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A New Hybrid Machine Learning Method for Stellar Parameter Inference

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ABSTRACT

The advent of machine learning (ML) is revolutionary to numerous scientific disciplines, with a growing number of examples in astronomical spectroscopic inference, as ML is more powerful than traditional techniques. Here we introduce a hybrid ML (HML) method combining automatic differentiation, interpolation, and Bayesian optimization to infer stellar parameters of stellar spectra. We study T_{eff} , $\log(g)$, and [Fe/H], but this method **could be extended** to other **parameters** such as $[\alpha/\text{Fe}]$ (alpha element abundance), C/O (carbon-oxygen ratio), and f_{sed} (sedimentation efficiency). We first use blase to semi-empirically recast spectra into sets of Voigt profiles. blase is run on 1,314 synthetic spectra from a rectilinear subset of the PHOENIX model grid $(T_{\text{eff}}: [2300, 12000] \text{ K}, \log(g): [2, 6], [\text{Fe/H}]: [-0.5, 0], \lambda: [8038, 12849] \text{ Å}). \text{ For } 128,723$ detected features, we map stellar parameters to spectral line parameters using linear interpolation. This creates the PHOENIX generator, enabling parallelized spectral synthesis. Gaussian Process minimization is used to infer stellar parameters by minimizing a root-mean-square loss function. Testing 210 noise-free models (T_{eff} : [3000, 11000] K, $\log(q)$: [2, 6], [Fe/H]: [-0.5, 0]), we find inference errors: T_{eff} : 93 K, $\log(g)$: 0.24, and [Fe/H]: 0.056 for T_{eff} < 7000 K, and T_{eff} : 347 K, $\log(g)$: 0.26, and [Fe/H]: 0.16 for $T_{\text{eff}} \geq 7000$ K. We also upload online an archive of blase models of the PHOENIX subset. This proof-of-concept study shows that semi-empirical HML is a viable alternative to traditional approaches in spectroscopic inference.

1. INTRODUCTION

Stellar spectra are exceedingly rich sources of infor-23 24 mation about the stars that produce them. Spectra en-25 code fundamental properties such as temperature, sur-26 face gravity, and chemical composition via their numer-27 ous absorption lines. Extrinsic properties such as the 28 stellar radial velocity or the projected equatorial rota-29 tion shift the wavelengths of lines and broaden their 30 widths, respectively. Observing the spectrum alters it 31 again; the resolution, bandwidth, and other properties 32 of the instrument change the fidelity at which we observe 33 the spectrum, and can limit our ability to extract fun-34 damental properties precisely. Observed stellar spectra 35 thus represent extremely complex, data, influenced by 36 multiple parameters, that have gone through multiple 37 transformations before reaching our detectors.

Modern astronomical spectroscopy takes advantage of intuitive, performant spectral models such as KORG (Wheeler et al. 2023), however it would

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41 further benefit from the paradigm of interpretabil-42 ity. The determination of fundamental properties, the 43 creation of Extreme Precision Radial Velocity (EPRV) 44 templates, and the application of composite spectral fit-45 ting could all leverage such an innovation. This am-46 bitious aim may be broadly referred to as a "founda-47 tional spectroscopy model for astrophysics", in reference 48 to the same category of models used for large language 49 models (LLMs) in artificial intelligence (AI). There are 50 many challenges with creating such a foundational spec-51 tral model for astrophysics. Physical inputs into the 52 spectral modeling process are imperfect, and simplify-53 ing assumptions in stellar (and substellar) atmospheres 54 are necessarily inexact due to factors such as asym-55 metries and unknown physics. Computational costs 56 make the training of such models challenging; how-57 ever, advances have been made with attention-58 based (Różański et al. 2023) and transformer-59 based (Leung & Bovy 2024) models, among oth-60 ers. Most prevailing solutions have had to choose ei-61 ther a model driven approach (such as this work), in 62 which precomputed models are taken as gospel, or a data 63 driven/empirical approach (such as Lux (Horta et al. 64 2025)), in which our knowledge of stellar physics is ignored or treated phenomenologically, depending on the application. Hybrid solutions exist and have been implemented, such as those presented in Leung & Bovy 2019 and Rains et al. 2024. Although a lingering challenge is always how to ideally balance the fusion of model driven and data driven paradigms.

