1) Euclidean distance $d(A,B) = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$

Give points

$P_1(0.4, 0.5)$ $P_2(0.2, 0.3)$ $P_3(0.1, 0.8)$ $P_4(0.21, 0.12)$ $P_5(0.6, 0.16)$

$P_6(0.33, 0.28)$ $P_7(0.11, 0.15)$

$P_1 - P_2 = \sqrt{(0.4-0.2)^2 + (0.5-0.3)^2} = 0.2828$

$P_1 - P_3 = \sqrt{(0.4-0.1)^2 + (0.5-0.08)^2} = 0.5161$

$P_1 - P_4 = \sqrt{(0.4-0.21)^2 + (0.5-0.12)^2} = 0.4249$

$P_1 - P_5 = \sqrt{(0.4-0.6)^2 + (0.5-0.16)^2} = 0.3945$

$P_1 - P_6 = \sqrt{(0.4-0.33)^2 + (0.5-0.28)^2} = 0.2309$

$P_1 - P_7 = \sqrt{(0.4-0.11)^2 + (0.5-0.15)^2} = 0.4545$

$P_2 - P_3 = \sqrt{(0.2-0.1)^2 + (0.3-0.08)^2} = 0.247$

$P_2 - P_4 = \sqrt{(0.2-0.21)^2 + (0.3-0.12)^2} = 0.1803$

$P_2 - P_5 = \sqrt{(0.2-0.6)^2 + (0.3-0.12)^2} = 0.4238$

$P_2 - P_6 = \sqrt{(0.2-0.33)^2 + (0.3-0.28)^2} = 0.1315$

$P_2 - P_7 = \sqrt{(0.2-0.11)^2 + (0.3-0.15)^2} = 0.1749$

$P_3 - P_4 = \sqrt{(0.1-0.21)^2 + (0.08-0.12)^2} = 0.1170$

$P_3 - P_5 = \sqrt{(0.1-0.6)^2 + (0.08-0.12)^2} = 0.5070$

$P_3 - P_6 = \sqrt{(0.1-0.33)^2 + (0.08-0.28)^2} = 0.3048$

$P_3 - P_7 = \sqrt{(0.1-0.11)^2 + (0.08-0.15)^2} = 0.0707$

$P_4 - P_5 = \sqrt{(0.21-0.6)^2 + (0.12-0.16)^2} = 0.2955$

$P_4 - P_6 = \sqrt{(0.21-0.33)^2 + (0.12-0.15)^2} = 0.2000$

$P_4 - P_7 = \sqrt{(0.21-0.11)^2 + (0.12-0.15)} = 0.1044$

$P_5 - P_6 = \sqrt{(0.60-0.33)^2 + (0.16-0.28)} = 0.4905$

$P_6 - P_7 = \sqrt{(0.33-0.11)^2 + (0.28-0.15)^2} = 0.2555$

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
P_1	0						
P_2	0.2828	0					
P_3	0.5161	0.2417	0.4270	0			
P_4	0.4249	0.1803	0.1170	0			
P_5	0.3945	0.4238	0.5064	0.3920	0		
P_6	0.2309	0.1315	0.3048	0.2000	0.2955	0	
P_7	0.9545	0.1749	0.0707	0.1044	0.4901	0.2555	0

MIN (Single linkage) clustering

Smaller distance → $P_3 - P_7 = 0.0707$

Now merge P_3 & P_7 → New cluster $C_1 (P_3, P_7)$

C_1 to P_1 = min (0.5167, 0.4545) = 0.4545

C_1 to P_2 = min (0.247, 0.1749) = 0.1749

C_1 to P_4 = min (0.170, 0.1044) = 0.1044

C_1 to P_5 = min (0.5064, 0.4901) = 0.4901

C_1 to P_6 = min (0.3048, 0.2555) = 0.2555

New distance Matrix

	C_1	P_1	P_2	P_3	P_5	P_6
C_1	0					
P_1	0.4545	0				
P_2	0.1749	0.2828	0			
P_4	0.1044	0.4299	0.1803	0		
P_5	0.4401	0.3945	0.4238	0.3920	0	
P_6	0.2555	0.2309	0.1315	0.2000	0.2955	0

