COGS 119 - Week 0

Fundamental structure : Array

A list of numbers arranged in rows or columns.

1) Vector
2) matrix

Vector

The Simplest array (one-dimensional) is a row or column of numbers

$x = [1 \quad 2 \quad 3 \quad 4] \rightarrow$ row vector

$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \rightarrow$ column vector

Matrix

A two-dimensional array (or more complex) : a collection of numbers arranged in rows and columns.

$$x = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \downarrow \text{rows} \Rightarrow \underbrace{2 \text{ rows}, 3 \text{ columns}}_{2 \times 3 \text{ matrix}}$$

$\xrightarrow{\hspace{2em}}$ columns

$$y = \begin{bmatrix} 10 & -4 & 6 \\ 5 & 2 & 8 \\ 3 & -7 & 12 \\ -9 & 4 & -15 \end{bmatrix} \Rightarrow \underbrace{4 \text{ rows}, 3 \text{ columns}}_{4 \times 3 \text{ matrix}}$$

Note: All the rows in a matrix must have the same number of elements.

Addressing Array Elements

Vector

- The address of an element in a vector is its position in the row or column.
- For a vector v, v(k) refers to the element in position k.

Example:

$$v = [\,3 \quad 12 \quad -4 \quad 5 \quad -8\,]$$

$v(1) = 3$, $v(3) = -4$, $v(5) = -8$.

Matrix

- The address of an element in a matrix is its position defined by the row number and column number where it is located.
- For matrix m, m(i,j) refers to the element in row i and column j.

Example:

$$m = \begin{bmatrix} -3 & 10 & 5 & -2 \\ 8 & -7 & 6 & 3 \\ 12 & 1 & 0 & 9 \end{bmatrix}$$

$m(1,1) = -3$ $m(2,1) = 8$ $m(3,4) = 9$

$m(1,2) = 10$ $m(2,3) = 6$ $m(4,3) =$ None

$m(1,4) = -2$ $m(3,2) = 1$ $m(3,5) =$ None.

Matrix Operations

Addition

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$\Rightarrow$ matrices need to be the same size

$$A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

Subtraction

$\Rightarrow$ matrices need to be the same size.

$$\begin{bmatrix} 3 & -8 \\ -5 & 7 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3-4 & -8-(-1) \\ -5-2 & 7-(-4) \end{bmatrix} = \begin{bmatrix} -1 & -7 \\ -7 & 11 \end{bmatrix}$$

Multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$$

$$\overset{\text{matrix}}{(2\times 2)} \times \overset{\text{matrix}}{(2\times 2)}$$

Step 1 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (1*3)+(2*2) & - \\ - & - \end{bmatrix}$

Step 2 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (1*3)+(2*2) & (1*4)+(2*2) \\ - & - \end{bmatrix}$

Step 3 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (1*3)+(2*2) & (1*4)+(2*2) \\ (3*3)+(4*2) & - \end{bmatrix}$

Step 4 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} (1*3)+(2*2) & (1*4)+(2*2) \\ (3*3)+(4*2) & (3*4)+(4*2) \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 12 & 20 \end{bmatrix}$

Rule about multiplication

The number of columns of the first matrix should be equal to the number of rows of the second matrix.

$$\underset{\text{matrix}}{(2 \times 3)} \; * \; \underset{\text{matrix}}{(3 \times 4)} \quad ✓ \quad \longrightarrow \quad (2 \times 4) \text{ matrix.}$$

$$\underset{\text{matrix}}{(2 \times 3)} \; * \; \underset{\text{matrix}}{(2 \times 3)} \quad X$$

Element-by-element Multiplication

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & -5 & -1 \end{bmatrix} .* \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2*0 & -2*-1 & 3*2 \\ 1*1 & -5*0 & -1*3 \end{bmatrix}$$

note the sign

$$= \begin{bmatrix} 0 & 2 & 6 \\ 1 & 0 & -3 \end{bmatrix}$$

matrices should be the same size.

Operations with scalars

$$2 + \begin{bmatrix} 7 & 8 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+7 & 2+8 \\ 2+2 & 2+3 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 3 & -2 \end{bmatrix} * (-3) = \begin{bmatrix} (-1)*(-3) & 2*(-3) \\ 3*(-3) & -2*(-3) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -9 & 6 \end{bmatrix}$$

Transpose Operator

Vector

It switches a row vector to a column vector, and a colum vector to a row vector.

$$A = \begin{bmatrix} 3 & 8 & -2 & 1 \end{bmatrix}, \quad A' = \begin{bmatrix} 3 \\ 8 \\ -2 \\ 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}, \quad B' = \begin{bmatrix} 2 & 3 & 8 \end{bmatrix}$$

Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

2×3 matrix $\longrightarrow$ 3×2 matrix

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad B' = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$