Spectroscopic surveys such as APOGEE (Majewski et al. 2017) use their own in-house pipelines to extract stellar parameters from spectra (ASPCAP (Gartract stellar parameters from spectra (ASPCAP (Garfor the exact al. 2016) in the case of APOGEE), and
these pipelines appear effective. The core assumption
for the vast majority of pipelines is that the data to be
analyzed is a list of pixels. Analysis pipelines may be
closed-source or limited to the scope of the survey itself,
causing a sort of siloing effect among surveys. This motivates the development of more universal, instrumentagnostic open-source frameworks that can apply broadly
to a range of spectral observations with relatively little
tuning.

Multiple efforts have been made in this direction, treating spectra in different ways. The standard practice is to treat the wavelength and flux as simply two arrays and use bespoke algorithms tailored to a small number of well-calibrated spectral lines to obtain fundamental stellar parameters and chemical compositions (e.g. the IGRINS YSO Survey (López-Valdivia et al. 2023) and the IRTF Spectral Library (Rayner et al. 2009)). Other whole-spectrum fitting abstractions decompose model spectra into an eigenbasis, implementing the data-model comparison stage as a tractable regression, such as starfish (Czekala et al. 2015).

Ideally, we want a system that can self-consistently 99 learn, a genuine AI foundational spectral model. Such a 100 system would enable the assignment of accurate stellar parameters, and could yield re-usable interpretable spec-102 tral models. blase, first presented in Gully-Santiago 103 & Morley 2022, took an important, albeit limited, step in this direction, treating spectra not as a set of pixels 105 or a set of eigenbasis coefficients but as a set of inter-106 pretable and traceable spectral lines, specifically Voigt 107 profiles. Each of these approaches has tradeoffs, but one 108 key scientific advantage of blase comes from its ability 109 to adapt to new information, while preserving some ad-110 herence to physics-based models. This intelligent capability stems from its ability to fit a theoretically unlim-112 ited number of nonlinear spectral line parameters with automatic differentiation (autodiff). Autodiff is a tech-114 nology that tracks transformations made to data using 115 the chain rule, even being able to differentiate control 116 flow transformations involving if-else blocks, for exam117 ple. With the gradient obtained from autodiff's chain 118 rule, we can optimize model parameters by just going against the direction of the gradient (because we usu-120 ally want to minimize something such as a loss function, 121 we go in the direction of greatest decrease, which is al-122 ways the negative of the gradient). Autodiff has been 123 used successfully in other astrophysical contexts such as 124 exojax (Kawahara et al. 2022) and wobble (Bedell 125 et al. 2019), but the recasting of spectra into sets of 126 inherently nonlinear spectral lines positions blase as a 127 unique and promising semi-empirical tool. The origi-128 nal blase paper demonstrated the ability to tune spec-129 tral lines with autodiff and 'clone' spectra, recasting 130 them as ML models defined by interpretable sets 131 of tuned spectral lines, however it was restricted 132 to a pre-selected static synthetic model.

Here we introduce the next logical step in the sequence of expanding interpretable spectroscopic machine learning ing from operating on a single grid point and towards an entire 3D grid of precomputed spectra. We rebrand this augmentation as blase3D.

Ideally this process would be monolithic, with the 139 end-to-end spectral inference code powered by a sin-140 gle autodifferentiable machine learning framework, like 141 PyTorch (Paszke et al. 2019) or JAX (Bradbury 142 et al. 2018). However, here we separate the prob-143 lem into three pieces, only the first currently being 144 autodifferentiable. In section 2, we scale out the 145 blase method to 1,314 precomputed synthetic spectral 146 model clones, yielding a downloadable archive of pretrained machine learning models with 128,723 unique 148 spectral lines. In section 3, we then fit manifolds 149 mapping stellar parameters to uniformly-derived spec-150 tral line parameters using regular grid linear interpola-151 tion. Finally, in section 4, we show how reconstruc-152 tions of the spectra using said manifolds can be used 153 for inferring stellar fundamental properties from spec-154 tra. This final step resembles the aims of "atmospheric 155 retrievals", but in principle should be faster, adaptive, 156 and reusable. An overview of this process is shown in 157 Figure 1.

