# Classical Time Series Analysis

Saran Ishika Maiti
Department of Statistics, Visva-Bharati
Santiniketan

# What is Time Series?

- *A Time series* is a set of observations, each one being recorded at a specific time. (Annual GDP of a country, Sales figure, water flow of a river overetc)
- *A discrete time series* is one in which the set of time points at which observations are made is a discrete set. (All above including irregularly spaced data)
- *Continuous time series* are obtained when observations are made continuously over some time intervals. *It is a theoretical Concept*.( Roughly, ECG graph).
- *A discrete valued time series* is one which takes discrete values. (No of accidents, No of transaction etc.).

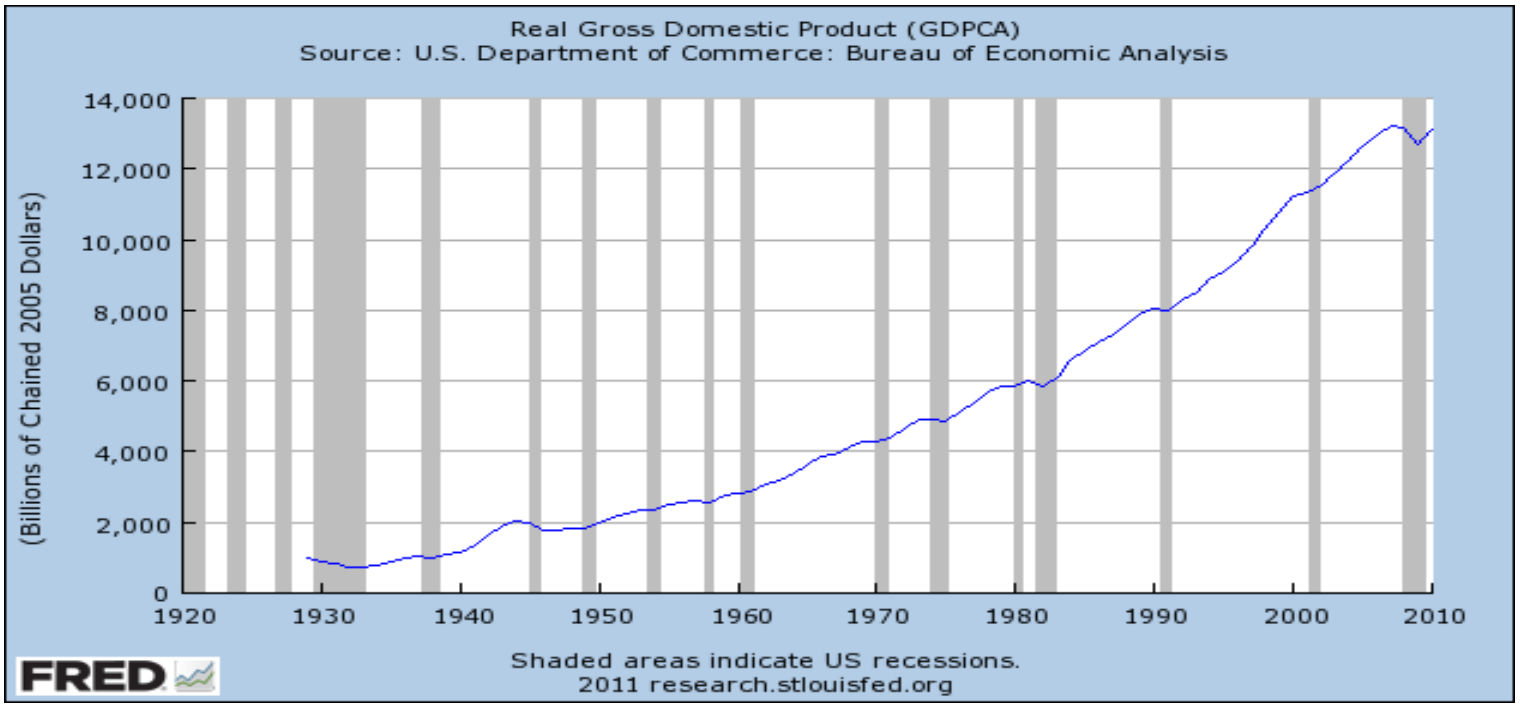
- ✓ A *discrete time series* consists of data points separated by time intervals that are greater than one second
- ✓ A *continuous time series* contains one data point usually measured per second

# Few Time series Plots

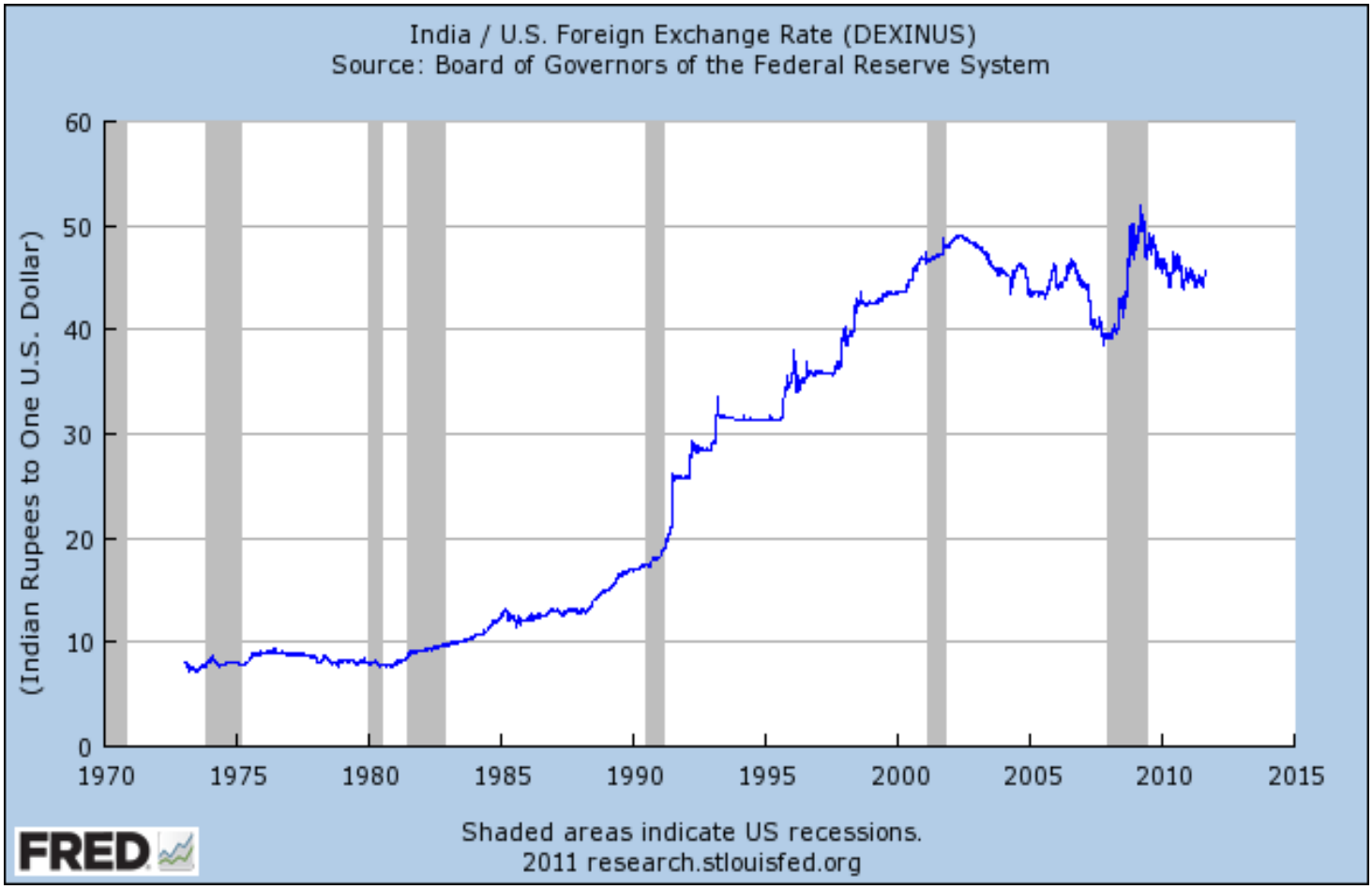
Remember for a time series plot time points are drawn along the major axis

Remember for a time series plot value of the variables will always be joined by a line

