ASSIGNMENT11.15_13Q

EE22BTECH11219 - Rada Sai Sujan

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Question

Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represents (i) travelling wave, (ii) a stationary wave or (iii) none at all:

1. $y = 2\cos(3x)\sin(10t)$
2. $y = 2\sqrt{x - vt}$
3. $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$
4. $y = \cos x \sin t + \cos 2x \sin 2t$

Solution: Theory

TRAVELLING WAVE	STATIONARY WAVE
$y(x,t) = A\sin(kx \pm \omega t)$	$y(x,t) = A\sin kx \cos \omega t$

PARAMETERS	DEFINITION
A	Amplitude
ω	Angular Velocity
x	Position
k	Wavenumber

Table: Travelling wave *vs* Stationary wave

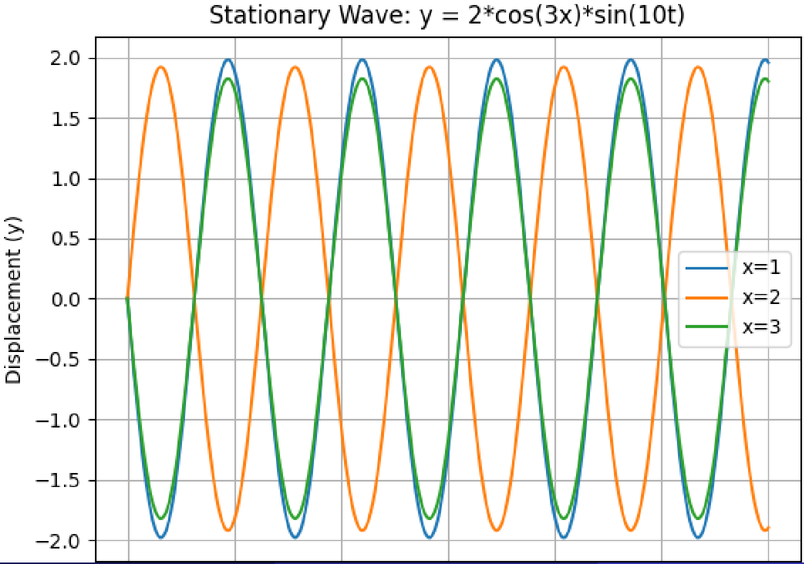
Theory

Let us assume an equation:

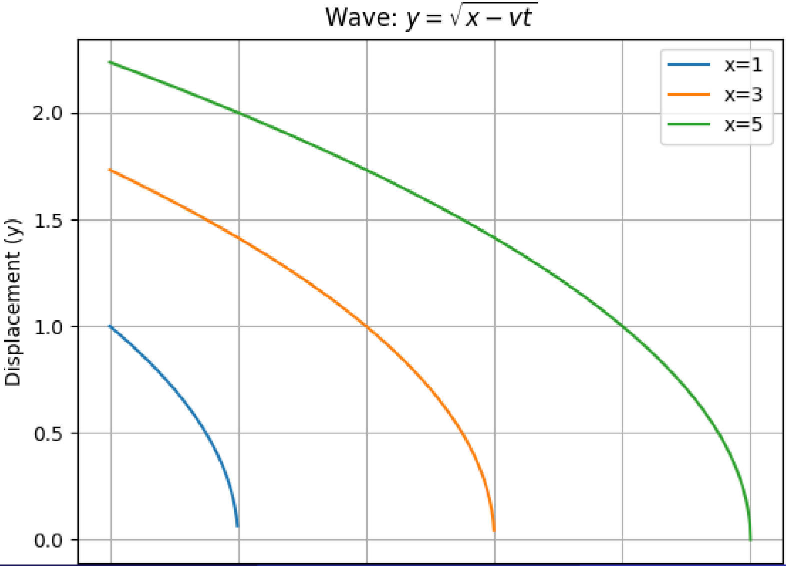
$$y = A(x)\cos\left(\omega t + \phi\left(x\right)\right) \tag{1}$$

Theory

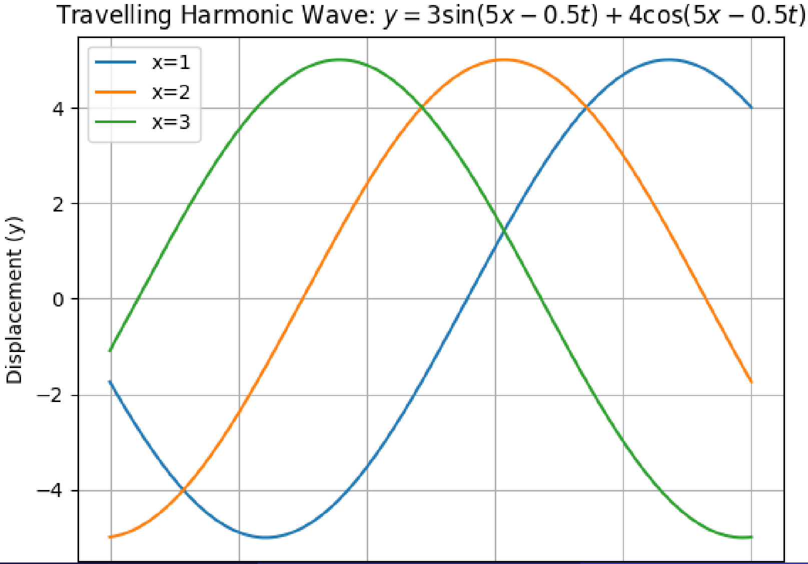
STATIONARY WAVE CONDITION	TRAVELLING WAVE CONDITION
(1) $A(x)$ should be a function of position x, and it can be expressed as $A(x) = A_0 cos(\omega t + \alpha)$ where A_0 is a constant, k is the wavenumber, x is the position and α is a phase constant.	(1) $A(x)$ should be a constant, and it can be expressed as $A(x) = A_0$ where A_0 is a constant number.
(2) $\phi(x)$ can be expressed as $\phi(x) = c$ where c is a constant.	(2) $\phi(x)$ is a linear expression in x or $\phi(x) = kx + \theta$ where k is the wavenumber and θ is the phaseconstant.


