

# GATE: EE - 47.2022

EE22BTECH11219 - Rada Sai Sujan

## QUESTION

Let an input  $x(t) = 2 \sin(10\pi t) + 5 \cos(15\pi t) + 7 \sin(42\pi t) + 4 \cos(45\pi t)$  is passed through an LTI system having an impulse response

$$h(t) = 2 \left( \frac{\sin(10\pi t)}{\pi t} \right) \cos(40\pi t)$$

The output of the system is

- (a)  $2 \sin(10\pi t) + 5 \cos(15\pi t)$
- (b)  $2 \sin(10\pi t) + 4 \cos(45\pi t)$
- (c)  $7 \sin(42\pi t) + 4 \cos(45\pi t)$
- (d)  $5 \sin(15\pi t) + 7 \cos(42\pi t)$

**Solution:**

Frequency components of input	Value
$f_1$	$\frac{10\pi}{2\pi} = 5\text{Hz}$
$f_2$	$\frac{15\pi}{2\pi} = 7.5\text{Hz}$
$f_3$	$\frac{42\pi}{2\pi} = 21\text{Hz}$
$f_4$	$\frac{45\pi}{2\pi} = 22.5\text{Hz}$

TABLE I  
FREQUENCY COMPONENTS

Given,

$$h(t) = 2 \left( \frac{\sin(10\pi t)}{\pi t} \right) \cos(40\pi t) \quad (1)$$

$$= \frac{\sin 50\pi t}{\pi t} - \frac{\sin 30\pi t}{\pi t} \quad (2)$$

$$= h_1(t) - h_2(t) \quad (3)$$

where,

$$h_1(t) = \frac{\sin 50\pi t}{\pi t} \quad (4)$$

$$h_2(t) = \frac{\sin 30\pi t}{\pi t} \quad (5)$$

Taking Fourier transform of  $h(t)$

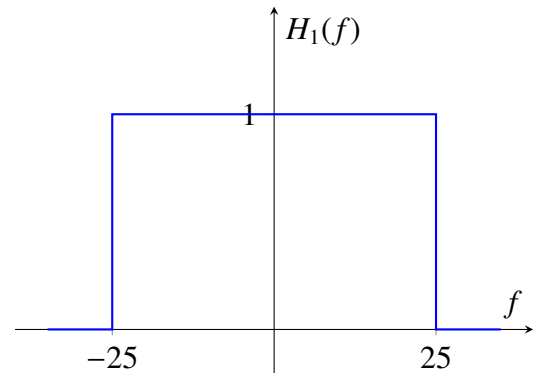
$$h(t) \xrightarrow{\mathcal{F}} H_1(f) - H_2(f) \quad (6)$$

where,

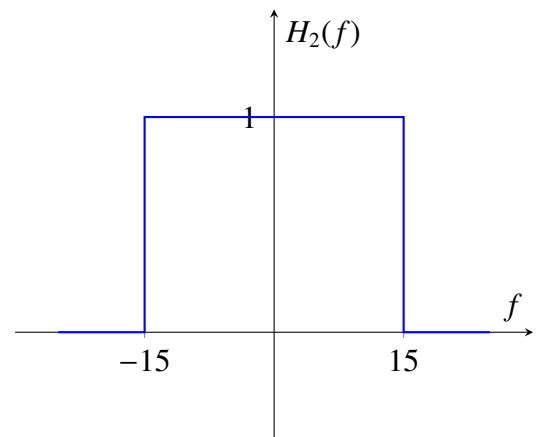
$$h_1(t) \xrightarrow{\mathcal{F}} H_1(f) \quad (7)$$

$$h_2(t) \xrightarrow{\mathcal{F}} H_2(f) \quad (8)$$

Plotting  $H_1(f)$  and  $H_2(f)$  we get,

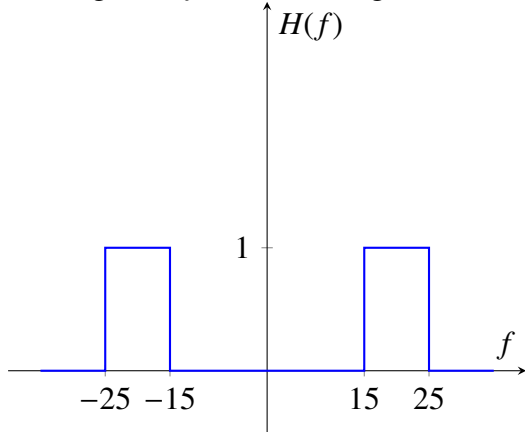


(9)



(10)

Plotting  $H(f)$  from Fig. 9 and Fig. 10



Therefore,  
the given system is a Bandpass filter with passband:

$$15 \leq |f| \leq 25 \quad (11)$$

Verifying Table I with (11), only  $f_3$  and  $f_4$  will be passed through the system.

$$\therefore y(t) = 7 \sin(42\pi t) + 4 \cos(45\pi t) \quad (12)$$

( $\because |H(f)| = 1$ , the amplitude of frequency components will be unchanged.)