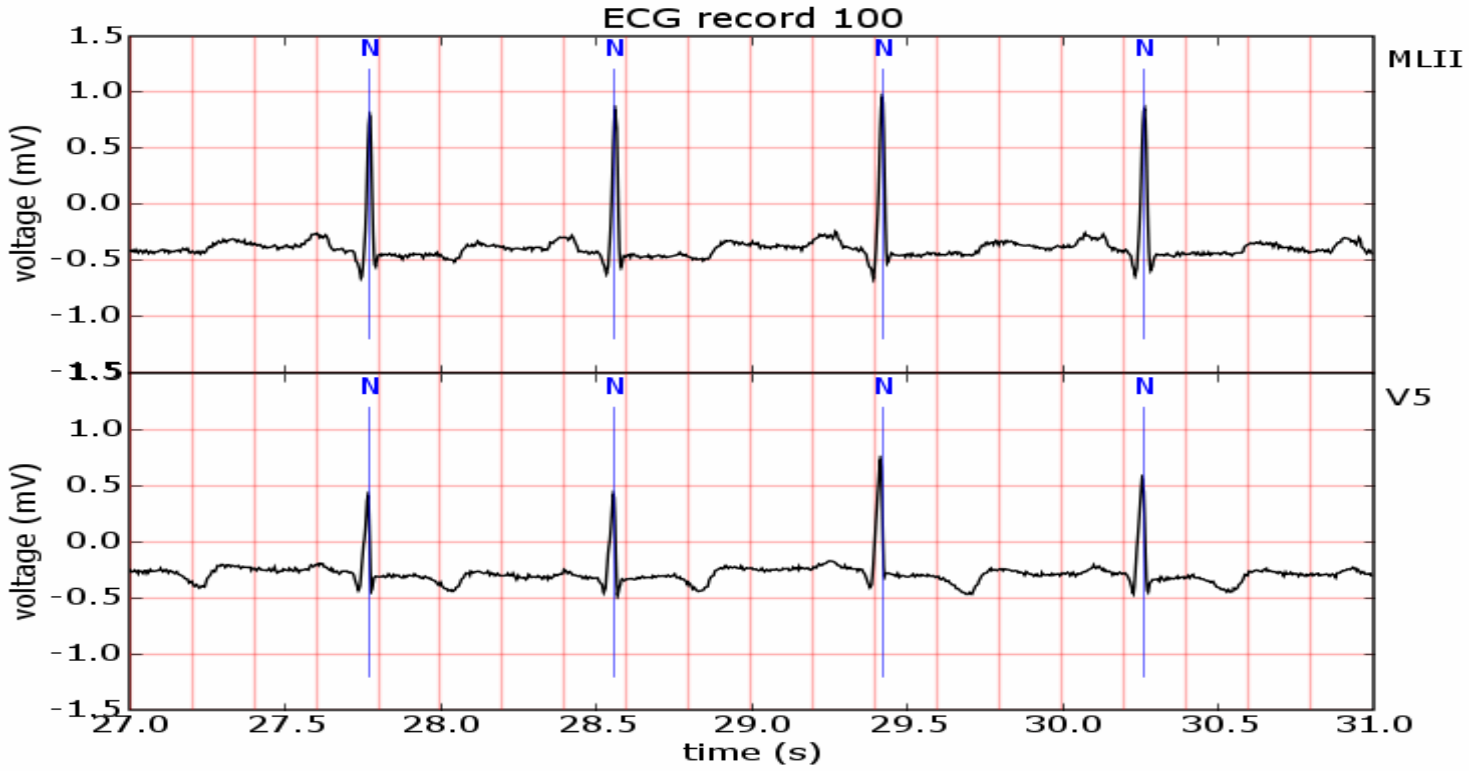
## Annual GDP of USA



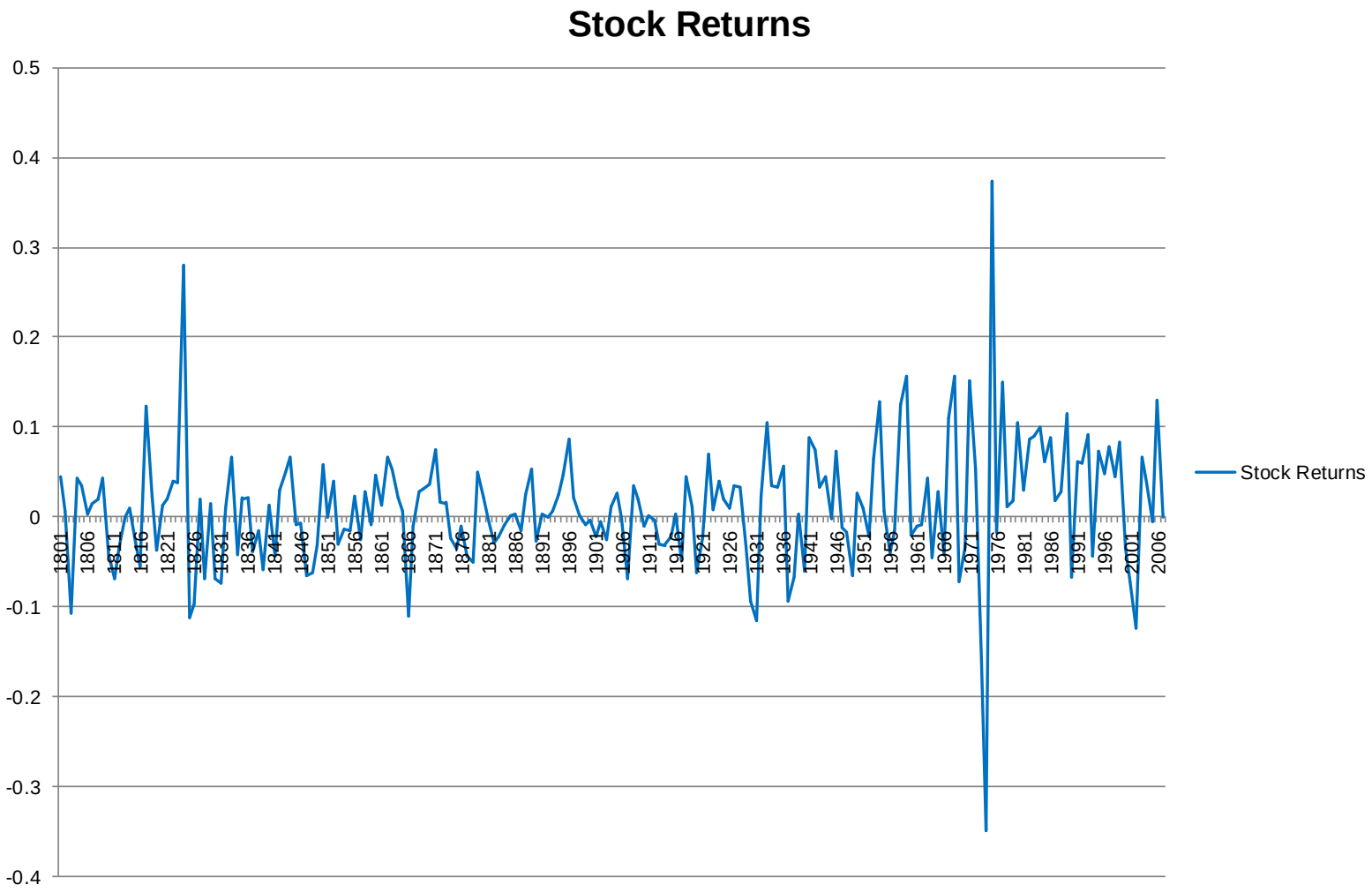
*A discrete time series* *is one in which the set of time points at which observations are made is a discrete set. (All above including irregularly spaced data)*



*Continuous time series* are obtained when observations are made continuously over some time intervals. (ECG graph).



# Continuous time series data (Stock returns):



*A discrete valued time series is one which takes discrete values. (No of accidents, No of transaction etc.).*

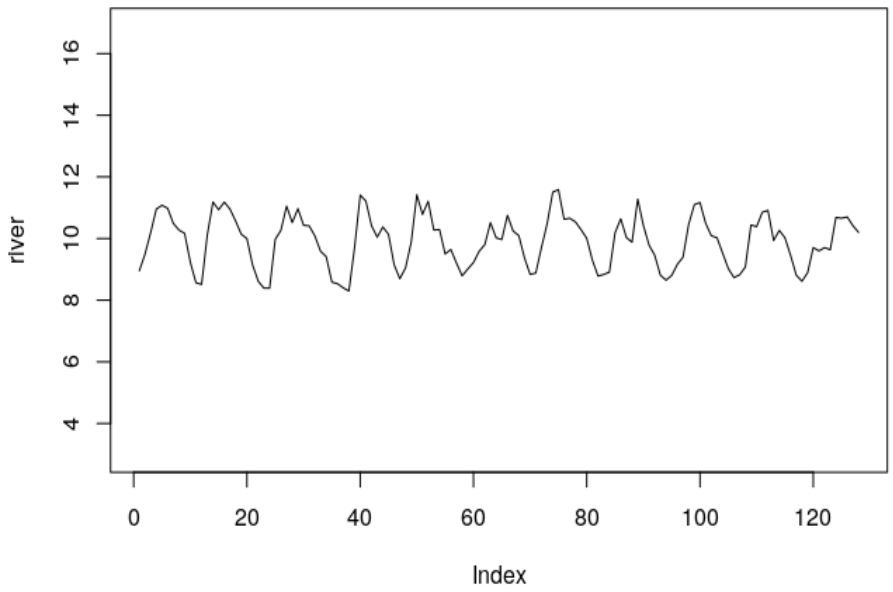
*Time series plot on car accident in U.K.*

# Few points on Plots

- Plot help us to summarize and reveal patterns in data
- Graphics help us to identify anomalies in data
- Plot helps us to present a huge amount of data in small space and makes huge data set coherent.
- To get all the features of a plot 'aspect ratio' of plot is very crucial.
- The ratio of height to width of a plot is called the aspect ratio.
- Aspect ratio makes a plot visually appealing and more balanced.

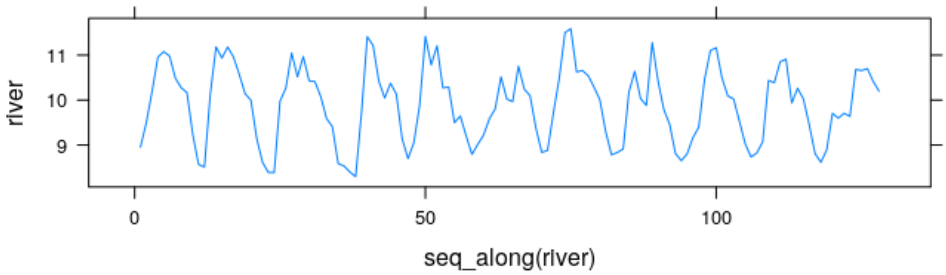
# Aspect ratio

- Aspect ratio of a square is 1:1 while aspect ratio of a ideal rectangular landscape format is 3:4.
- Generally in time series plot aspect ratio is expected to be  around 0.618.
- However, for long time series data aspect ratio should be around 0.25.
- To understand the impact of aspect ratio see the following two plots




Aspect ratio= **5**

Aspect ratio= **.2**

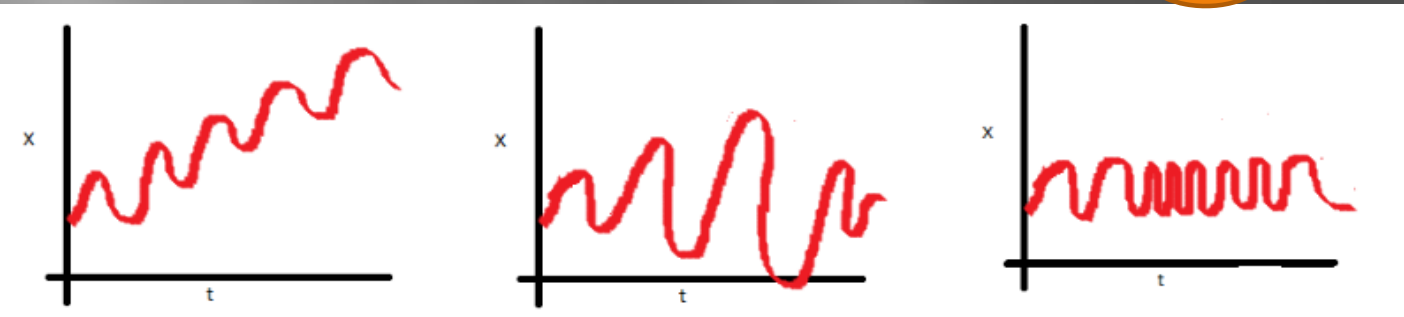


# Stability of time series: Concept of Stationarity

- Stationarity means that the statistical properties (usually mean, variance, covariance) of a time series (or rather the process generating it) do not change over time. Unless a time series is stationary it is not ready for further analysis as well as future prediction is not possible.

From a time series plot one can roughly guess on time series stability aka stationarity

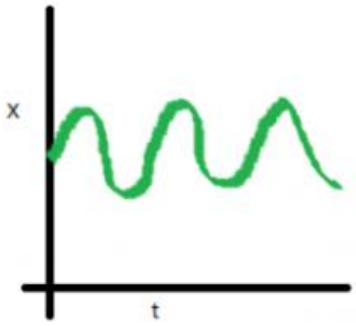
**Look at the three plots below**



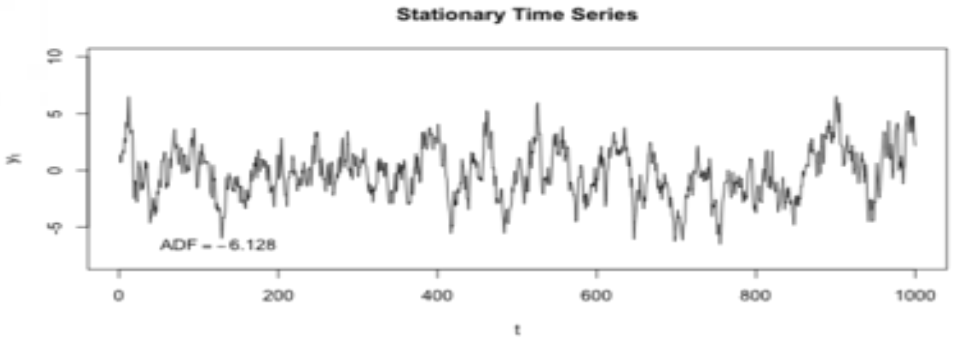

1. Mean increases with time-a clear upward trend
2. No trend but span of each cycle varying
3. Spread becomes closer as time increases

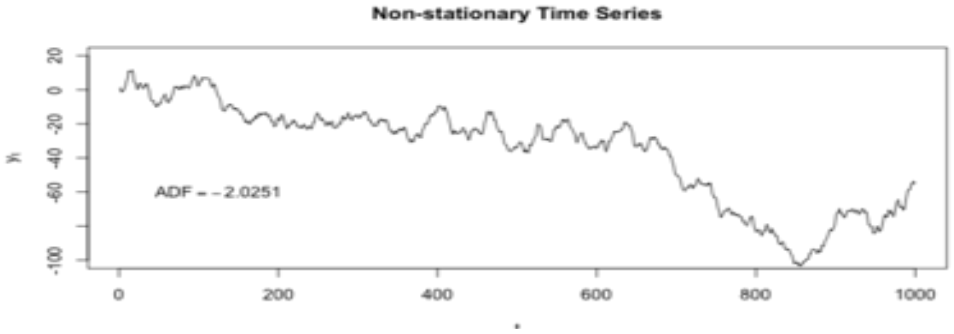
**Nonstationarity: Unstable time series**

How does a stationary time plot look then ?




In this case mean , variance and covariance are constant with time



# Two Different Approaches of Explanation

- Old/Classical/Traditional Approach
- Modern/Stochastic Process Approach

It is not that modern approach is better than the classical approach.

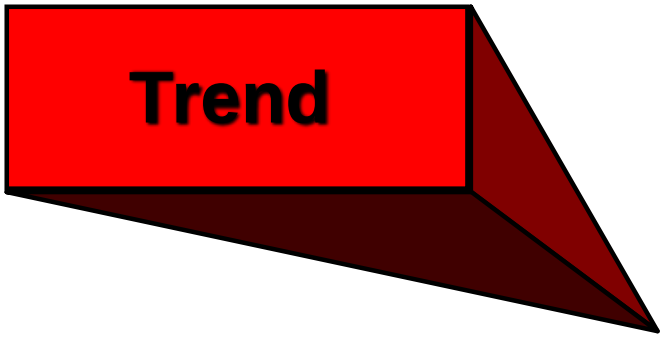
Looking into the same time series from different perspective.

OLD WINE IN NEW BOTTLES.