Smallest distance → $P_4 - C_1 = 0.1044$

merge $C_1 \rightarrow P_4$ → new cluster $C_2\{P_3, P_7, P_4\}$

C_2 to P_1 : min(0.3161, 0.4543, 4249) = 0.4249

C_2 to P_2 : min(0.2417, 0.1749, 0.1803) = 0.17499

C_2 to P_5 : min(0.5062, 0.4901, 0.3920) = 0.3920

C_2 to P_6 : min(0.3048, 0.2555, 0.209) = 0.2000

	C_2	P_1	P_2	P_5	P_6
C_2	0				
P_1	0.4249	0			
P_2	0.1749	0.2828	0		
P_5	0.3920	0.3945	0.4238	0	
P_6	0.2000	0.2309	0.1315	0.2955	0

Smallest distance = 0.1375 $\{P_2, P_6\}$

merge P_2 & P_6 → $C_3 = \{P_2, P_6\}$

C_3 to C_2 : min(0.1749, 0.2000) = 0.1749

C_3 to P_1 : min(0.2828, 0.2309) = 0.2309

C_3 to P_5 : min(0.4258, 0.2955) = 0.2955

New Clusters C_2, C_3, P_1, P_5

	C_2	C_3	P_1	P_5
C_2	0			
C_3	0.1749	0		
P_1	0.4249	0.2309	0	
P_5	0.3920	0.2955	0.3945	0

Smallest distance = 0.1749 $\{C_2, C_3\}$

merge : $\{P_3, P_1, P_4\} + \{P_2, P_6\} \rightarrow P_4 = \{P_2, P_3, P_4, P_6, P_7\}$.

C_4 to P_1 : min(04299, 0.2309, 0.2828, 05161, 0.4545)

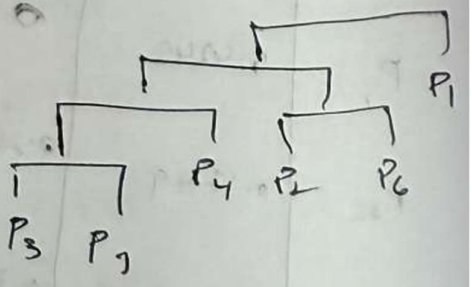
= 0.2309

C_4 to P_5 : min (0.3921, 0.2955) 0.4238; 0.5064, 0.4901)

= 0.2955

New Cluster = $\{C_4, P_1, P_5\}$

	C_4	P_1	P_5
C_4	0		
P_1	0.2307	0	
P_5	0.2955	0.3945	0



smallest distane = 0.2309

merge C_4 & P_1

$C_5 = \{P_1, P_2, P_3, P_4, P_6, P_7\}$

$P_1 - P_5 = 0.3745$

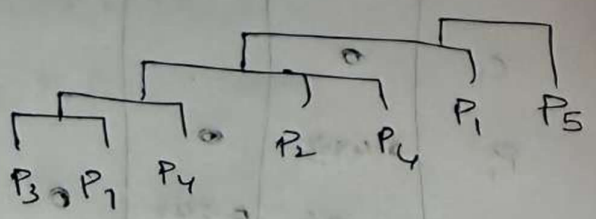
$P_2 - P_5 = 0.4230$

$P_3 - P_5 = 0.5064$

$P_4 - P_5 = 0.3921$

$P_6 - P_5 = 0.2955$

$P_7 - P_5 = 0.4901$; min = (2955)



Clusters (C_5, P_5)

