

# GATE: EE - 7.2021

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## QUESTION

Two discrete-time linear time-invariant systems with impulse responses  $h_1[n] = \delta[n - 1] + \delta[n + 1]$  and  $h_2[n] = \delta[n] + \delta[n - 1]$  are connected in cascade, where  $\delta[n]$  is the Kronecker delta. The impulse response of the cascaded system is

- (a)  $\delta[n - 2] + \delta[n + 1]$
- (b)  $\delta[n - 1]\delta[n] + \delta[n + 1]\delta[n - 1]$
- (c)  $\delta[n - 2] + \delta[n - 1] + \delta[n] + \delta[n + 1]$
- (d)  $\delta[n]\delta[n - 1] + \delta[n - 2]\delta[n + 1]$

(GATE 2021 EE)

## Solution:

From the Z-transformation pairs,

$$\delta[n] \xleftrightarrow{Z} 1 \quad (1)$$

$$x(n - k) \xleftrightarrow{Z} z^{-k} X(z) \quad (2)$$

$$x_1(n) * x_2(n) \xleftrightarrow{Z} X_1(z) X_2(z) \quad (3)$$

If  $h_1(n)$  and  $h_2(n)$  are cascade connected then the resultant impulse can be given by:

$$h(n) = h_1(n) * h_2(n) \quad (4)$$

$$\Rightarrow H(z) = H_1(z) H_2(z) \quad (5)$$

$$H(z) = (z^{-1} + z)(1 + z^{-1}) \quad (6)$$

$$= (z^{-1} + z^{-2} + z + 1), \quad |z| \neq 0 \quad (7)$$

Using the Z-transformation pairs to find the the inverse Z-transform,

$$h(n) = \delta[n - 2] + \delta[n - 1] + \delta[n] + \delta[n + 1] \quad (8)$$