2. CLONING THE PHOENIX MODEL GRID

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2.1. The PHOENIX Subset

For the purposes of this study, we chose the widely-adopted PHOENIX synthetic spectral model grid (Husser et al. 2013). Our approach can be straightforwardly applied to any other model grid in the future, including substellar atmosphere grids such as Sonora (Marley et al. 2021; Karalidi et al. 2021; Morley et al. 2024; Mukherjee et al. 2024), but for now we limit our scope to a rectilinear subset of the PHOENIX grid,

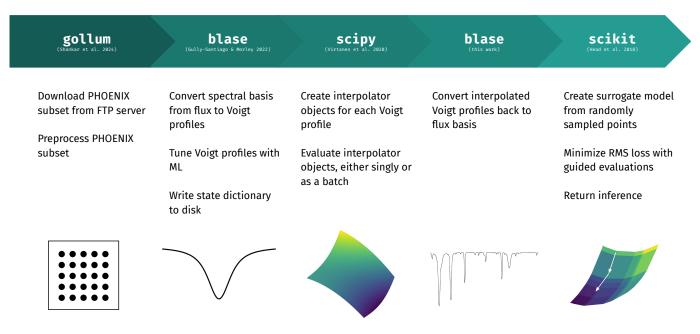


Figure 1. Overview of the process used in this study.

168 focusing on near solar metallicities and a broad range of
169 effective temperature and surface gravity, with details
170 given in Table 1. This approach is taken due to the
171 computational cost and storage impact of inter172 polation algorithms. Future versions with more
173 advanced models will be able to cover a broader
174 scope and bypass the limitation of a rectilinear
175 subset.

2.2. Preprocessing with gollum

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First, the PHOENIX subset was programmatically retrieved with gollum (Shankar et al. 2024), which downloaded spectra from the PHOENIX FTP server. The spectra were then put through a short preprocessing pipeline similar to that from Gully-Santiago & Morley 2022.

- 1. Blackbody Division: Since the $T_{\rm eff}$ of each spectrum is known, the **corresponding** blackbody spectrum was divided out.
- 2. Continuum Normalization: The spectra were further normalized by dividing them by a 5th order polynomial continuum fit using a peak-finding filter in order to eliminate curvature that would inhibit line modeling.

Mathematically, we can express the preprocessing as follows:

$$\bar{S} = \frac{S}{BQ_5} \tag{1}$$

where \bar{S} is the preprocessed spectrum, S is the original spectrum, B is the blackbody spectrum, and Q_n is the nth order polynomial continuum fit. Arithmetic operations between arrays are assumed to be elementwise in all following notation.

2.3. Line Identification with blase

The next step was to convert the PHOENIX subset 200 into a physically interpretable intermediate representa-201 tion: a table of spectral line properties rather than an 202 array of fluxes. We used blase, which models spectral 203 lines as Voigt profiles and tunes the profiles to mimic the 204 original PHOENIX spectrum with back propagation. 205 Back propagation, put simply, is the process of moving 206 in the autodiff gradient field to update ML model param-207 eters (like mentioned earlier, usually against the gradi-208 ent because we minimize loss functions). Four parame-209 ters were optimized: the line center μ , the log-amplitude $_{210}$ ln(a), the Gaussian width σ , and the Lorentzian width γ . 211 The optimization used the Adam optimizer (Kingma & 212 Ba 2017) with a learning rate of 0.05 over 100 epochs. 213 Gully-Santiago & Morley 2022 recommends this 214 setup as the minimum, and for a proof-of-concept 215 implementation, we found it best to leave it as is. 216 In addition, we limited two custom parameters: wing 217 cut to 6,000 and prominence to 0.005. Wing cut (in 218 **pixels**) is a parameter that determines the extent of 219 the Voigt profile to evaluate, saving computational re-220 sources by not evaluating negligible line wings. Promi-221 nence (in normalized flux counts) sets a lower limit 222 for the amplitude of detected lines, which saves resources

Parameter	Symbol	Interval	Sampling
Alpha Element Abundance	α	[0]	N/A
Iron Abundance	$[\mathrm{Fe/H}]$	[-0.5, 0]	0.5
Effective Temperature	$T_{ m eff}$	[2300, 12000] K	100 K until 7000 K, then 200 K
Surface Gravity	$\log(g)$	[2, 6]	0.5
Wavelength	λ	[8038, 12849] Å	$\mathbf{R}=500,000$