	C_5	P_5
C_5	0	
P_5	0.2955	0

Smallest distance = 0.2955

merge Cluster = $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$

Average linkage clustering

Clusters: $\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5\}, \{P_6\}, \{P_7\}$

from distance matrix

Smallest distance $P_3 - P_7 = 0.0707$

merge $\{P_3\} + \{P_7\} \rightarrow C_1 = \{P_3, P_7\}$

$C_1 - P_1 = (0.576 + 0.4543)/2 = 0.4853$

$C_1 - P_2 = (0.247 + 0.141)/2 = 0.2083$

$C_1 - P_4 = (0.1170 + 0.1044)/2 = 0.1107$

$C_1 - P_5 = (0.5064 + 0.4901)/2 = 0.4983$

$C_1 - P_6 = (0.3048 + 0.2555)/2 = 0.2802$

New clusters : $C_1, P_1, P_2, P_4, P_5, P_6$

	C_1	P_1	P_2	P_4	P_5	P_6
C_1	0					
P_1	0.4953	0				
P_2	0.2085	0.2828	0			
P_4	0.1107	0.4249	0.1803	0		
P_5	0.4983	0.3945	0.4238	0.3920	0	
P_6	0.2802	0.2309	0.1315	0.2000	0.2955	0

Smallest distance = 0.1107

Merge $C_1 + P_4 \Rightarrow C_2 = \{P_3, P_4, P_4\}$

$C_2 - P_1 = (0.516 + 0.4545 + 0.4249)/3 = 0.4652$

$C_2 - P_2 = (0.2417 + 0.1749 + 0.1803)/3 = 0.1990$

$C_2 - P_5 = (0.3064 + 0.4901 + 0.3920) = 0.462$

$C_2 - P_6 = (0.2548 + 0.2555 + 0.2000) = 0.2534$

New clusters: C_2, P_1, P_2, P_5, P_6

	C_2	P_1	P_2	P_5	P_6
C_2	0				
P_1	0.4652	0			
P_2	0.1990	0.2828	0		
P_5	0.4629	0.3945	0.4238	0	
P_6	0.2534	0.2309	0.1315	0.2955	0

Smallest distance = 0.1315 $(P_2 \& P_6)$

New cluster $C_3 = \{P_2, P_6\}$

$C_3 - C_2 = 1.35216 = 0.2262$

$C_3 - P_1 = (0.28 - 0.2309)/2 = 0.2567$

$C_3 - P_5 = (0.4231 + 0.2951)/2 = 0.3597$

New cluster: C_2, C_3, P_1, P_5

	C_2	C_3	P_1	P_5
C_2	0			
C_3	0.2262	0		
P_1	0.4652	0.2569	0	
P_5	0.4629	0.3597	0.3945	0

Smallest distance = 0.2262 $\{C_2 - C_3\}$

merge C_2 & C_3

$C_2 = \{P_3, P_7, P_4\}$

$C_3 = \{P_2, P_6\}$

$C_4 = \{P_2, P_3, P_4, P_6, P_7\}$

$C_4 - P_1 = (0.2228 + 0.516 + 0.4249 + 0.2309 + 0.4545)/5$

$= 0.3818$

$C_4 - P_5 = 0.4238 + 0.3064 + 0.3921 + 0.2955 + 0.4901)/5$

$= 0.4216$

New cluster: C_4, P_1, P_5

	C_4	P_1	P_5
C_4	0		
P_1	0.3818	0	
P_5	0.4216	0.3945	0

Smallest distance $C_4, P_1 = 0.3818$

New cluster $C_5 = \{P_1, P_2, P_3, P_4, P_6, P_7\}$

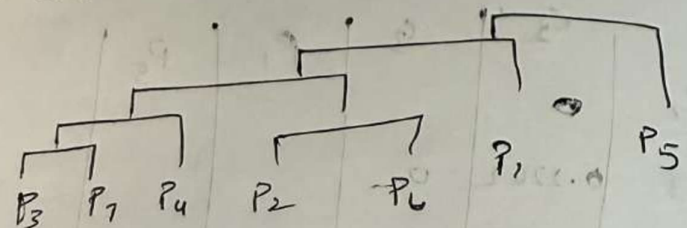
$C_5 - P_5 = (0.3945 + 0.4238 + 0.567 + 0.3921 + 0.2955 + 0.4901)/6$

