

# GATE: CE - 29.2022

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## QUESTION

Consider the following recursive iteration scheme for different values of variable P with the initial guess  $x_1 = 1$ :

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{P}{x_n} \right), \quad n = 1, 2, 3, 4, 5$$

For  $P = 2$ ,  $x_5$  is obtained to be 1.414, rounded off to 3 decimal places. For  $P = 3$ ,  $x_5$  is obtained to be 1.732, rounded off to 3 decimal places.

If  $P = 10$ , the numerical value of  $x_5$  is \_\_\_\_\_.  
(round off to three decimal places)

## Solution:

Applying A.M  $\geq$  G.M inequality,

$$\frac{x_n + \frac{P}{x_n}}{2} \geq \sqrt{P} \quad (1)$$

$$\Rightarrow x_{n+1} \geq \sqrt{P} \quad (2)$$

Solving the equation,

$$2x_{n+1}x_n - x_n^2 - P = 0 \quad (3)$$

Applying Z-transform we get,

$$X(z) * X(z) = \frac{PZ^{-1}}{(1 - z^{-1})(2 - z^{-1})} \quad (4)$$

$$= P \left( \frac{z^{-1}}{1 - z^{-1}} - \frac{z^{-1}}{2 - z^{-1}} \right) \quad (5)$$

From the transformation pairs,

$$x_{n-a} \xleftrightarrow{Z} z^{-a} X(z) \quad (6)$$

$$x_{n_1} \times x_{n_2} \xleftrightarrow{Z} X_1(z) * X_2(z) \quad (7)$$

$$\frac{u(n-1)}{a^n} \xleftrightarrow{Z} \frac{z^{-1}}{a - z^{-1}} \quad (8)$$

Now, applying inverse Z-transform,

$$x_n^2 = P \left( u(n-1) - \frac{u(n-1)}{2^n} \right) \quad (9)$$

$$\Rightarrow x_n^2 = P \left( 1 - \frac{1}{2^n} \right) \quad [\because n \geq 1] \quad (10)$$

Similarly,

$$x_{n+1}^2 = P \left( 1 - \frac{1}{2^{n+1}} \right) \quad (11)$$

$$\Rightarrow x_{n+1} - x_n > 0 \quad (12)$$

Hence, the system is convergent as it is increasing and bounded,

Now finding the limit of the sequence,

$$x^2 = \lim_{n \rightarrow \infty} P \left( 1 - \frac{1}{2^n} \right) \quad (13)$$

$$\Rightarrow x = \pm \sqrt{P} \quad (14)$$

From (2) and (14),

$$x_{n+1} = \sqrt{P} \quad (15)$$

Therefore, for  $P = 10$  the value of  $x_5$  is,

$$x_5 = \sqrt{10} \quad (16)$$

$$\therefore x_5 = 3.162 \quad (17)$$