GATE: EE - 7.2021

EE22BTECH11219 - Rada Sai Sujan

Question

Two discrete-time linear time-invarient systems with impulse responses $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascaded system is

(a) $\delta[n-2] + \delta[n+1]$
(b) $\delta[n-1]\delta[n] + \delta[n+1]\delta[n-1]$
(c) $\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$
(d) $\delta[n]\delta[n-1] + \delta[n-2]\delta[n+1]$

(GATE 2021 EE)

Solution:
From the Z-transformation pairs,

$$\delta[n] \overset{Z}{\longleftrightarrow} 1 \tag{1}$$

$$x(n-k) \overset{Z}{\longleftrightarrow} z^{-k}X(z) \tag{2}$$

$$x_1(n) * x_2(n) \overset{Z}{\longleftrightarrow} X_1(z)\,X_2(z) \tag{3}$$

If $h_1(n)$ and $h_2(n)$ are cascade connected then the resultant impulse can be given by:

$$h(n) = h_1(n) * h_2(n) \tag{4}$$

$$\implies H(z) = H_1(z)\,H_2(z) \tag{5}$$

$$H(z) = \left(z^{-1} + z\right)\left(1 + z^{-1}\right) \tag{6}$$

$$= \left(z^{-1} + z^{-2} + z + 1\right), \quad |z| \neq 0 \tag{7}$$

Using the Z-transformation pairs to find the the inverse Z-transform,

$$h(n) = \delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1] \tag{8}$$

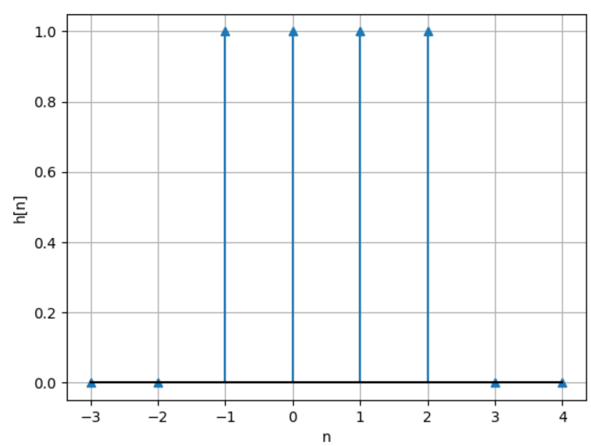

Fig. 1. h(n) vs n graph