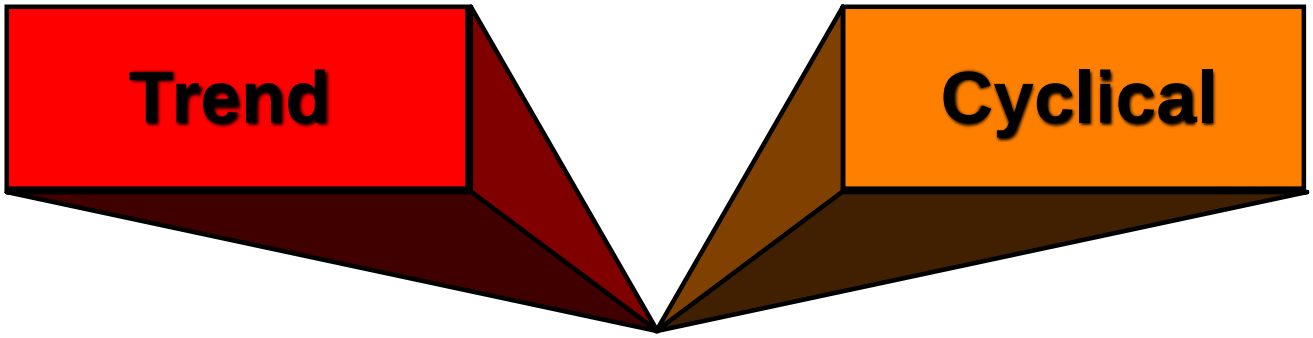
By classical Approach

# Time Series Components

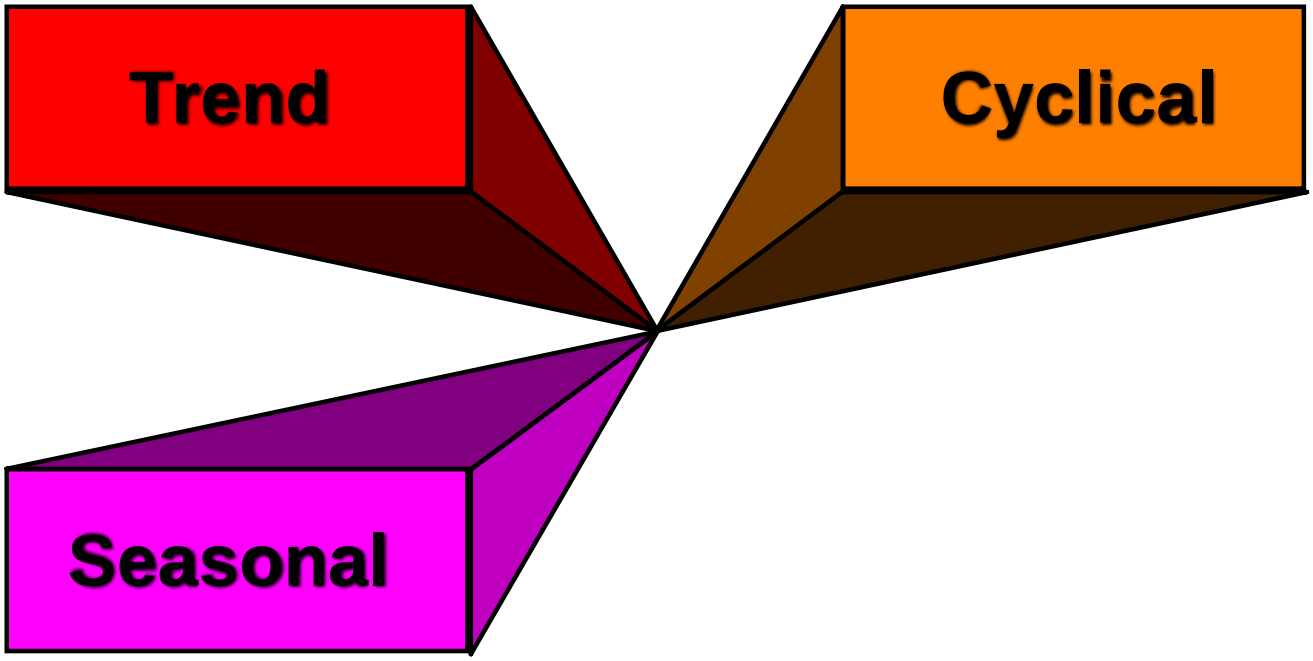
# Time Series Components



# Time Series Components

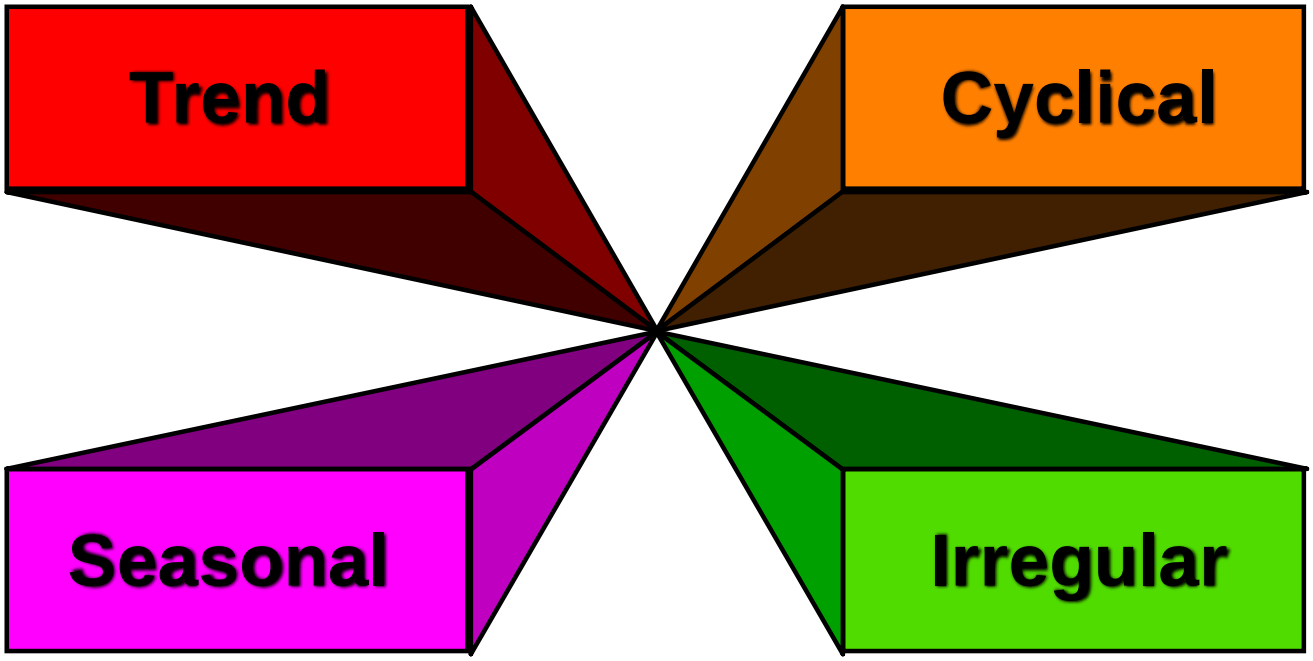


# Time Series Components



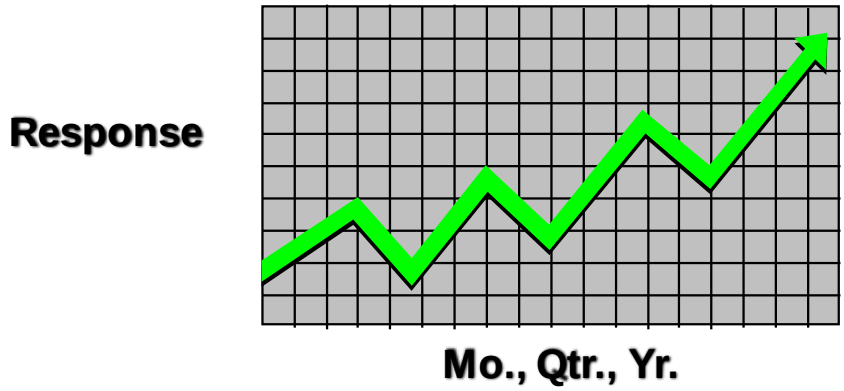
# Time Series Components

1. Trend 2. Periodic Component 3. Irregular/error component
2. Under periodic component two components-cyclical, seasonal



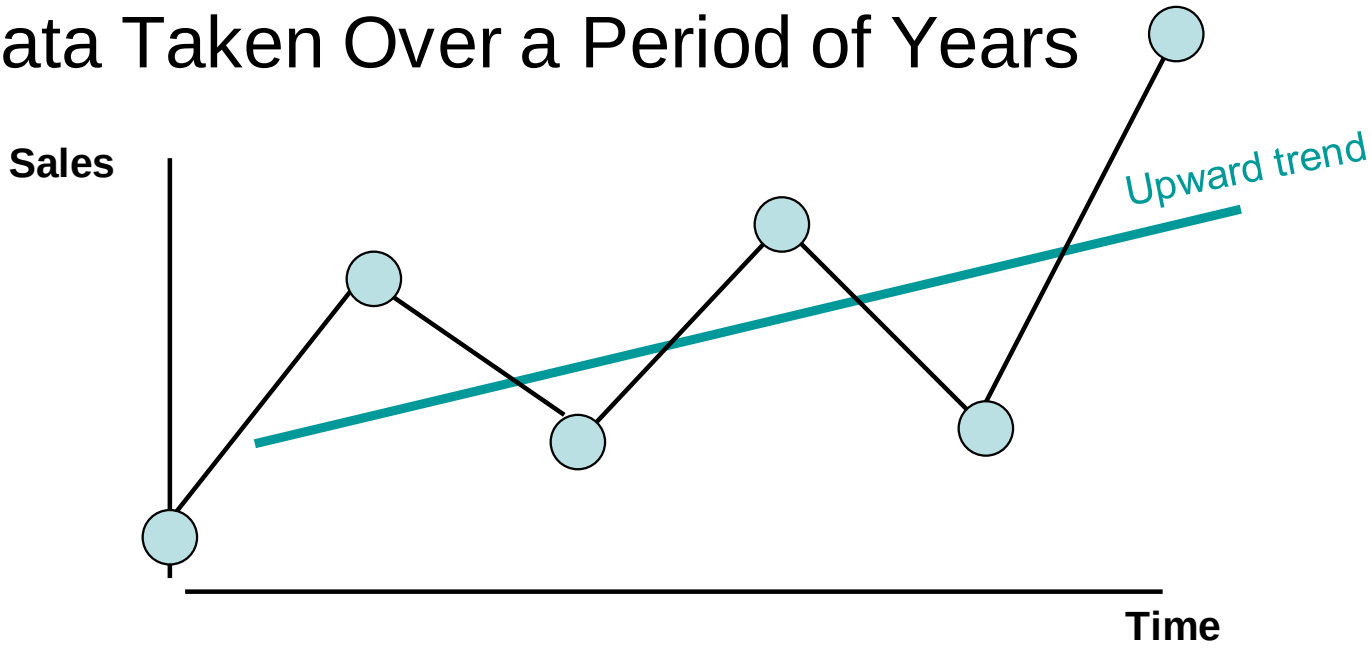
# Trend Component

- Persistent, smooth,regular, long-term movement, usually overall upward or downward pattern
- Several years duration



