

# Assignment 10.5.3\_13Q

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# Question

Find the sum of the first 15 multiples of 8.

# Solution: Theory

PARAMETER	VALUE	DESCRIPTION
$x(0)$	8	First term
$d$	8	common difference
$x(n)$	$[8 + 8n]u(n)$	General term of the series

Table: Parameter Table1

# Theory

For an AP,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \quad (1)$$

$$\Rightarrow X(z) = \frac{8}{1 - z^{-1}} + \frac{8z^{-1}}{(1 - z^{-1})^2} \quad (2)$$

$$= \frac{8}{(1 - z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$y(n) = x(n) * u(n) \quad (4)$$

$$\Rightarrow Y(z) = X(z) U(z) \quad (5)$$

$$Y(z) = \left( \frac{8}{(1 - z^{-1})^2} \right) \left( \frac{1}{1 - z^{-1}} \right) \quad (6)$$

$$= \frac{8}{(1 - z^{-1})^3}, \quad |z| > 1 \quad (7)$$

Using Contour Integration to find the inverse Z-transform,

$$y(14) = \frac{1}{2\pi j} \oint_C Y(z) z^{13} dz \quad (8)$$

$$= \frac{1}{2\pi j} \oint_C \frac{8z^{13}}{(1 - z^{-1})^3} dz \quad (9)$$

We can observe that the pole is repeated 3 times and thus  $m = 3$ ,

$$R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (10)$$

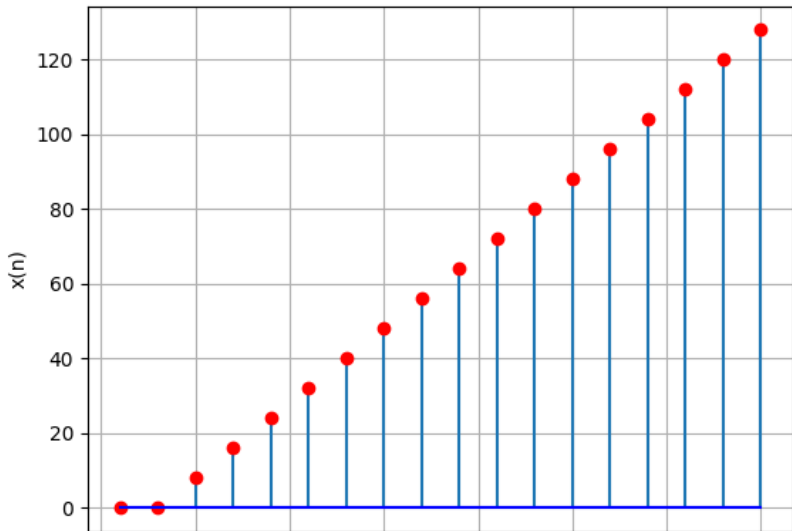
$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left( (z-1)^3 \frac{8z^{16}}{(z-1)^3} \right) \quad (11)$$

$$= 4 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{16}) \quad (12)$$

$$= 960 \quad (13)$$

$$\therefore \boxed{y(14) = 960} \quad (14)$$

# Graph



# Code

```
1 #include <stdio.h>
2 int main() {
3     FILE *file = fopen("data.txt", "w");
4     if (file == NULL) {
5         printf("Error opening file!\n");
6         return 1; }
7     int a = 8;
8     int d = 8;
9     int start_n = -2;
10    int end_n = 15;
11    fprintf(file, "%d %d\n", a, d);
12    for (int n = start_n; n <= end_n; ++n) {
13        int x_n = (a + d * n) > 0 ? (a + d * n) : 0;
14        fprintf(file, "%d %d\n", n, x_n);
15    }
16    fclose(file);
17    return 0;
18 }
```



# Code

```
1 import matplotlib.pyplot as plt
2 import numpy as np
3
4 # Read a and d values from the file
5 a, d = np.loadtxt("data.txt", max_rows=1, dtype=int)
6
7 # Read data (excluding the first line with a and d values)
8 data = np.loadtxt("data.txt", skiprows=1)
9 n_values, ap_values = data[:, 0], data[:, 1]
10
11 # Calculate the sum using NumPy vectorized approach
12 sum_x_n = np.sum(np.maximum(0, a + d * n_values))
13
14 # Plot the data
15 plt.stem(n_values, ap_values, basefmt='b-', linefmt='d-', markerfmt='ro')
16 plt.xlabel('n')
17 plt.ylabel('x(n)')
18 plt.grid(True)
19 plt.savefig('a.png')
20
```