GATE: CH - 45.2023

EE22BTECH11219 - Rada Sai Sujan

March 4, 2024

Question

Level (h) in a steam boiler is controlled by manipulating the flow rate (F) of the break-up(fresh) water using a proportional (P) controller. The transfer function between the output and the manipulated input is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}$$

The measurement and the valve transfer functions are both equal to 1. A process engineer wants to tune the controller so that the closed loop response gives the decaying oscillations under the servo mode. Which one of the following is the CORRECT value of the controller gain to be used by the engineer?

Question

1. 0.25
2. 2
3. 4
4. 6

Solution: Theory

PARAMETER	DESCRIPTION
G_c	Proportional controller's transfer function
G_f	Valve transfer function
G_p	Process transfer function
G_M	Measurement transfer function
$G(s)$	Open loop transfer function
$T(s)$	Transfer function of system

Table: PARAMETER TABLE 1

Block Diagram

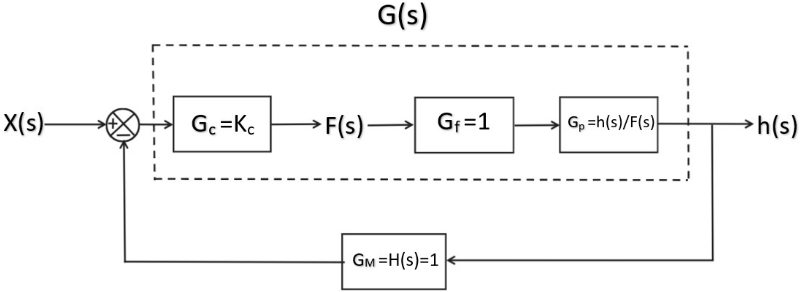


Figure: Closed loop Block diagram

Theory

Closed loop signal transfer function of the above block diagram can be given by,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{1}$$

Theory

From Fig. 1 and Table 1 for a unit impulse, $X(s) = 1$

$$h(s) = T(s) \times X(s) \tag{2}$$

$$h(s) = \frac{(1-s)K_c}{8s^2 + (4-K_c)s + K_c} \tag{3}$$

$$\implies h(s) = \frac{(1-s)K_c}{8(s-s_1)(s-s_2)} \tag{4}$$

Theory

Where,

$$s_1 = \frac{(K_c - 4)}{16} + \sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}} \tag{5}$$

$$s_2 = \frac{(K_c - 4)}{16} - \sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}} \tag{6}$$

From (4) we get,

$$h(s) = \frac{K_c}{8(s_1 - s_2)}\left(\frac{1 - s_1}{s - s_1} - \frac{1 - s_2}{s - s_2}\right) \tag{7}$$

Theory

Now taking the inverse laplace transform we have,

$$h(t) = \frac{K_c}{8(s_1 - s_2)} \left[(1 - s_1) e^{s_1 t} - (1 - s_2) e^{s_2 t} \right] u(t) \tag{8}$$

$$\implies h(t) = e^{\frac{K_c - 4}{16}} \left(A_1 e^{\sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}}} - A_2 e^{-\sqrt{\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8}}} \right) u(t) \tag{9}$$

Where,

$$A_1 = \frac{K_c}{8}\left(\frac{1-s_1}{s_1-s_2}\right) \tag{10}$$

$$A_2 = \frac{K_c}{8}\left(\frac{1-s_2}{s_1-s_2}\right) \tag{11}$$

Now applying the condition for underdamped oscillations,

$$\left(\frac{K_c - 4}{16}\right)^2 - \frac{K_c}{8} < 0 \tag{12}$$

$$\implies K_c \in \left(20 - \sqrt{384}, 20 + \sqrt{384}\right) \tag{13}$$

Theory

For the system to be stable,

$$\frac{K_c - 4}{8} < 0 \tag{14}$$

$$\implies K_c < 4 \tag{15}$$

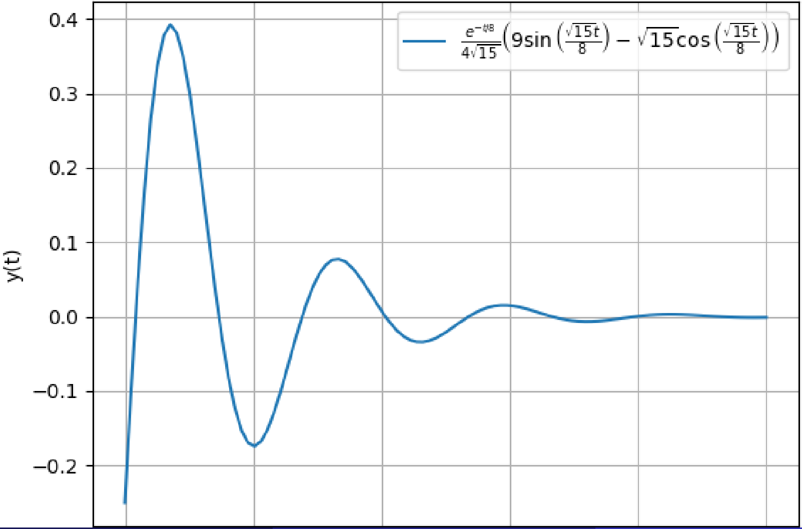
From (13) and (15)

$$K_c \in (0.4, 4) \tag{16}$$

(16) represents the ROC,($R_e\{s\} < 0$)

$$\implies K_c = 2 \tag{17}$$

Theory



$\frac{e^{-t/8}}{4\sqrt{15}}\left(9\sin\left(\frac{\sqrt{15}t}{8}\right) - \sqrt{15}\cos\left(\frac{\sqrt{15}t}{8}\right)\right)$
y(t)
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2

Code

```c
#include <stdio.h>
#include <math.h>

#define NUM_POINTS 100
#define FILENAME "yt_data.txt"

int main() {
    FILE *fp;
    fp = fopen(FILENAME, "w");
    if (fp == NULL) {
        printf("Error opening file.\n");
        return 1;
    }

    double t_values[NUM_POINTS];
    double y_values[NUM_POINTS];
    for (int i = 0; i < NUM_POINTS; i++) {
        t_values[i] = i * 50.0 / (NUM_POINTS - 1);
        y_values[i] = (exp(-t_values[i]/8) / (4 * sqrt(15))) * (9 * sin(sqrt(15) * t_values[i] / 8) - sqrt(15) * cos(sqrt(15) * t_values[i] / 8));
        fprintf(fp, "%.6f %.6f\n", t_values[i], y_values[i]);
    }

    fclose(fp);
    return 0;
}
```

Code

```python
import numpy as np
import matplotlib.pyplot as plt

# Load data from file
data = np.loadtxt('yt_data.txt')
t_values = data[:, 0]
y_values = data[:, 1]

# Plot the graph
plt.plot(t_values, y_values, label=r'$\frac{e^{-t/8}}{4\sqrt{15}} \left(9\sin\left(\frac{\sqrt{15}t}{8}\right) - \sqrt{15}\cos\left(\frac{\sqrt{15}t}{8}\right)\right)$')
plt.xlabel('t')
plt.ylabel('y(t)')
plt.legend()
plt.grid(True)
plt.savefig('b.png')
```