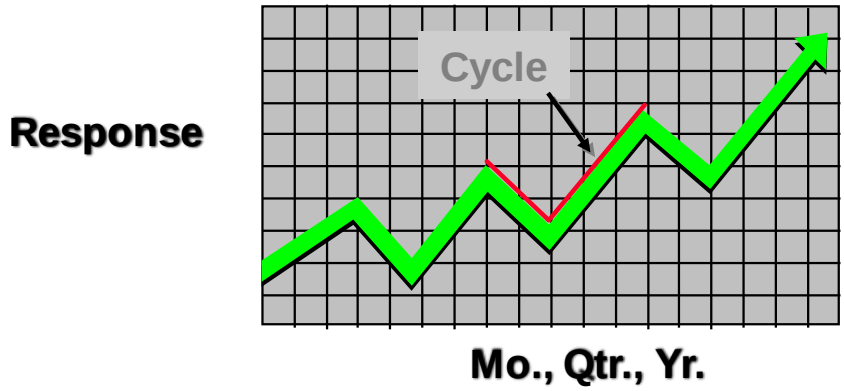
# Trend Component

- Overall Upward or Downward Movement
- Data Taken Over a Period of Years



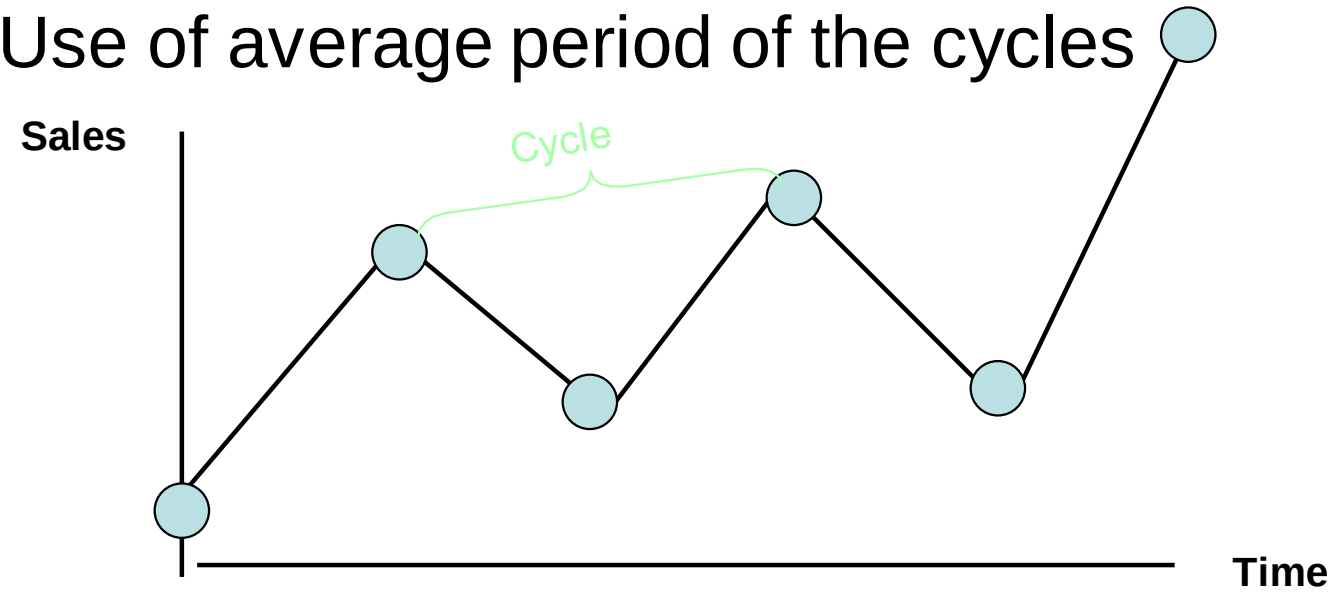
# Cyclical Component

- Repeating up & down movements
- With uniform period and a uniform amplitude (height)
- Usually 2-10 years duration



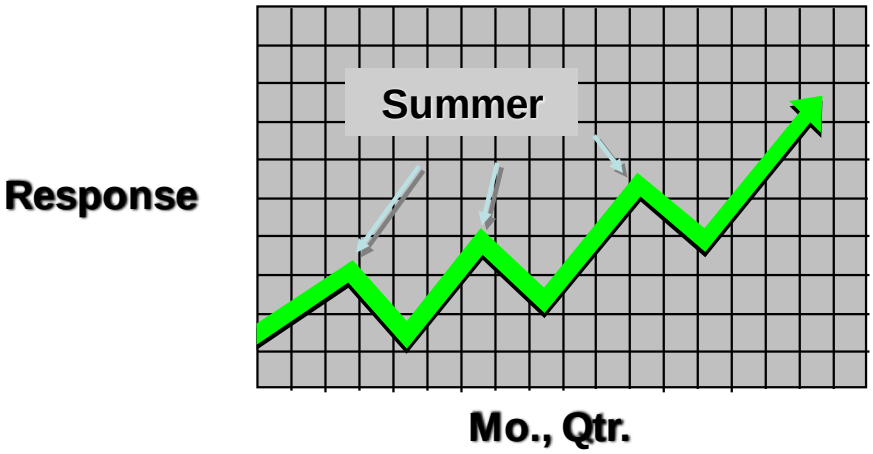
# Cyclical Component

- Upward or Downward Swings
- Cycles in economic time series are <u>not</u> strictly periodic
- Use of average period of the cycles



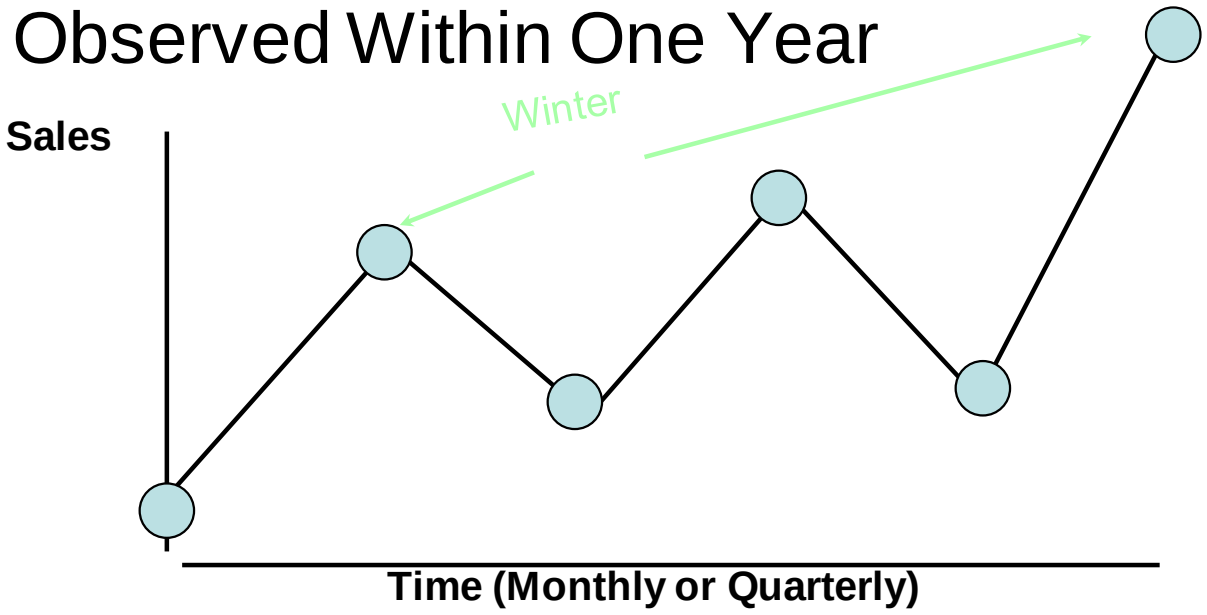
# Seasonal Component

- Regular pattern of up & down fluctuations
- Period is not longer than one year
- Occur due to weather, customs etc.
- Daily, monthly, quarterly or yearly data



# Seasonal Component

- Upward or Downward Swings
- Regular Patterns
- Observed Within One Year



# Irregular Component

- Erratic, unsystematic, ‘residual’ fluctuations,unexplained
- Due to random variation or unforeseen events
  - Union strike
  - War
  - Natural calamities e.g. flood, storm etc.
- Short duration & nonrepeating

# Looking into the Time Series

Any time series is composed of any combinations of four components.

- $T_t$ : Secular Trend.
- $S_t$: Seasonality.
- $C_t$ : Cyclical Component.
- $I_t$ : Irregular component.

Mathematically, $Y_t = f(T_t, S_t, C_t, I_t)$ where f being a function

- In classical approach this function f can be either simple multiplicative or simple additive

# Variations in Time Series…..

- Secular Trend: Long Term change in the mean level.

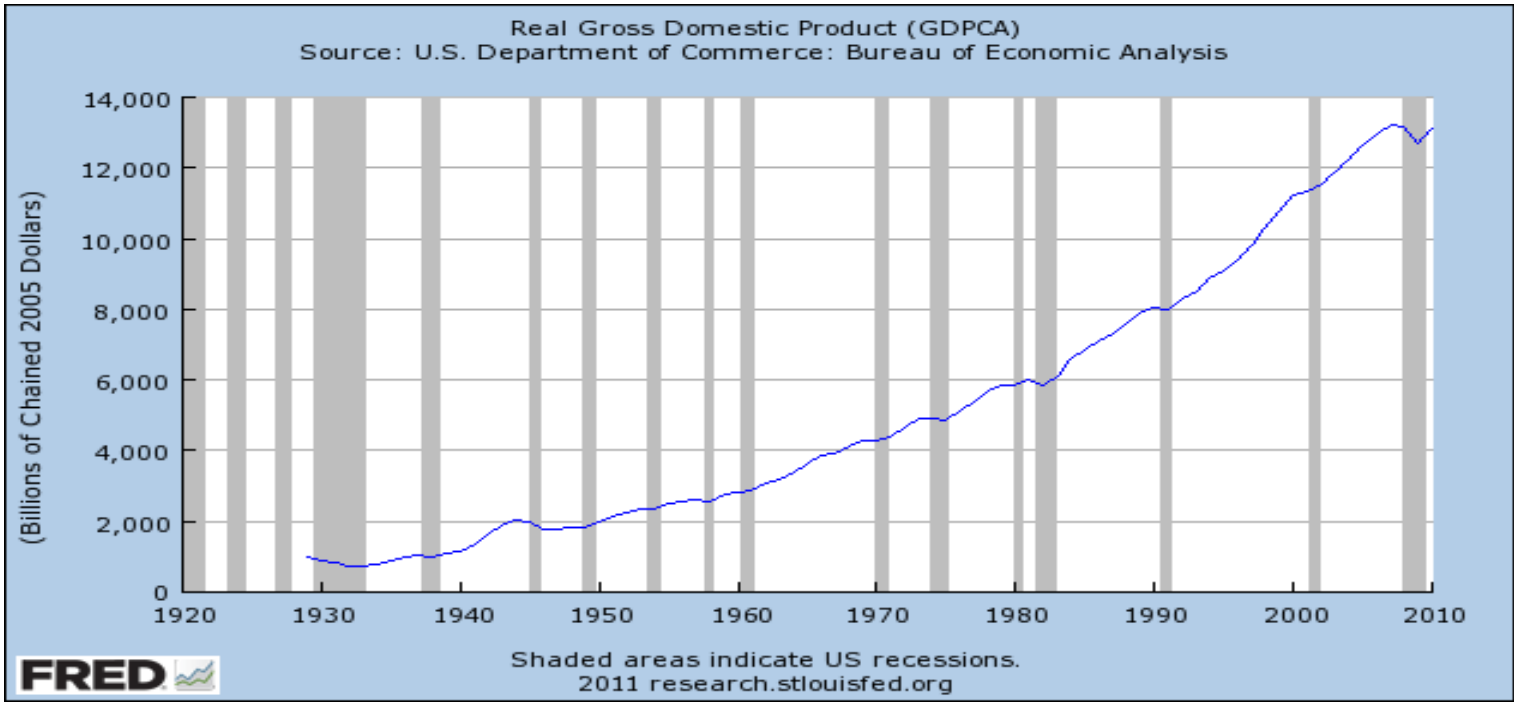
  The smooth, regular, long term movement.

  Problem: How long is long?

  Some part of a oscillating series with a very long cycle may look like an upward or downward trend.

# Example of Trend

Quarterly  GDP of USA



# Variation In Time series….

- Seasonal variation: Periodic movement of a time series where the period is not longer than one year. Seasonality is always of a fixed and known period. Hence, seasonal time series are sometimes called **periodic** time series.

  A periodic movement in a series is one which recurs or repeats at regular intervals of time.

  Examples:

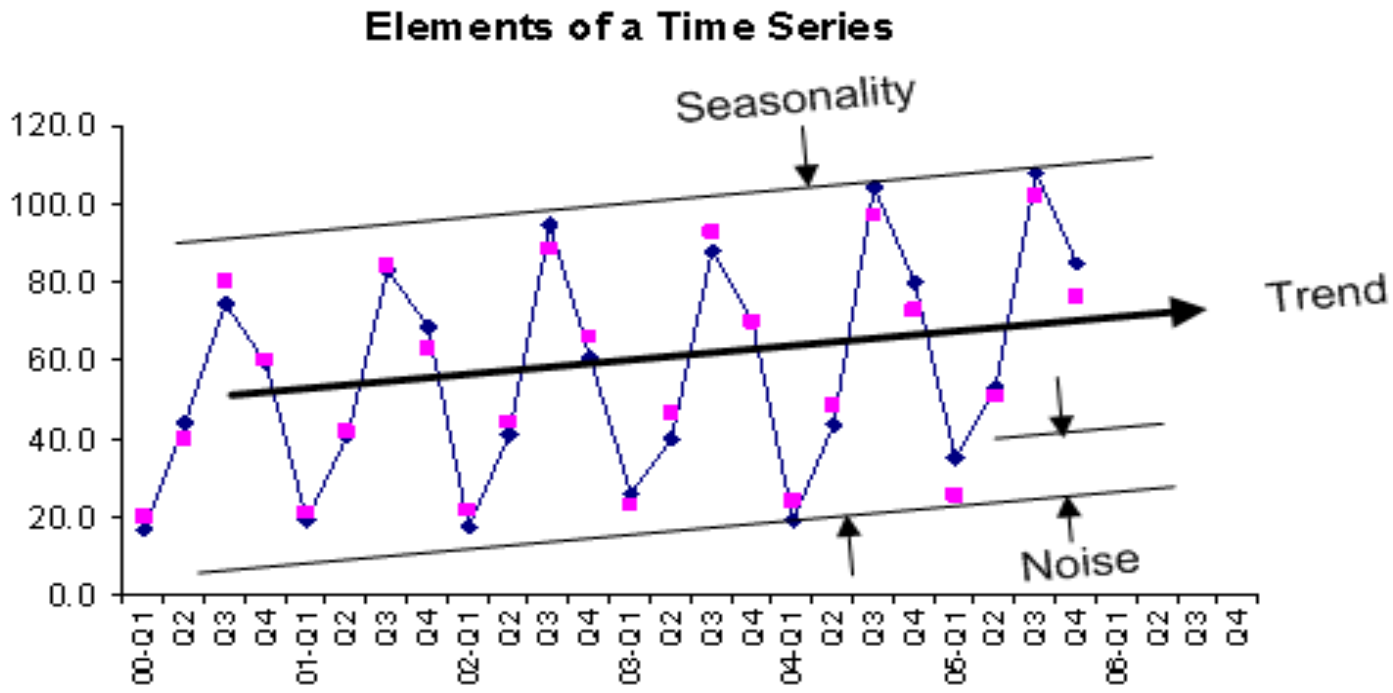
  1.The passenger traffic during 24 hours.

