GATE: ME - 14.2022

EE22BTECH11219 - Rada Sai Sujan

Appendix

Complex Fourier series:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nft} \tag{1}$$

where c_n is the exponential fourier coefficient.

$$c_n = \frac{1}{T}\int_0^T x(t)e^{-j2\pi nft}\,dt \tag{2}$$

where T is the time period of function $x(t)$.

Trignometric fourier series:

$$e^{j2\pi nft} = cos(2\pi nft) + jsin(2\pi nft) \tag{3}$$

Substituting (3) in (1)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n\left(\cos(2\pi nft) + j\sin(2\pi nft)\right) \tag{4}$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n\cos(2\pi nft)) + (b_n\sin(2\pi nft)) \tag{5}$$

where a_0, a_n and b_n are trigonometric fourier series.

$$a_0 = c_0 \tag{6}$$

$$= \frac{1}{T}\int_0^T x(t)\,dt \tag{7}$$

$$a_n = 2Re(c_n) \tag{8}$$

$$= \frac{2}{T}\int_0^T x(t)\cos(2\pi nft)\,dt \tag{9}$$

$$b_n = -2Im(c_n) \tag{10}$$

$$= \frac{2}{T}\int_0^T x(t)\sin(2\pi nft)\,dt \tag{11}$$

Therefore, Fourier series expansion of the function $x(t)$ in the interval $[-L, L]$ can be given by:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} b_n\sin\left(\frac{n\pi t}{L}\right) \tag{12}$$

where,

$$f = \frac{1}{2L} \tag{13}$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(t)\;dt \tag{14}$$

$$a_n = \frac{1}{2L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right)\,dt \tag{15}$$

$$b_n = \frac{1}{2L}\int_{-L}^{L} f(t)\sin\left(\frac{n\pi t}{L}\right)\,dt \tag{16}$$

$$\tag{17}$$