GATE: CH - 45.2023

EE22BTECH11219 - Rada Sai Sujan

Question

Level (h) in a steam boiler is controlled by manipulating the flow rate (F) of the break-up(fresh) water using a proportional (P) controller. The transfer function between the output and the manipulated input is

$$\frac{h(s)}{F(s)} = \frac{0.25(1-s)}{s(2s+1)}$$

The measurement and the valve transfer functions are both equal to 1. A process engineer wants to tune the controller so that the closed loop response gives the decaying oscillations under the servo mode. Which one of the following is the CORRECT value of the controller gain to be used by the engineer?

(a) 0.25
(b) 2
(c) 4
(d) 6

Solution:

PARAMETER	VALUE	DESCRIPTION
G_c	K_c	Proportional controller's transfer function
G_f	1	Valve transfer function
G_p	$\frac{0.25(1-s)}{s(2s+1)}$	Process transfer function
G_M	1	Measurement transfer function

TABLE I
PARAMETER TABLE 1

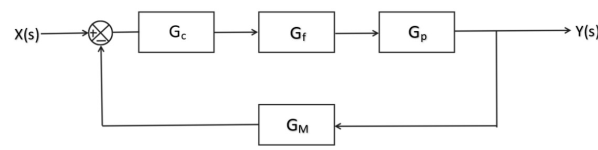

Fig. 1. Block diagram

PARAMETER	VALUE	DESCRIPTION
$X(s)$	$X(s)$	Laplace transform of input signal
$Y(s)$	$Y(s)$	Laplace transform of Output signal
$G(s)$	$G_cG_fG_p$	Open loop transfer function
$H(s)$	G_M	Feedback
$K(s)$	$\frac{Y(s)}{X(s)}$	Transfer function of system

TABLE II
PARAMETER TABLE 2

Output signal transfer function of the above block diagram can be given by,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \tag{1}$$

From Table II for a unit impulse, X(s)=1

$$Y(s) = T(s) \times X(s) \tag{2}$$

$$Y(s) = \frac{(1-s)K_c}{8s^2 + (4-K_c)s + K_c} \tag{3}$$

$$\implies Y(s) = \frac{(1-s)K_c}{8(s-s_1)(s-s_2)} \tag{4}$$

Where,

$$s_1 = \frac{(K_c-4)}{16} + \sqrt{\left(\frac{K_c-4}{16}\right)^2 - \frac{K_c}{8}} \tag{5}$$

$$s_2 = \frac{(K_c-4)}{16} - \sqrt{\left(\frac{K_c-4}{16}\right)^2 - \frac{K_c}{8}} \tag{6}$$

From (4) we get,

$$Y(s) = \frac{K_c}{8(s_1-s_2)}\left(\frac{1-s_1}{s-s_1} - \frac{1-s_2}{s-s_2}\right) \tag{7}$$

Now taking the inverse laplace transform we have,

$$y(t) = \frac{K_c}{8(s_1-s_2)}\left[(1-s_1)e^{s_1 t} - (1-s_2)e^{s_2 t}\right] \tag{8}$$

$$\implies y(t) = e^{\frac{K_c-4}{16}}\left(A_1 e^{\sqrt{\left(\frac{K_c-4}{16}\right)^2 - \frac{K_c}{8}}} - A_2 e^{-\sqrt{\left(\frac{K_c-4}{16}\right)^2 - \frac{K_c}{8}}}\right) \tag{9}$$

Where,

$$A_1 = \frac{K_c}{8}\left(\frac{1-s_1}{s_1-s_2}\right) \tag{10}$$

$$A_2 = \frac{K_c}{8}\left(\frac{1-s_2}{s_1-s_2}\right) \tag{11}$$

Now applying the condition for underdamped oscillations,

$$\left(\frac{K_c-4}{16}\right)^2 - \frac{K_c}{8} < 0 \tag{12}$$

$$\implies K_c \in \left(20 - \sqrt{384}, 20 + \sqrt{384}\right) \tag{13}$$

For the system to be stable,

$$\frac{K_c-4}{8} < 0 \tag{14}$$

$$\implies K_c < 4 \tag{15}$$

From (13) and (15)

$$K_c \in (0.4, 4) \tag{16}$$

$$\implies K_c = 2 \tag{17}$$

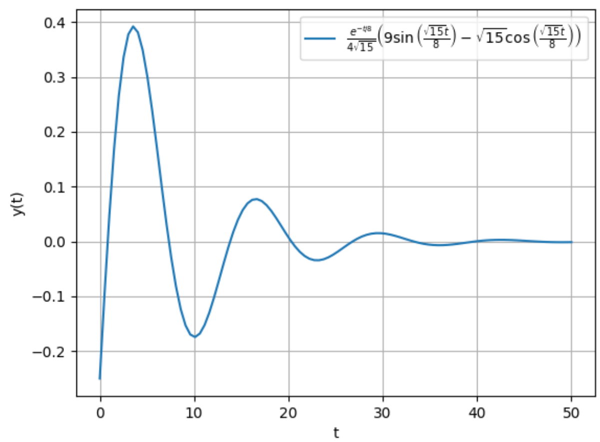

Fig. 2. y(t) vs t graph