# Seasonal variation….

2. Sales of a departmental store during the 12 months of a year.
3. Unemployment rate is high in winter and low in summer.
4. Sales of wine/chocolates/apparels will be high at Christmas time and low at some other time.

# Quarterly Sales of Ice-cream

Q1-Dec-Jan



# Purpose of Studying Seasonality

- Efficient running of a shop; keeping inventory ready for the boom season; employing more number of employee for the boom period.
- Seasonal variation in vegetable price series is less prominent for developed country as compared to developing countries!! Infrastructure---cold-storage, food processing industry.

# Variation of Time Series…..

- Cyclical Component: The oscillatory component in a time series where the period of oscillation is more than one year.

  One complete period is called a cycle.

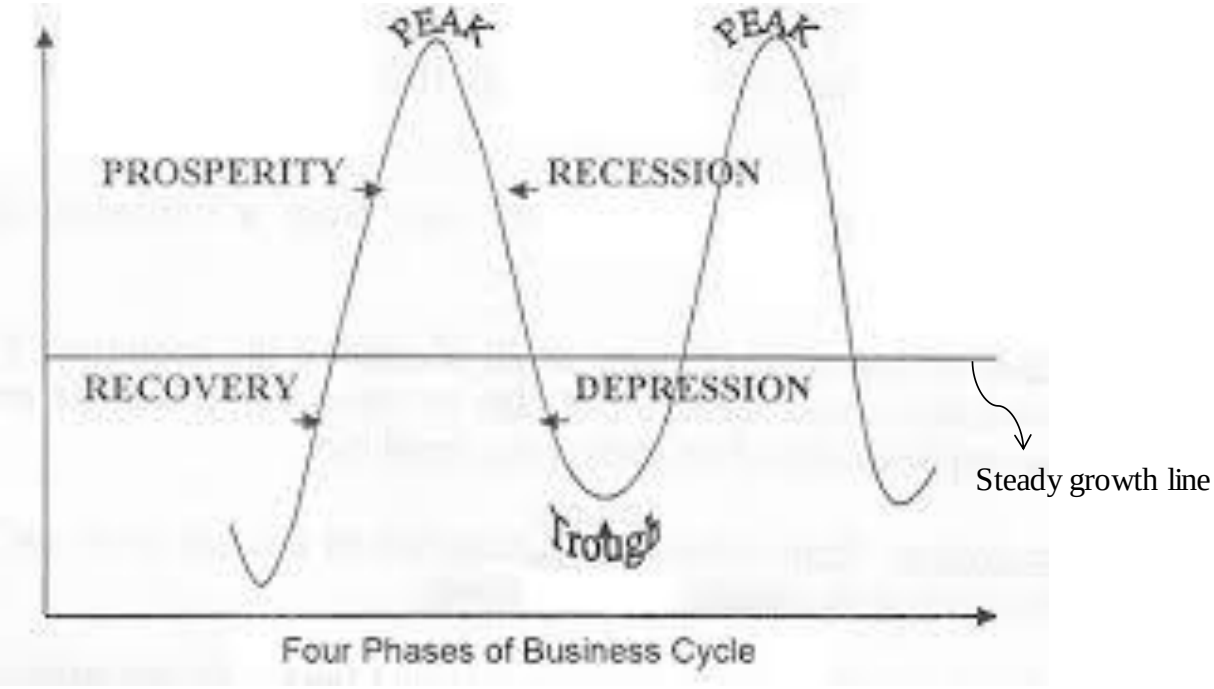
  Length of cycle, intensity of cycle, occurrence of cycle are not fixed and unpredictable.

  The duration of these fluctuations is usually of at least 2 years.

  Think of business cycles which usually last several years, but where the length of the current cycle is unknown beforehand

  EXAMPLE: Every economy goes through prosperity (boom) and depression.

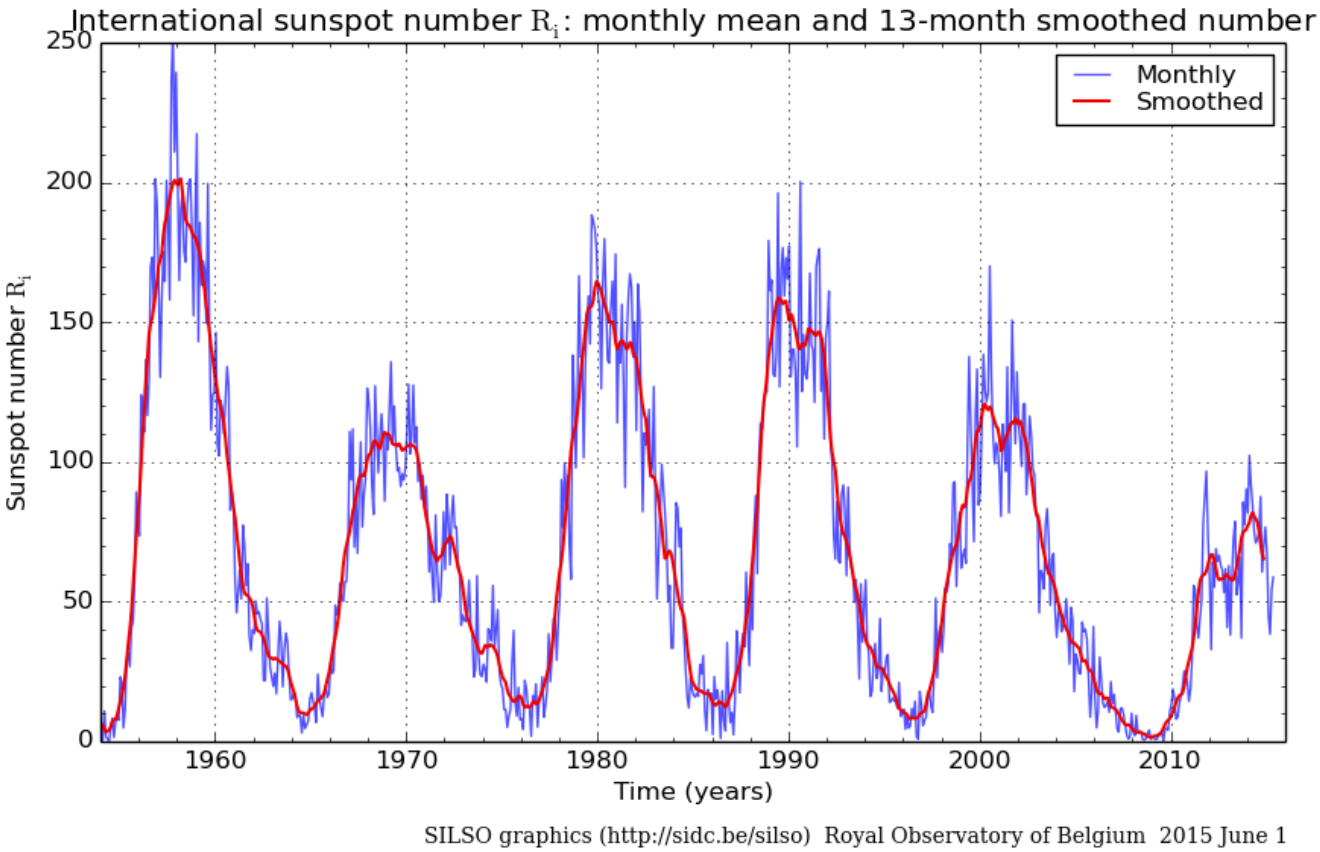
# General look of a business cycle




Regarding US economy, National Bureau of Economic research defined eleven **cycles** spanning from 1945 to the year 2009, with the average **cycle** lasting a bit over 5-1/2 years.

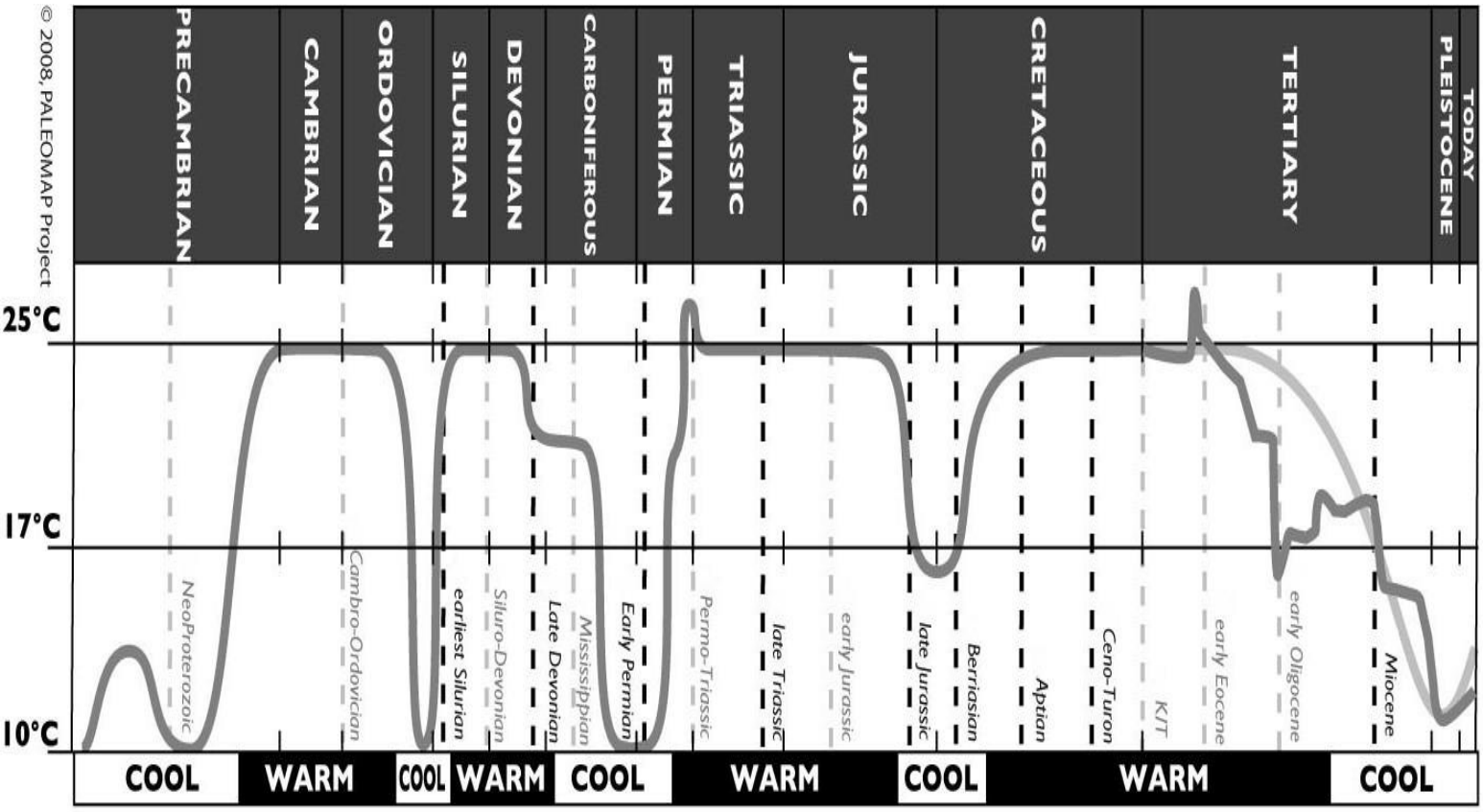
# Time series data (Number of sunspots) showing cycles:

**Sunspots** are temporary phenomena on the **Sun**'s photosphere that appear as spots darker,cooler than the surrounding areas




The **number** of **sunspots** that can be seen on the surface of sun increases and decreases in a regular pattern known as a solar cycle with a maximum **number** of **sunspots** occurring every **11.5** years

