GATE: ME - 14.2022

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Question

The fourier series expansion of x^3 in the interval $-1 \leq x \leq 1$with periodic continuation has

(a) only sine terms
(b) only cosine terms
(c) both sine and cosine terms
(d) only sine terms and a non-zero constant

(GATE 2022 ME)

Solution:

Fourier series expansion of the function $f(x)$ in the interval $[-L, L]$ can be given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \tag{1}$$

where,

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\, dx \tag{2}$$

$$a_n = \frac{1}{2L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right) dx \tag{3}$$

$$b_n = \frac{1}{2L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right) dx \tag{4}$$

$$\tag{5}$$

Therefore, the expansion can be given by:

$$x^3 = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + \sum_{n=1}^{\infty} b_n \sin(n\pi x) \tag{6}$$

Since x^3 is an odd function,

$$a_0 = a_n = 0 \tag{7}$$

$$b_n = \frac{1}{2}\int_{-1}^{1} x^3 \sin(n\pi x)\, dx \tag{8}$$

$$= (-1)^{n+1}\left(\frac{2}{n\pi} - \frac{12}{(n\pi)^3}\right) \tag{9}$$

$$\implies x^3 = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \tag{10}$$

$\therefore$It contains only sine terms.