$= 0.4171$

clusters = C_5, P_5

merge = $C_5 \& P_5$

Final Cluster = $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$



2) Points (2,1) (3,1) (3,3) (4,1) (5,1) (6,7) (1,3) (2,5)

$k=3$

Centroid1 = (2,4)

Centroid2 = (4,1)

Centroid3 = (5,1)

$ED = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

P(2,4)

$d - C_1(2,1) = \sqrt{(2-2)^2+(1-1)^2} = 0$

$d - C_2(4,1) = \sqrt{(2-4)^2+(1-1)^2} = 2$

$d - C_3(5,1) = \sqrt{(2-5)^2+(1-1)^2} = 3$

Assign to C_1

P(3,1)

$d - C_1(2,1) = \sqrt{(3-2)^2+(1-1)^2} = 1$

$d - C_2(4,1) = \sqrt{(3-4)^2+(1-1)^2} = 1$

$d - C_3(5,1) = \sqrt{(3-5)^2+(1-1)^2} = 2$

4 Assign to 4

P(3,3)

$d - C_1(2,1) = \sqrt{(3-2)^2+(3-1)^2} = 2.361$

$d - C_2(4,1) = \sqrt{(3-4)^2+(3-1)^2} = 2.361$

$d - C_3(5,1) = \sqrt{(3-5)^2+(3-1)^2} = 2.8284$

P(4,1)

$d - C_1(2,1) = \sqrt{(4-2)^2 + (1-1)^2} = 2$

$d - C_2(4,1) = \sqrt{(4-4)^2 + (1-1)^2} = 0$

$d - C_3(5,1) = \sqrt{(4-5)^2 + (1-1)^2} = 1$

↳ Assign to C_2

P(5,1)

$d - C_1(2,1) = \sqrt{(5-2)^2 + (1-1)^2} = 3$

$d - C_2(4,1) = \sqrt{(5-4)^2 + (1-1)^2} = 1$

$d - C_3(5,1) = \sqrt{(5-5)^2 + (1-1)^2} = 0$

P(6,7)

$d - C_1(2,1) = \sqrt{(6-2)^2 + (7-1)^2} = 7.214$

$d - C_2(4,1) = \sqrt{(6-4)^2 + (7-1)^2} = 6.3246$

$d - C_3(5,1) = \sqrt{(6-5)^2 + (7-1)^2} = 6.0828$

↳ Assign to C_3

P(1,3)

$d - C_1(2,1) = \sqrt{(1-2)^2 + (3-1)^2} = 2.236$

$d - C_2(4,1) = \sqrt{(1-4)^2 + (3-1)^2} = 3.6056$

$d - C_3(5,1) = \sqrt{(1-5)^2 + (3-1)^2} = 4.4721$

↳ Assign to C_1

P(2,5)

$d - C_1(2,1) = \sqrt{(2-2)^2 + (5-1)^2} = 4$

$d - C_2(4,1) = \sqrt{(2-4)^2 + (5-1)^2} = 4.472$

$d - C_3(5,1) = \sqrt{(2-5)^2 + (5-1)^2} = 5$

↳ Assign to C_1

Cluster c_1 : (2,1), (3,1) (3,3) (1,3) (2,5)

Cluster c_2 : (4,1)

Cluster c_3 : (5,1), (6,7)

New Centroids

c_1 new : Points (2,1) (3,1) (3,3) (1,3) (2,5)

$$\text{New Centroid} = \left[\frac{2+3+3+1+2}{5}, \frac{1+1+3+3+5}{5}\right]$$

$$= [2.2, 2.6]$$

New Centroid 2 : Points (4,1)

Centroid remains (4,1) New c_2 = (4,1)

c_3 New points - (5,1) (6,7)

$$\text{New Centroids} = \left[\frac{5+6}{2}, \frac{1+7}{2}\right]$$

$$= [5.5, 4]$$

New Centroids

New c_1 : [2.2, 2.6]

New c_2 : [4,1]

New c_3 : [5.5, 4]