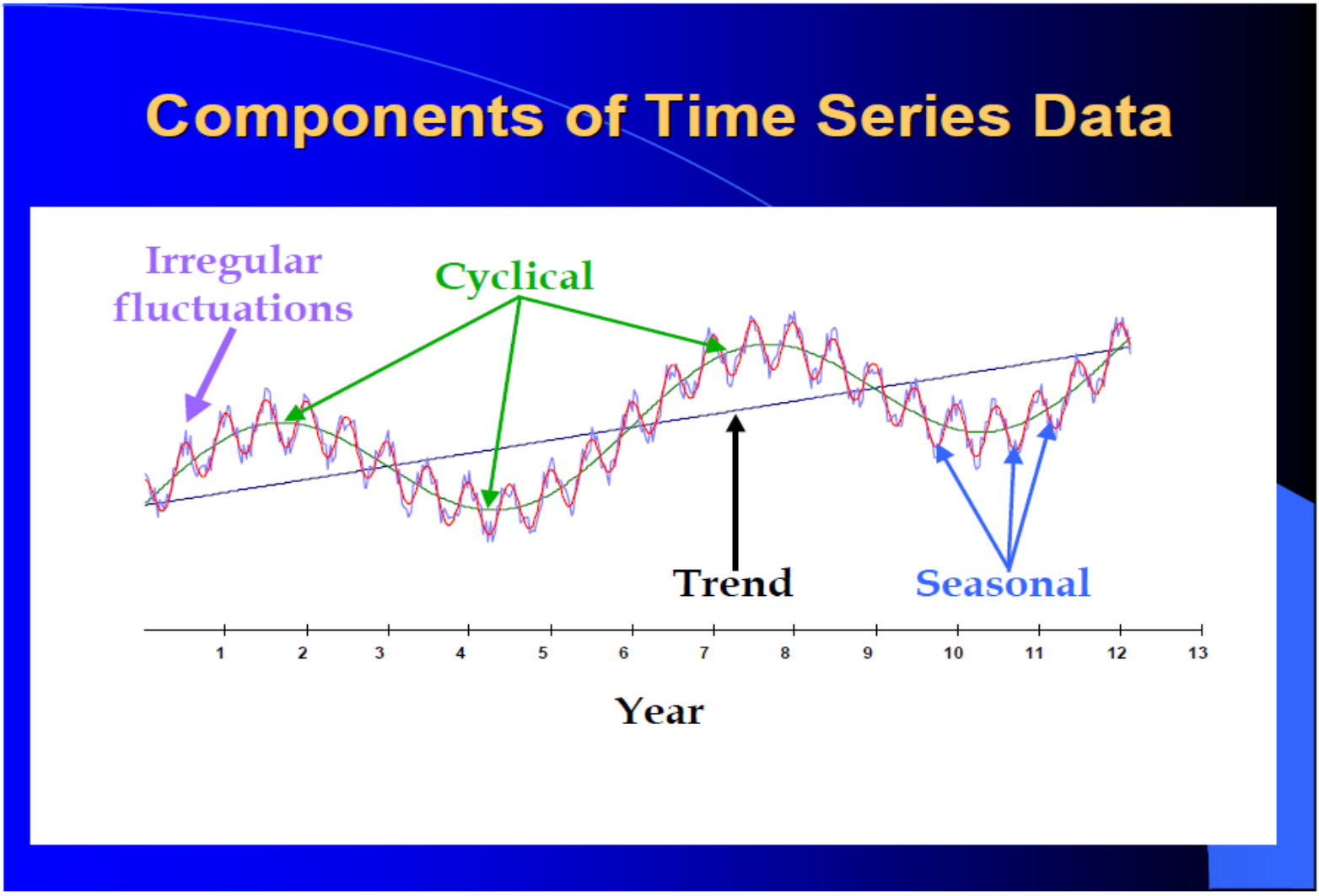
# Global average temperature: Each cycle has length of few million years.



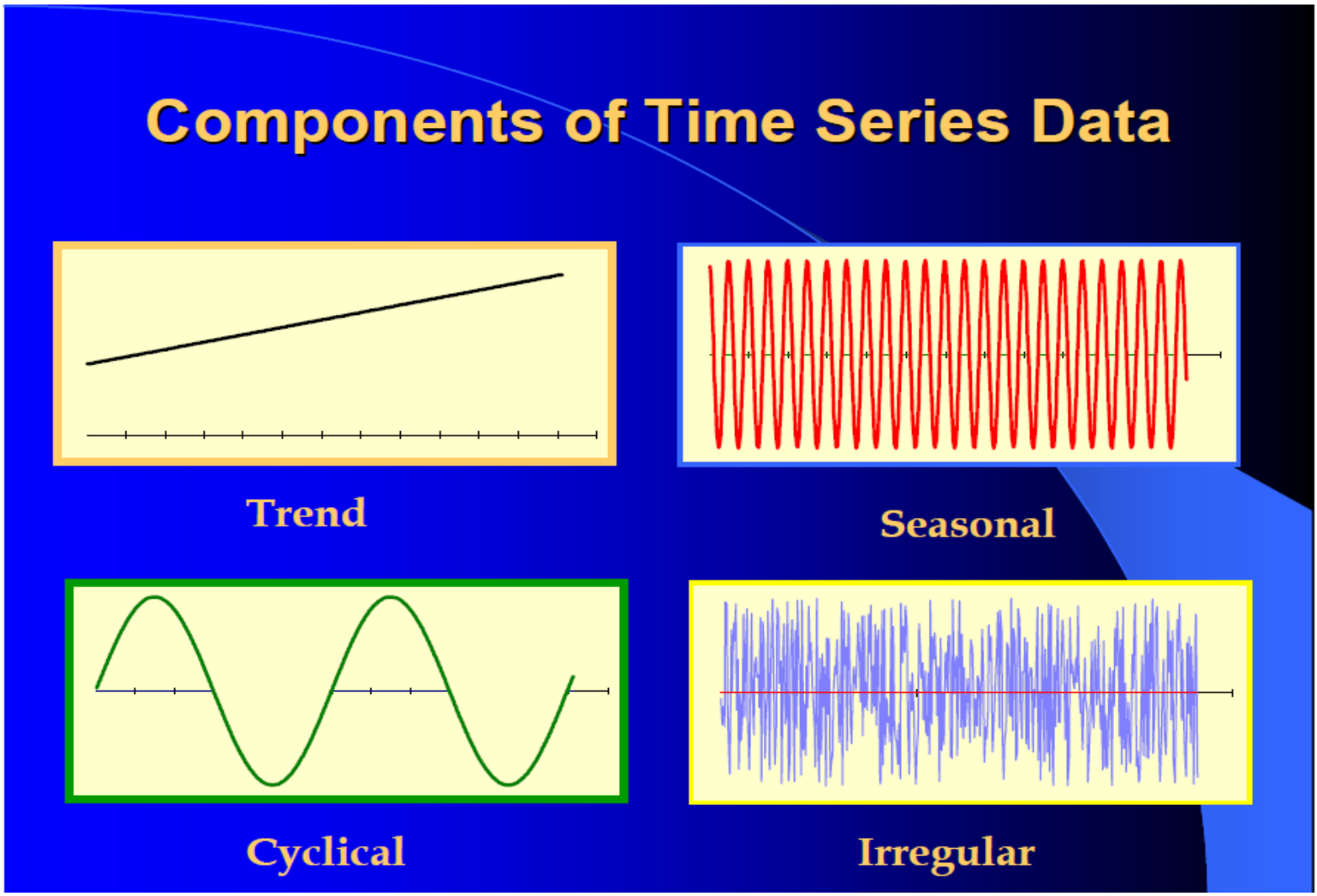
# Irregular Component

- The left over part which are completely unaccountable or are due to unforeseen events as wars, floods, strike.

