

GATE: EE - 47.2022

EE22BTECH11219 - Rada Sai Sujan

QUESTION

Let an input $x(t) = 2 \sin(10\pi t) + 5 \cos(15\pi t) + 7 \sin(42\pi t) + 4 \cos(45\pi t)$ is passed through an LTI system having an impulse response

$$h(t) = 2 \left(\frac{\sin(10\pi t)}{\pi t} \right) \cos(40\pi t)$$

The output of the system is

- (a) $2 \sin(10\pi t) + 5 \cos(15\pi t)$
- (b) $2 \sin(10\pi t) + 4 \cos(45\pi t)$
- (c) $7 \sin(42\pi t) + 4 \cos(45\pi t)$
- (d) $5 \sin(15\pi t) + 7 \cos(42\pi t)$

Solution:

Frequency components of input	Value
f_1	$\frac{10\pi}{2\pi} = 5\text{Hz}$
f_2	$\frac{15\pi}{2\pi} = 7.5\text{Hz}$
f_3	$\frac{42\pi}{2\pi} = 21\text{Hz}$
f_4	$\frac{45\pi}{2\pi} = 22.5\text{Hz}$

TABLE I
FREQUENCY COMPONENTS

Given,

$$h(t) = 2 \left(\frac{\sin(10\pi t)}{\pi t} \right) \cos(40\pi t) \quad (1)$$

$$= \frac{\sin 50\pi t}{\pi t} - \frac{\sin 30\pi t}{\pi t} \quad (2)$$

$$= h_1(t) - h_2(t) \quad (3)$$

where,

$$h_1(t) = \frac{\sin 50\pi t}{\pi t} \quad (4)$$

$$h_2(t) = \frac{\sin 30\pi t}{\pi t} \quad (5)$$

Taking Fourier transform of $h(t)$

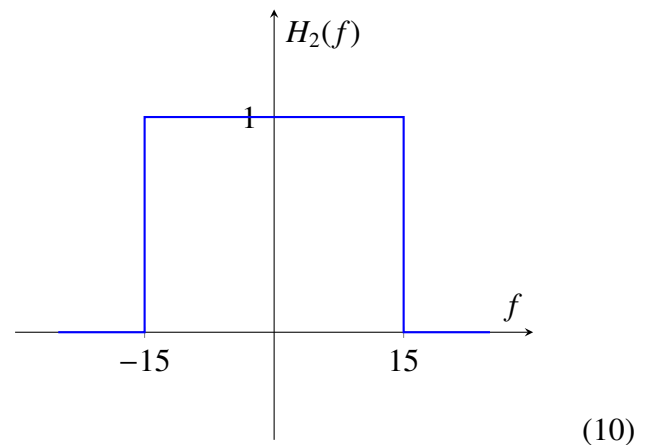
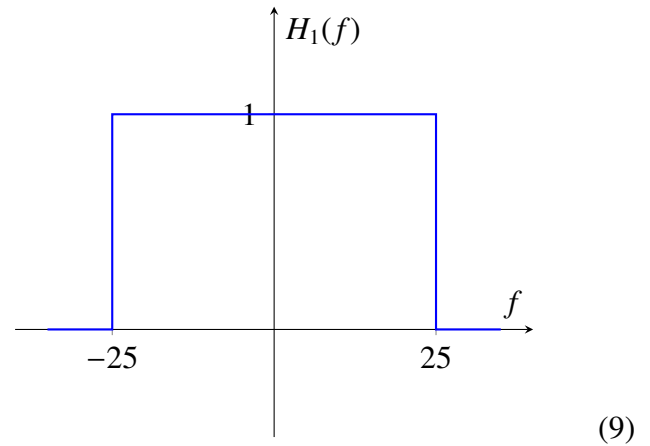
$$h(t) \xleftrightarrow{\mathcal{F}} H_1(f) - H_2(f) \quad (6)$$

where,

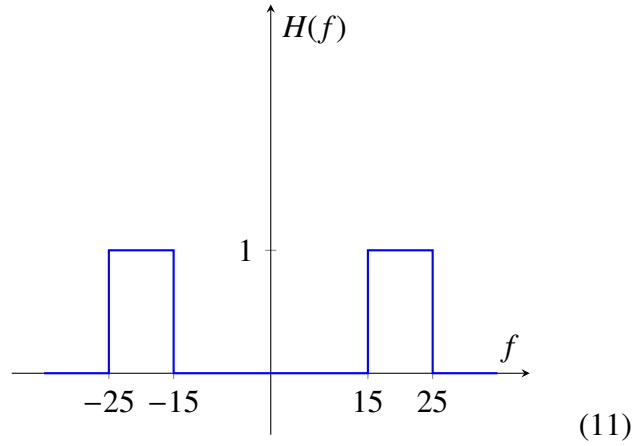
$$h_1(t) \xleftrightarrow{\mathcal{F}} H_1(f) \quad (7)$$

$$h_2(t) \xleftrightarrow{\mathcal{F}} H_2(f) \quad (8)$$

Plotting $H_1(f)$ and $H_2(f)$ we get,



Plotting $H(f)$ from (9) and (10)



Therefore, the given system is a Bandpass filter with passband:

$$15 \leq |f| \leq 25 \quad (12)$$

Verifying Table I with (12), only f_3 and f_4 will be passed through the system.

$$\therefore y(t) = 7 \sin(42\pi t) + 4 \cos(45\pi t) \quad (13)$$

($\because |H(f)| = 1$, the amplitude of frequency components will be unchanged.)