Table 1. The subset of the PHOENIX grid used in this study. These limits were imposed to reduce the computational cost of the algorithms and to ensure a rectilinear parameter space in order to work with scipy's RegularGridInterpolator (Virtanen et al. 2020). The wavelength limits in particular roughly line up with that of the Habitable Zone Planet Finder (HPF) spectrograph (Mahadevan et al. 2012). This subset is comprised of 1,314 individual spectra: $73 T_{\rm eff}$ values, $9 \log(g)$ values, and $2 [{\rm Fe/H}]$ values.

by disregarding shallow lines, so in our case we disregard lines with amplitude less than 0.005. In short,
our choices for wing cut and prominence decrease the
computational cost of blase's cloning process at the expense of decreasing its accuracy slightly. blase uses the
pseudo-Voigt approximation, which saves on computational cost while remaining accurate to about 1% (Ida
to et al. 2000). The pseudo-Voigt approximation uses a
weighted average of a Gaussian and Lorentzian as opposed to a convolution. blase's pseudo-Voigt profile
implementation uses the following:

$$\tilde{\mathbf{V}}_{\mu}(\lambda) = a \left[\eta \mathbf{L}(\lambda - \mu'; f) + (1 - \eta) \mathbf{G}(\lambda - \mu'; f) \right]$$

$$\eta = \sum_{n=0}^{2} \mathbf{u}_{n} \left(\frac{2\gamma}{f} \right)^{n+1}$$
(2b)

$$f = 32 \sum_{n=0}^{5} \mathbf{v}_n \left(\sqrt{2 \ln(2)} \sigma \right)^{5-n} (\gamma)^n$$
 (2c)

$$\mathbf{u} = \begin{bmatrix} 1.36603 \\ -0.47719 \\ 0.11116 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 2.69269 \\ 2.42843 \\ 4.47163 \\ 0.07842 \\ 1 \end{bmatrix}$$

where **L** and **G** are abbreviations for Lorentzian and Gaussian profiles, respectively. Notice that we use μ' instead of μ in the formula. This is because blase optionally allows the line center to shift slightly durage ing optimization, and it is this shifted center which is used in computation. The individual Voigt profiles are still indexed by μ for cross-model line identification, explained in the next section. Once optimization was complete, the list of identified lines was saved to a 'state dictionary': a common representation for pre-trained machine learning models that can be stored to disk for the reuse later. These are stored in the .pt file format for each of the 1,314 PHOENIX subset grid points. The total disk space these files take up is 465 MB (382 MB) when downloaded as a zip archive). For reference, the

249 storage space the PHOENIX subset takes up on disk is 250 approximately 8.1 GB. This represents a data compres-251 sion factor of around 20 just by recasting the spectrum 252 with blase. The state dictionaries are available on 253 Zenodo at https://zenodo.org/records/11246174 254 (Shankar 2024).

3. INTERPOLATING MANIFOLDS

3.1. Cross-Model Line Identification

As previously mentioned, blase tunes the line centers of detected lines. This means that from one PHOENIX spectrum to the next, the same line could have a slightly different line center. Since the goal of this study is to interpolate the properties of each line, we needed to identify the presence of a particular line across the PHOENIX subset, associating the same line with every occurrence. We decided to do this by using the line centers μ of the detected lines pre-optimization.

Pre-optimized line centers were required to be 267 equal in order to group lines together. The val-268 ues of μ pre-optimization are recorded by blase 269 to 0.01 Å (picometer) precision; if they instead 270 used the full available floating point precision, 271 this method would likely detect singular (un-272 groupable) instances of spectral lines across the 273 grid, completely negating the premise of this 274 study. Of course, this method can be confused ₂₇₅ by species that produce spectral lines at very 276 close wavelengths and other confounding factors, 277 likely resulting in some false positives and nega-278 tives. However, we deem it sufficient to approx-279 imate this way, as an in-depth treatment would 280 be time-consuming and computationally expen-281 **sive.**

Now with each spectral line indexed by μ , we had four parameters to interpolate: μ' , $\ln(a)$, σ , and γ . Note that since we are dealing with the parameters of Voigt profiles, we can see in Equation 2 that even if the interpolation method is linear, a final spectral reconstruction will vary nonlinearly in flux.

Spectral lines were often only detected in some spectra 289 from the PHOENIX subset. In Figure 2, we show that 290 different grid points **show** differing counts of detected 291 spectral lines.

