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GATE: EE - 7.2021

EE22BTECH11219 - Rada Sai Sujan

QUESTION

Two discrete-time linear time-invarient systems with impulse responses $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascaded system is

(a)
$$\delta[n-2] + \delta[n+1]$$

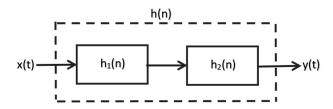
(b)
$$\delta[n-1]\delta[n] + \delta[n+1]\delta[n-1]$$

(c)
$$\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$$

(d)
$$\delta[n]\delta[n-1] + \delta[n-2]\delta[n+1]$$

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Solution:



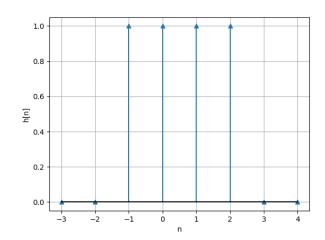


Fig. 2. h(n) vs n graph

Fig. 1. Block Diagram

From the Z-transformation pairs,

$$\delta[n] \stackrel{\mathcal{Z}}{\longleftrightarrow} 1$$
 (1)

$$x(n-k) \stackrel{\mathcal{Z}}{\longleftrightarrow} z^{-k}X(z)$$
 (2)

$$x_1(n) * x_2(n) \stackrel{\mathcal{Z}}{\longleftrightarrow} X_1(z) X_2(z)$$
 (3)

If $h_1(n)$ and $h_2(n)$ are cascade connected then the resultant impulse can be given by:

$$h(n) = h_1(n) * h_2(n)$$
 (4)

$$\implies H(z) = H_1(z) H_2(z) \tag{5}$$

$$H(z) = (z^{-1} + z)(1 + z^{-1})$$
 (6)

$$= (z^{-1} + z^{-2} + z + 1), \quad |z| \neq 0$$
 (7)

Using the Z-transformation pairs to find the the inverse Z-transform,

$$h(n) = \delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$$
 (8)