# Complete visualization



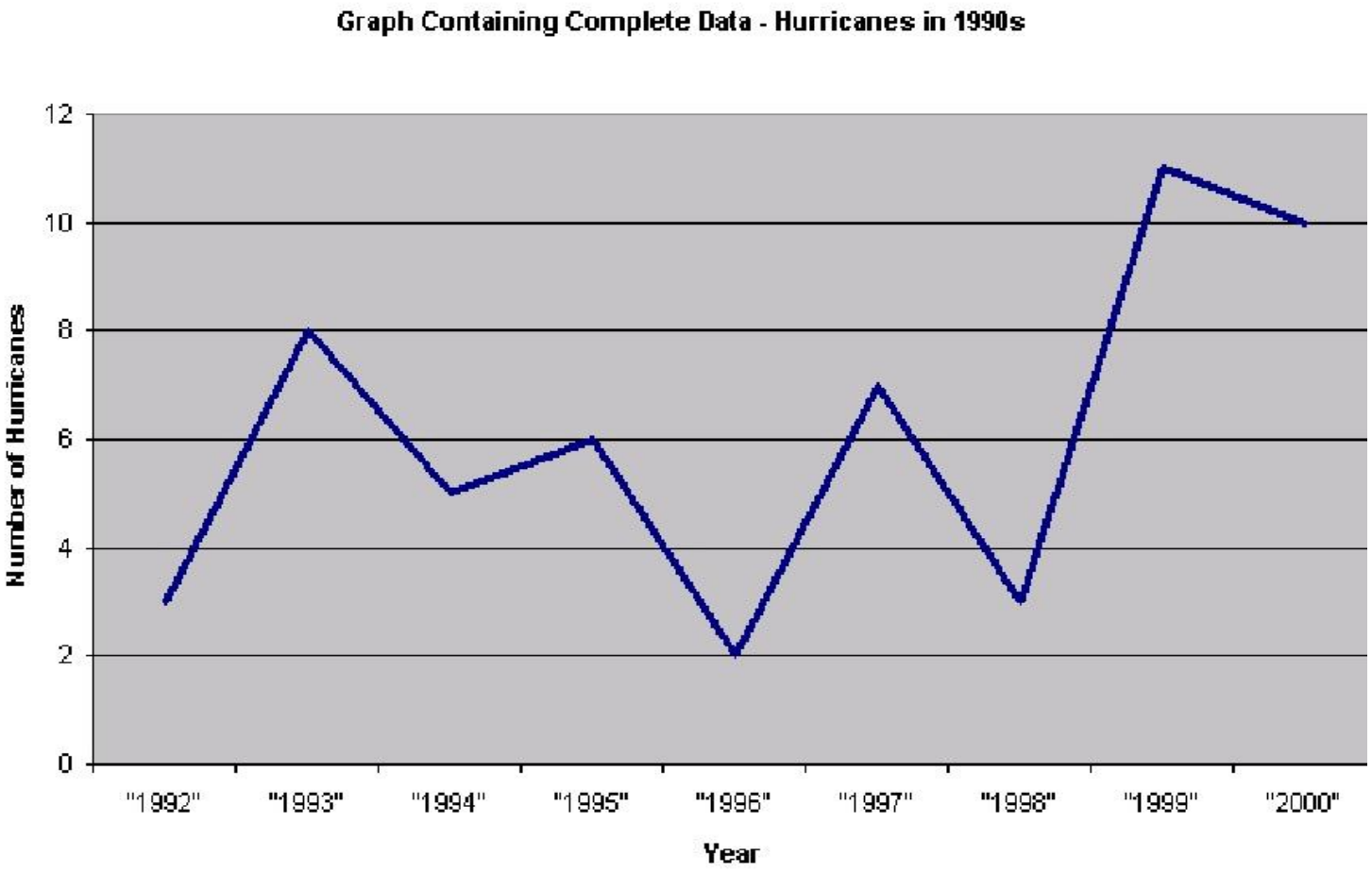
# Components at a glance



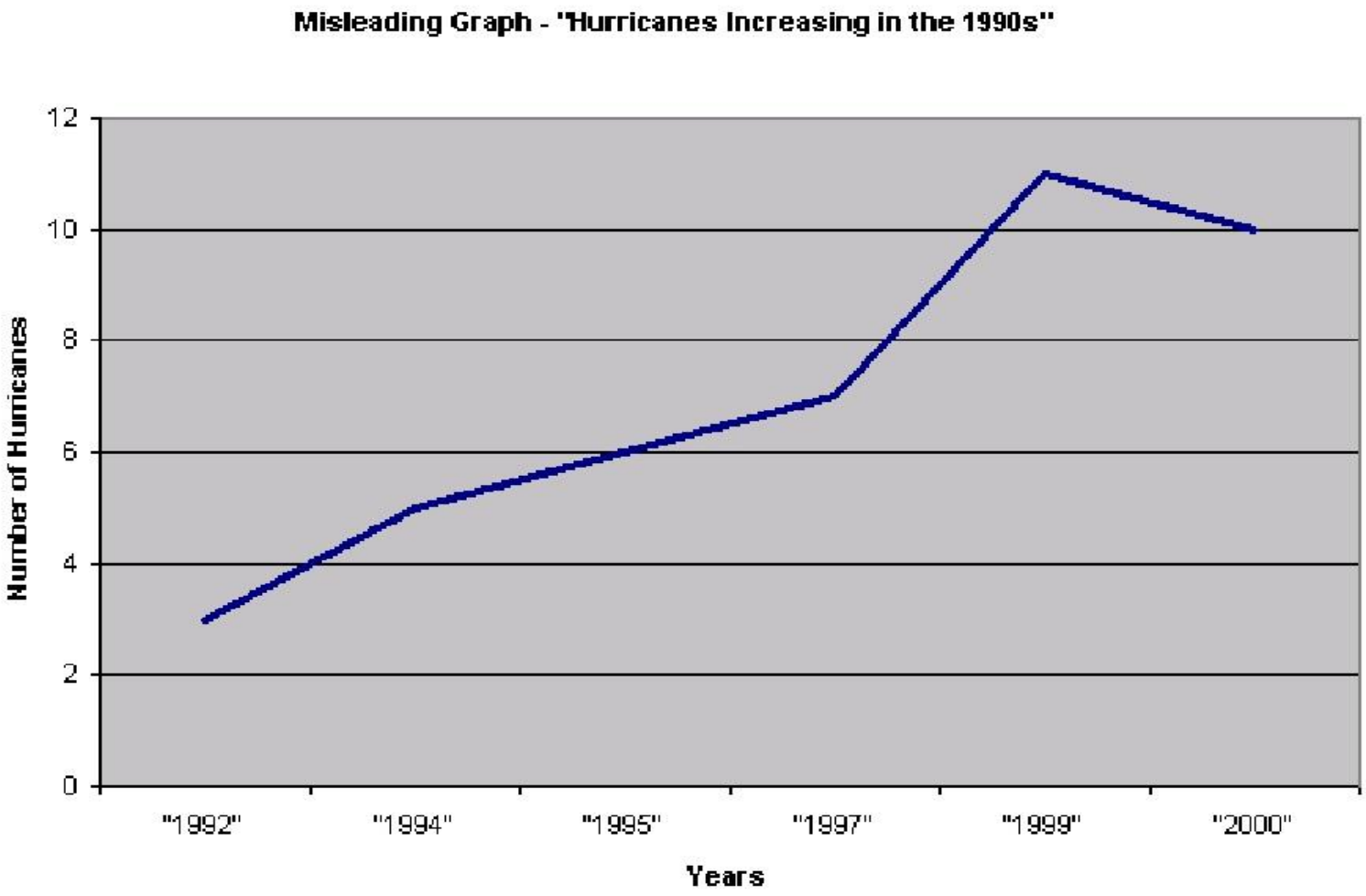
# Danger with Plot

- Generally a plot speaks a thousands words.
- However, plot needs to be studied carefully.
- Following plots elaborate---plot does not speak so many words as well----need for mathematical modeling.

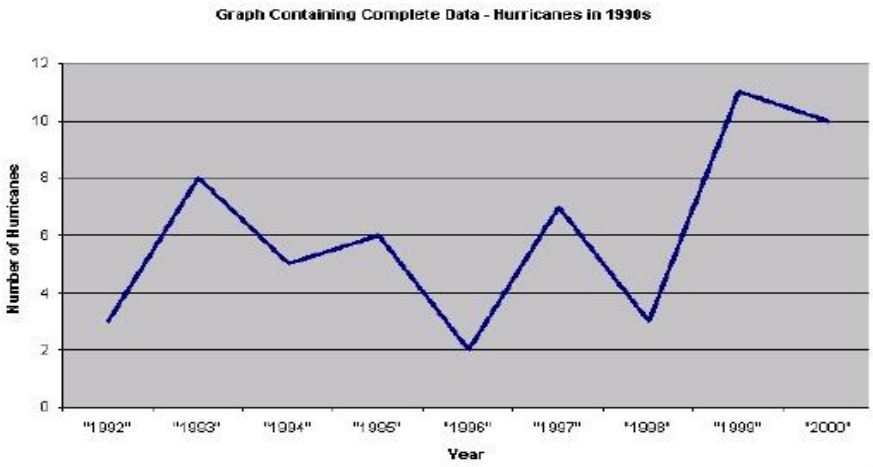
# A Time series plot:

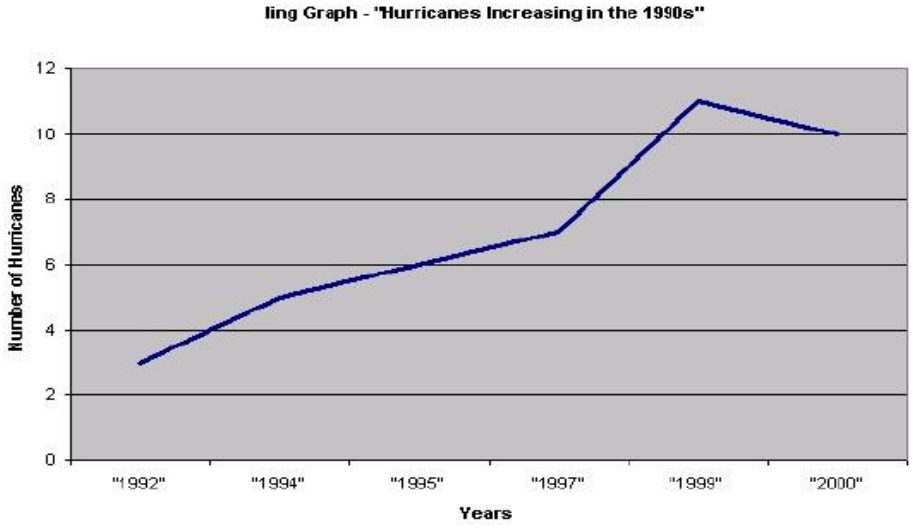


# Effect of omission of data on the Time series plot:

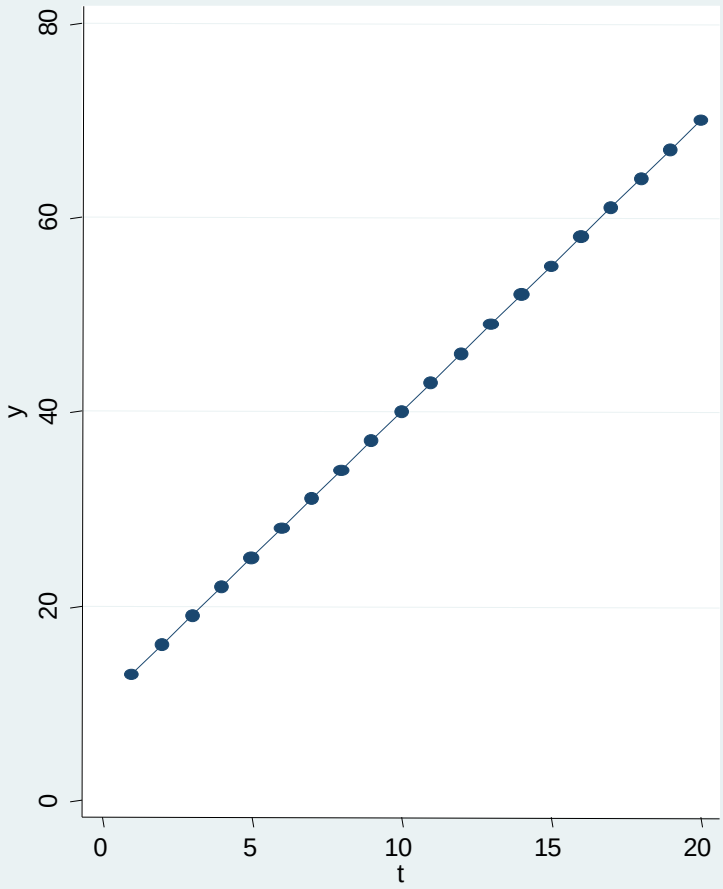


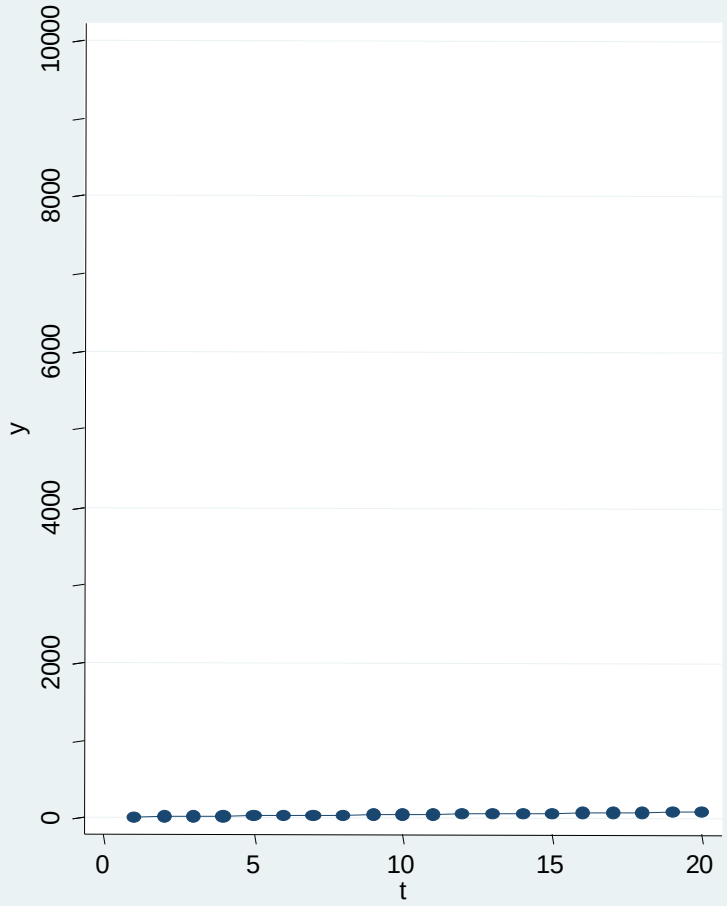
# Effect of omission of data on the Time series plot:



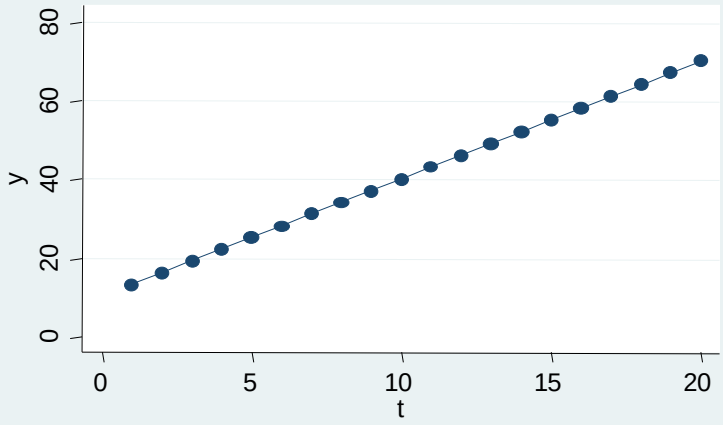



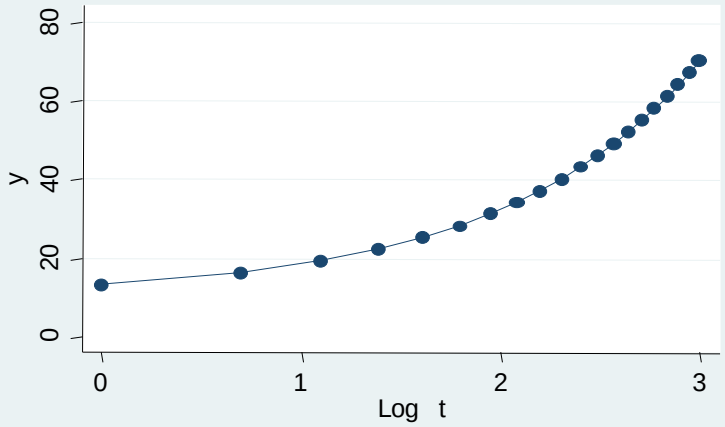
# Trend could be misleading due to scale effect:

