\pi t) + 4\cos(45\pi t) \tag{13}$$

($\because |H(f)| = 1$, the amplitude of frequency components will be unchanged.)