Stationary Wave: y = 2*cos(3x)*sin(10t)
Displacement (y)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
x=1
x=2
x=3

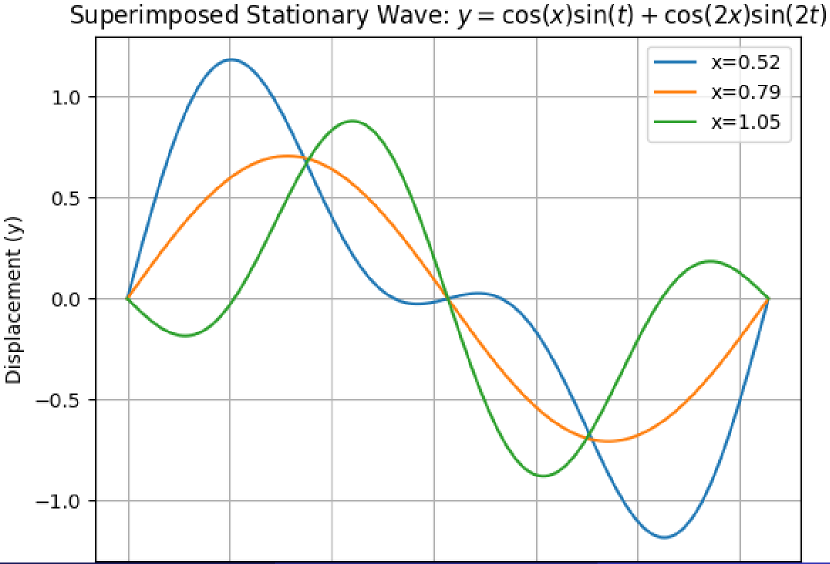
Graph



Graph


Travelling Harmonic Wave: $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$
x=1
x=2
x=3
Displacement (y)
4
2
0
−2
−4

Graph



Superimposed Stationary Wave: $y = \cos(x)\sin(t) + \cos(2x)\sin(2t)$
x=0.52
x=0.79
x=1.05
Displacement (y)
1.0
0.5
0.0
−0.5
−1.0

Theory

Fig. 1 and Fig. 3 are self explanatory for stationary and travelling waves.
Fig. 2 and Fig. 4 are neither stationary nor travelling waves.

Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define the time range from 0 to 3 seconds with small time intervals
t = np.linspace(0, 3, 1000)

# Define three different values for x
x_values = [1, 2, 3]

# Create a figure and axis with grid lines
fig, ax = plt.subplots()
ax.grid(True)

# Plot the graph for each x value
for x in x_values:
    y = 2 * np.cos(3 * x) * np.sin(10 * t)
    ax.plot(t, y, label=f'x={x}')

# Add labels and a legend
ax.set_xlabel('Time (seconds)')
ax.set_ylabel('Displacement (y)')
ax.set_title('Stationary Wave: y = 2*cos(3x)*sin(10t)')
ax.legend()

# Show the plot
```

Code

```
# Define the time range from 0 to 5 seconds with small time intervals
t = np.linspace(0, 5, 1000)

# Define three different values for x
x_values = [1, 3, 5]

# Create a figure and axis with grid lines
fig, ax = plt.subplots()
ax.grid(True)

# Plot the graph for each x value
for x in x_values:
    y = np.sqrt(x - v * t)
    ax.plot(t, y, label=f'x={x}')

# Add labels and a legend
ax.set_xlabel('Time (seconds)')
ax.set_ylabel('Displacement (y)')
ax.set_title('Wave: $y = \sqrt{x - vt}$')
ax.legend()

# Show the plot
plt.savefig('b.png')
```

Code

```python
t = np.linspace(0, 10, 1000)

# Define three different values for x
x_values = [1, 2, 3]

# Create a figure and axis with grid lines
fig, ax = plt.subplots()
ax.grid(True)

# Plot the graph for each x value
for x in x_values:
    y = 3 * np.sin(5 * x - 0.5 * t) + 4 * np.cos(5 * x - 0.5 * t)
    ax.plot(t, y, label=f'x={x}')

# Add labels and a legend
ax.set_xlabel('Time (seconds)')
ax.set_ylabel('Displacement (y)')
ax.set_title('Travelling Harmonic Wave: $y = 3\sin(5x - 0.5t) + 4\cos(
ax.legend()

# Show the plot
plt.savefig('c.png')
```

Code

```python
t = np.linspace(0, 2 * np.pi, 1000)

# Define three different values for x
x_values = [np.pi/6, np.pi/4, np.pi/3]

# Create a figure and axis with grid lines
fig, ax = plt.subplots()
ax.grid(True)

# Plot the graph for each x value
for x in x_values:
    y = np.cos(x) * np.sin(t) + np.cos(2 * x) * np.sin(2 * t)
    ax.plot(t, y, label=f'x={x:.2f}')

# Add labels and a legend
ax.set_xlabel('Time (seconds)')
ax.set_ylabel('Displacement (y)')
ax.set_title('Superimposed Stationary Wave: $y = \cos(x)\sin(t) + \cos
ax.legend()

# Show the plot
plt.savefig('d.png')
```