# GATE: EE - 47.2022

EE22BTECH11219 - Rada Sai Sujan

## Question

Let an input $x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t)$ is passed through an LTI system having an impulse response

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t)$$

The output of the system is

(a) $2\sin(10\pi t) + 5\cos(15\pi t)$
(b) $2\sin(10\pi t) + 4\cos(45\pi t)$
(c) $7\sin(42\pi t) + 4\cos(45\pi t)$
(d) $5\sin(15\pi t) + 7\cos(42\pi t)$

**Solution:**

Given,

$$h(t) = 2\left(\frac{\sin(10\pi t)}{\pi t}\right)\cos(40\pi t) \tag{1}$$

$$= \frac{\sin 50\pi t}{\pi t} - \frac{\sin 30\pi t}{\pi t} \tag{2}$$

$$= h_1(t) - h_2(t) \tag{3}$$

Taking Fourier transform of $h(t)$

$$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(f) \tag{4}$$

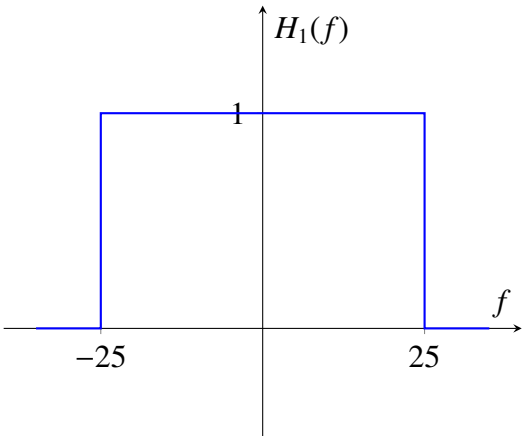
$$\implies h(t) \overset{\mathcal{F}}{\longleftrightarrow} H_1(f) - H_2(f) \tag{5}$$

where,

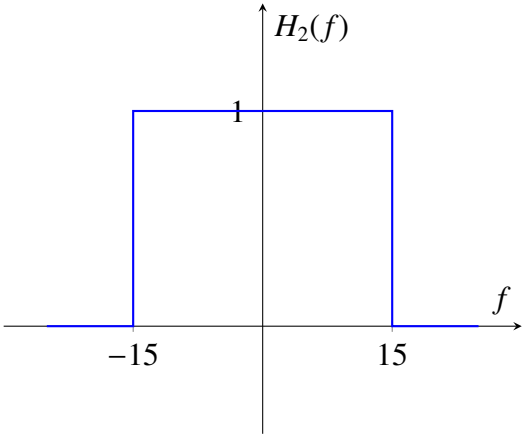
$$h_1(t) \overset{\mathcal{F}}{\longleftrightarrow} H_1(f) \tag{6}$$

$$h_2(t) \overset{\mathcal{F}}{\longleftrightarrow} H_2(f) \tag{7}$$

Plotting $H_1(f)$ and $H_2(f)$ we get,

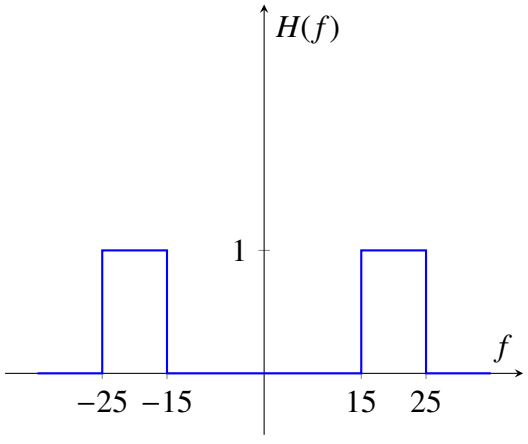



(8)




(9)

Plotting $H(f)$ from (8) and (9)




(10)

Therefore, the given system is a Bandpass filter with passband:

$$15 \leq |f| \leq 25 \tag{11}$$

Given input impulse signal is:

$$x(t) = 2\sin(10\pi t) + 5\cos(15\pi t) + 7\sin(42\pi t) + 4\cos(45\pi t) \tag{12}$$

Veryfying Table I with (11), only $f_3$ and $f_4$ will

| Frequency components of input | Value |
|---|---|
| $f_1$ | $5Hz$ |
| $f_2$ | $7.5Hz$ |
| $f_3$ | $21Hz$ |
| $f_4$ | $22.5Hz$ |

TABLE I
FREQUENCY COMPONENTS

be passed through the system.

$$\therefore y(t) = 7\sin(42\pi t) + 4\cos(45\pi t) \tag{13}$$

($\because |H(f)| = 1$, the amplitude of frequency components will be unchanged.)