

# GATE: ME - 14.2022

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## QUESTION

The fourier series expansion of  $x^3$  in the interval  $-1 \leq x \leq 1$  with periodic continuation has

- (a) only sine terms
- (b) only cosine terms
- (c) both sine and cosine terms
- (d) only sine terms and a non-zero constant

(GATE 2022 ME)

## Solution:

Fourier series expansion of the function  $x(t)$  in the interval  $[-L, L]$  can be given by:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{L}\right) \quad (1)$$

where,

$$a_0 = \frac{1}{2L} \int_{-L}^L f(t) dt \quad (2)$$

$$a_n = \frac{1}{2L} \int_{-L}^L f(t) \cos\left(\frac{n\pi t}{L}\right) dt \quad (3)$$

$$b_n = \frac{1}{2L} \int_{-L}^L f(t) \sin\left(\frac{n\pi t}{L}\right) dt \quad (4)$$

(5)

Therefore, the expansion can be given by:

$$x^3 = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + \sum_{n=1}^{\infty} b_n \sin(n\pi x) \quad (6)$$

Since  $x^3$  is an odd function,

$$a_0 = a_n = 0 \quad (7)$$

$$b_n = \frac{1}{2} \int_{-1}^1 x^3 \sin(n\pi x) dx \quad (8)$$

$$= (-1)^{n+1} \left( \frac{2}{n\pi} - \frac{12}{(n\pi)^3} \right) \quad (9)$$

$$\Rightarrow x^3 = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right) \quad (10)$$

$\therefore$  It contains only sine terms.