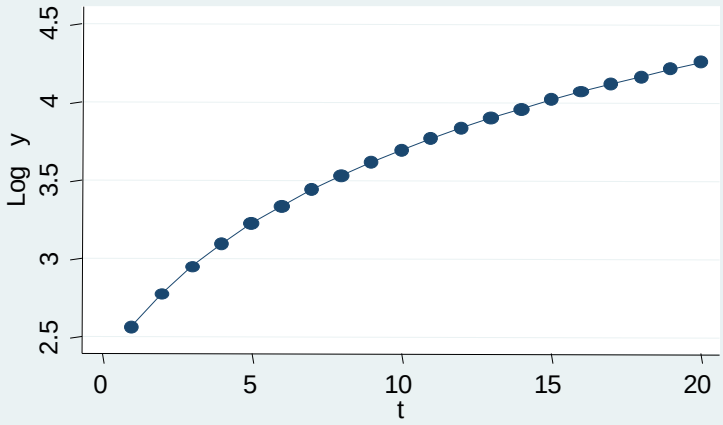


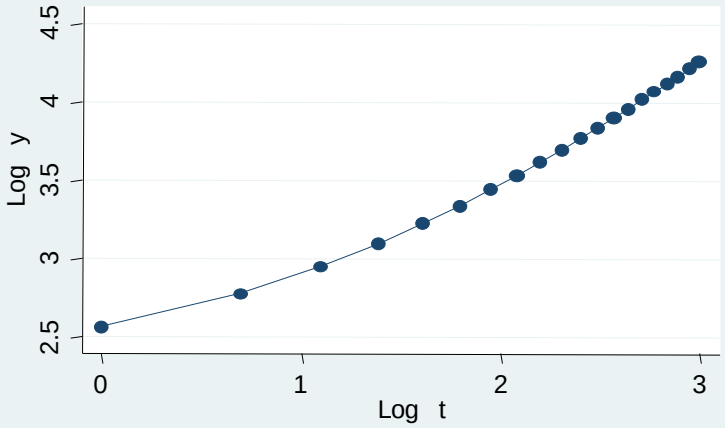



# Confusing kind of trend due to other type of scaling



# Formal (classical) Treatment of Time Series Data

Decomposition of a time series ($Y_t$) into its components:

1) Trend ($T_t$)
2) Cycle ($C_t$)
3) Seasonal ($S_t$)
4) Irregular ($I_t$)

Different combinations of the above components:

A: $Y_t = T_t \times C_t \times S_t \times I_t$ ⟶ Multiplicative Model (Most Popular)

B: $Y_t = T_t + C_t + S_t + I_t$ ⟶ Additive Model (Less Popular)

C: $Y_t = T_t \times C_t \times S_t + I_t$ ⟶ Mixed Model

D: Other Combinations ⟶ Mixed Model

# Formal (classical) Treatment of Time Series Data

## When to use Additive Model

- In some time series, the amplitude of both the seasonal and irregular variations do not change as the level of the trend rises or falls.

- In such cases, an additive model is appropriate. In the additive model, the observed time series is considered to be the sum of three independent components.

- Each of the three components has the same units as the original series.

# Formal (classical) Treatment of Time Series Data

## When to use Multiplicative Model

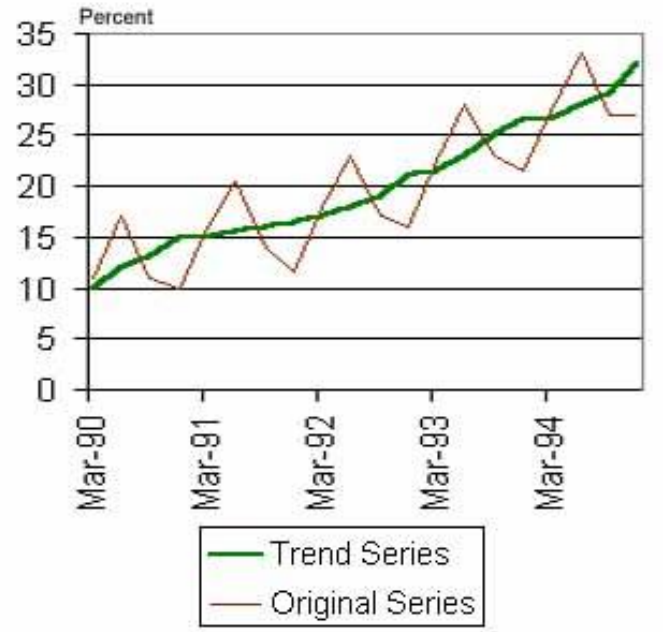
- In many time series, the amplitude of both the seasonal and irregular variations increase as the level of the trend rises.

- In this situation, a multiplicative model is usually appropriate.

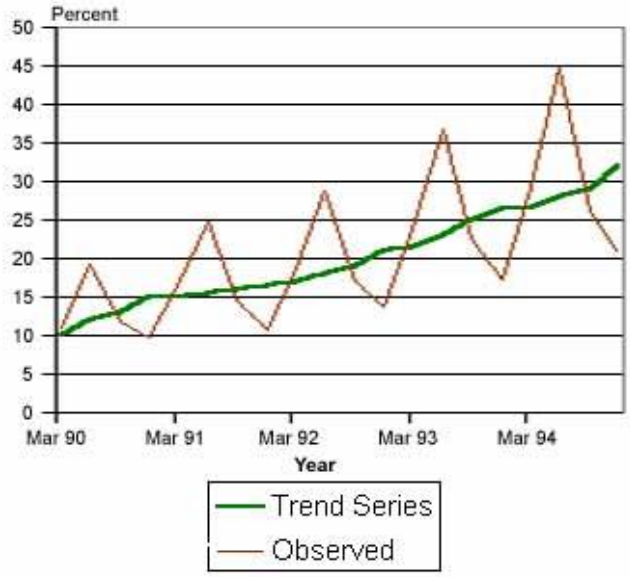
- Under this model, the trend has the same units as the original series, but the seasonal and irregular components are unit less factors, distributed around 1.

# ADDITIVE VS MULTIPLICATIVE SEASONALITY

Series for Which an Additive Model is Appropriate




Series for Which a Multiplicative Model Appropriate



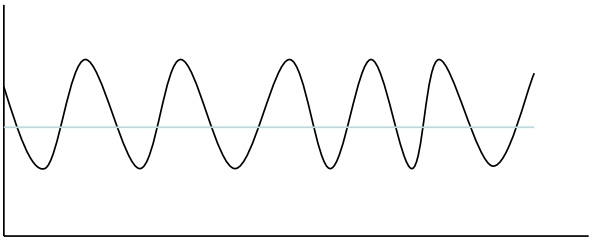
# ADDITIVE VS MULTIPLICATIVE SEASONALITY

- Seasonal components can be additive in nature or multiplicative. For example, during the month of December the sales for a particular toy may increase by 1 million dollars every year. Thus, we could *add* to our forecasts for every December the amount of 1 million dollars (over the respective annual average) to account for this seasonal fluctuation. In this case, the seasonality is *additive*.
- Alternatively, during the month of December the sales for a particular toy may increase by 40%, that is, increase by a *factor* of 1.4. Thus, when the sales for the toy are generally weak, than the absolute (dollar) increase in sales during December will be relatively weak (but the percentage will be constant); if the sales of the toy are strong, than the absolute (dollar) increase in sales will be proportionately greater. Again, in this case the sales increase by a certain *factor*, and the seasonal component is thus *multiplicative* in nature (i.e., the multiplicative seasonal component in this case would be 1.4).

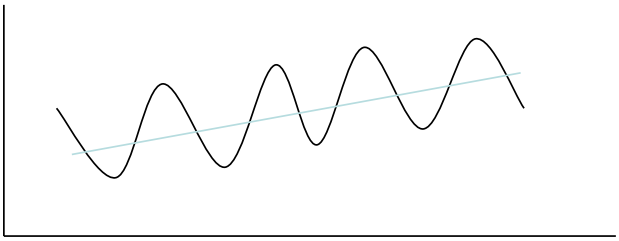
# ADDITIVE VS MULTIPLICATIVE SEASONALITY

- In plots of the series, the distinguishing characteristic between these two types of seasonal components is that in the additive case, the series shows steady seasonal fluctuations, regardless of the overall level of the series; in the multiplicative case, the size of the seasonal fluctuations vary, depending on the overall level of the series.

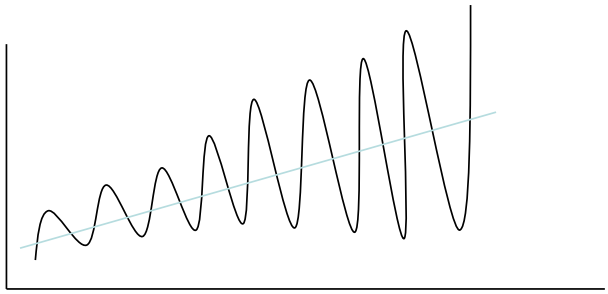
# Additive and Multiplicative Models



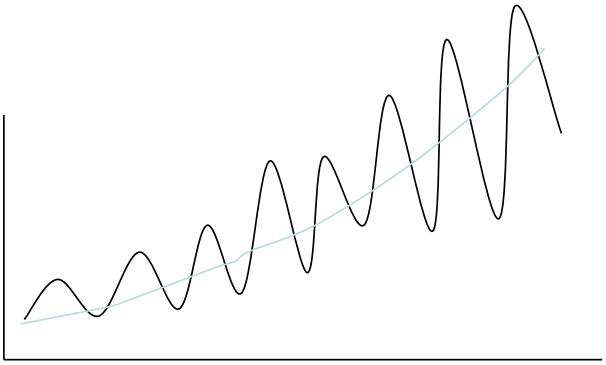
1. No trend and additive seasonal variability



2. Additive seasonal variability with trend

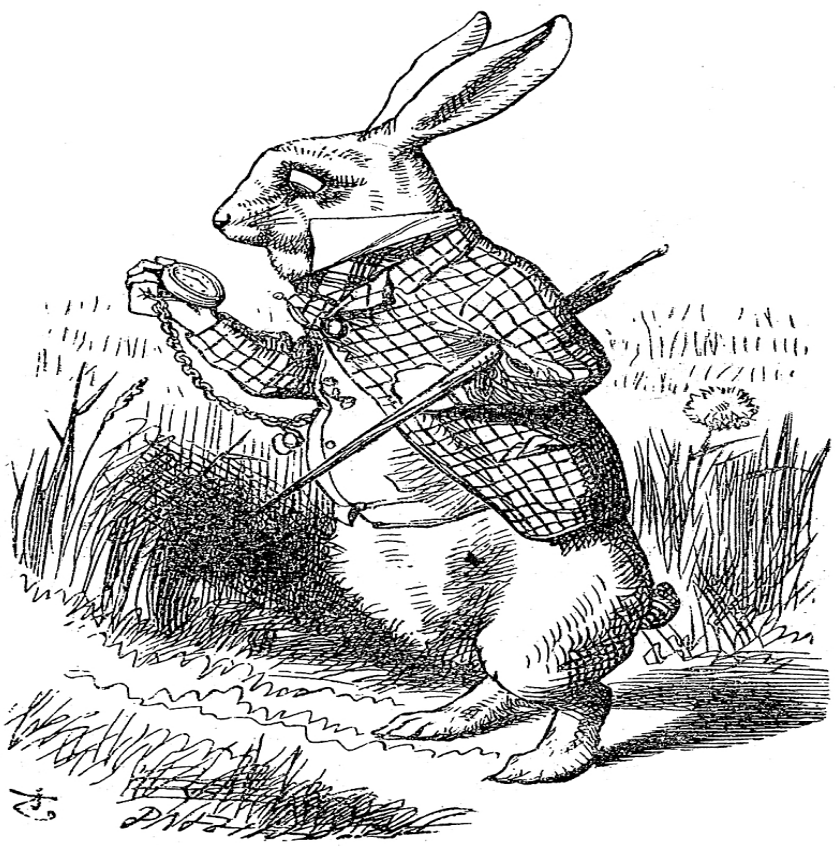


3. Multiplicative seasonal variability with a trend.



4. Multiplicative seasonal variability with a non-linear trend.

# Any Questions?



If Not Lets have fun with R

# Few time series data files available in R data repository

sunspots, LakeHuron, nottem, UKDriversdeath, Nile, JohnsonJohnson, fdeaths, EUStockMarket etc.

They are either Annual/quarterly/monthly/ daily data

- Data description, variable value, no of observations
- Convert the data in time series format (what additional features can you see? Start, end, frequency) , class of the data will be changed from numeric to time series (ts) .

- From frequency you have the idea on time gap of the data,i.e. annual (frequency=1), monthly (frequency=12), quarterly(frequency=4), you can set the frequency as well.
- From a large time series data you can take a part of data by “start” and “end” command.
- Then plot the data
- From plot check stationarity and presence of seasonality just by visual inspection.
- Decompose the time series plot as well.

# Example

Install “stats” , “tseries” package

- y<-sunspots
- y1<-ts(y)
- Class(y1)
- y1<ts(y,start=c(1770,2),end=c(1831,8),frequency=12)
- plot.ts(y1,col="blue",main="sunspots")
- Z<-decompose(y1,type=c(“multiplicative”, “additive”))
- plot(z)
- U can only plot seasonal component
- Plot(decompose$seasonal)

# Exercise

- Take a data “nottem” from r repository
- Describe the data
- What is the variable of interest here?
- What is the frequency of the data?
- Is it stationary?
- Extract first 30 observations
- Plot all the observations
- Decompose the data
- Is there cyclical component?
- Is the model additive/multiplicative? Can you say it visually?