

GATE: ME - 14.2022

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QUESTION

The fourier series expansion of x^3 in the interval $-1 \leq x \leq 1$ with periodic continuation has

- (a) only sine terms
- (b) only cosine terms
- (c) both sine and cosine terms
- (d) only sine terms and a non-zero constant

(GATE 2022 ME)

Solution:

Fourier series expansion of the function $x(t)$ in the interval $[-L, L]$ can be given by:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{L}\right) \quad (1)$$

where,

$$a_0 = \frac{1}{2L} \int_{-L}^L f(t) dt \quad (2)$$

$$a_n = \frac{1}{2L} \int_{-L}^L f(t) \cos\left(\frac{n\pi t}{L}\right) dt \quad (3)$$

$$b_n = \frac{1}{2L} \int_{-L}^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt \quad (4)$$

(5)

Therefore, the expansion can be given by:

$$t^3 = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi t) + \sum_{n=1}^{\infty} b_n \sin(n\pi t) \quad (6)$$

Since t^3 is an odd function,

$$a_0 = a_n = 0 \quad (7)$$

$$b_n = \frac{1}{2} \int_{-1}^1 t^3 \sin(n\pi t) dt \quad (8)$$

$$= (-1)^{n+1} \left(\frac{2}{n\pi} - \frac{12}{(n\pi)^3} \right) \quad (9)$$

$$\Rightarrow t^3 = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{L}\right) \quad (10)$$

\therefore It contains only sine terms.