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The number of detected spectral lines changes 293 as a function of stellar parameter for different rea-294 sons. Perhaps for astrophysical reasons: stellar atmo-295 spheres genuinely do not produce that line at detectable 296 strength at the given temperature and surface gravity. 297 Alternatively, blase's fitting or our line identifica-298 tion assumptions led to artifacts. Whatever the 299 cause, these missing lines have immediate practical con-300 sequences. Rectilinear interpolation schemes break in 301 regions where a line does not appear; you can't interpo-302 late a quantity that simply doesn't exist.

To solve this, we artificially populated missing grid 304 points with log-amplitudes of -1000, which retained in-305 terpolator stability while nullifying the evaluated line. To prove the feasibility of this computational 307 trick, we can do a simple thought experiment. Moving 1 K in $T_{
m eff}$ away from a grid point on the 309 boundary of detectability would reduce a line's amplitude by a factor of over 20,000 (or more precisely, e^{10}). This is an extremely large re-312 duction for an extremely small deviation from grid point, meaning that to any realistic pre-314 cision, the line immediately vanishes as soon as 315 blase no longer detects it.

Examples of the appearance of missing sections in 317 heatmaps where a line does not appear can be seen 318 in Figure 3 and Figure 4, which show heatmaps $_{319}$ of the 11617.66 Å C I and the 10123.66 Å He 320 II lines (identified with NIST (Kramida et al. 321 2023)) varying over a solar metallicity slice of 322 the PHOENIX subset stellar parameter space. 323 Across the entire PHOENIX subset, blase detected $_{324}$ 128,723 spectral lines with unique μ . On average, a 325 PHOENIX spectrum has 9,167 detected spectral $_{326}$ lines, with the minimum being 252 ($T_{
m eff}=12000$ 327 K, $\log(g) = 6$, [Fe/H] = -0.5) and the maximum 328 being 34,551 ($T_{
m eff} = 2300$ K, $\log(g) = 3$, [Fe/H] $_{329} = 0$). Every one of these lines can be visualized as a 330 manifold mapping a 3D stellar parameter vector to a 331 4D spectral line parameter vector, and every one was 332 interpolated to map stellar parameters to spectral 338 line properties.

3.2. Continuously Evaluable Manifolds

For each line, the inputs to the interpolator were the 337 three input parameters T_{eff} , $\log(g)$, and [Fe/H], and the output was a list of four parameters, μ' , $\ln(a)$, σ , and γ . For each line, one of these interpolator objects was 341 created using linear interpolation, and these interpola-342 tors were aggregated into a single list, which was then 343 written to disk in the .pkl file storage format. These 344 interpolators generate multiple manifolds representing 345 the following mapping:

$$\begin{bmatrix} T_{\text{eff}} \\ \log(g) \\ [\text{Fe/H}] \end{bmatrix} \rightarrow \begin{bmatrix} \mu' \\ \ln(a) \\ \sigma \\ \gamma \end{bmatrix}$$
 (3)

346 These interpolators could now be evaluated at any point 347 lying within the domain of the PHOENIX subset, turn-348 ing a discretely sampled PHOENIX subset into a con-349 tinuously evaluable function, sometimes called a spec-350 tral emulator. With the given size of the PHOENIX 351 subset, the interpolator list takes up 13.2 GB of disk 352 space. This is larger than the original PHOENIX 353 subset by about 63% and than the state dictio-354 nary clone archive by a factor of over 34. This is 355 due to the fact that we are now pickling entire 356 function objects that must memorize the data. 357 Future work with more space-efficient models 358 and clever storage strategies could reduce stor-359 age space requirements in exchange for I/O per-360 formance. This evaluation is able to reconstruct an ex-361 isting PHOENIX spectrum or alternatively interpolate 362 a new spectrum within the domain of the PHOENIX 363 subset, so we call this the PHOENIX generator.

The spectral reconstruction process is done by iterat-365 ing over the PHOENIX generator, evaluating each in-366 terpolator at the given coordinates, then reshaping the 367 data into the same format that PyTorch uses for state 368 dictionaries. During the iteration, if the interpolated 369 log-amplitude of the line is less than -100, the line is 370 excluded from the state dictionary. We do this to avoid 371 artifacts in the manifolds due to the artifical popula-372 tion of log-amplitudes of -1000 where grid points were 373 missing.

Finally, the state dictionary is fed into blase's 375 SparseLinearEmulator, which reconstructs the spec-376 trum by constructing a forward model based on the in-377 put state dictionary. Any nan values are set to 1 (which 378 we can do because the spectra are all normalized), and 379 the spectral reconstruction is complete. In Figure 5, 380 we see a comparison between a PHOENIX gen-381 erator solar spectrum and two nearby native 382 PHOENIX grid point spectra at the 10123.66 Å 383 He II spectral line.

We can observe in Figure 6 that our He II line from 386 a reconstructed solar spectrum is not simply a pixel in-387 terpolation between the nearest PHOENIX grid point 389 spectra. It is interpolating the spectral line properties

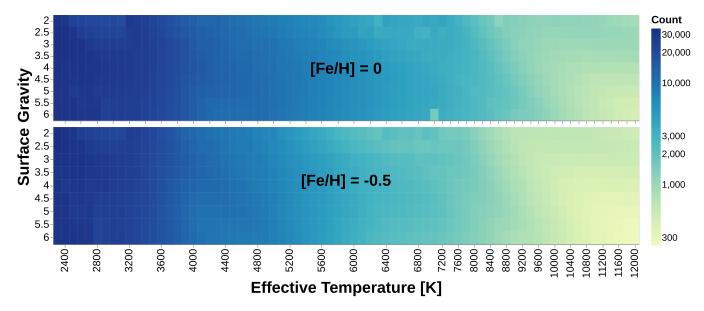


Figure 2. Number of detected spectral lines at each grid point of the PHOENIX subset. We can see that the number of detected lines decreases with increasing $T_{\rm eff}$. Remember that from $T_{\rm eff}=7000$ K onward, PHOENIX's sampling increment changes from 100 K to 200 K. Due to some combination of computational artifacts from our line identification assumptions or innate behavior of PHOENIX grid spectra themselves, outliers such as the obvious one here at $T_{\rm eff}=7000$ K and $\log(g)=6$ dex appear. Further exploration into why this happens is not in the scope of this study; for inference we are concerned with the lines themselves, not overall grid point behavior.

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³⁹⁰ using hundreds of thousands of manifolds, each repre-³⁹¹ senting a nonlinear parameter in the shape of the spec-³⁹² tral line.

3.3. Spectral Reconstruction Time

A typical use case for the PHOENIX generator may 394 be to batch reconstruct spectra from an array of input stellar parameters. Therefore, there is some motivation to reduce the computational cost of this procedure to 398 tractable levels. We evaluate the computational time 399 needed to use the PHOENIX generator in two distinct 400 ways. First, for a single input, which would be relevant 401 in serial applications. Second, for an array of multiple 402 inputs, which would be relevant in parallel applications. 403 scipy's RegularGridInterpolator API allows for the 404 passing in of an entire array of input coordinates to be 405 evaluated at once. However using blase to reconstruct 406 the spectrum from our interpolated state dictionary is 407 always done serially, leading to what is actually more of 408 a pseudo-parallel evaluation, but extremely performant 409 nonetheless.

Performance results are shown in Figure 7, and we can see that the PHOENIX generator's parallel mode is much faster than serial mode. While the generator takes around 15 seconds per spectrum in serial mode, this drops to 1 second per spectrum in parallel mode as soon as 30 reconstructions. The speedup factor between the two methods in creases as more spectra are generated, reaching a

419 factor of 20 at 60 reconstructions. Assuming lin420 ear time scaling with wavelength range, accord421 ing to KORG benchmarks (Wheeler et al. 2023),
422 which generate spectra over a 500 Å wavelength
423 range, the PHOENIX generator in serial mode
424 is approximately as fast as KORG and ~30 times
425 faster than Turbospectrum (Gerber et al. 2023).
426 The speedup factor in parallel mode compared to
427 KORG is then the same variable speedup factor
428 from serial mode (20 times faster at 60 recon429 structions).

The computer used for this test has specifications shown in Table 2 (note that spectral reconstruction does not currently leverage the GPU):

CPU	AMD EPYC 7513 (32c/64t, 3.65 GHz boost)
RAM	256 GB
GPU	Nvidia A100 [40GB] (×2)

Table 2. Specifications of the machine used for cloning and reconstruction testing. This machine is owned by Caroline Morley.

4. BAYESIAN INFERENCE AND TESTING

4.1. Gaussian Process Minimization

The goals of this study's inference algorithm are to infer the stellar parameters $T_{\rm eff}$, $\log(g)$, and

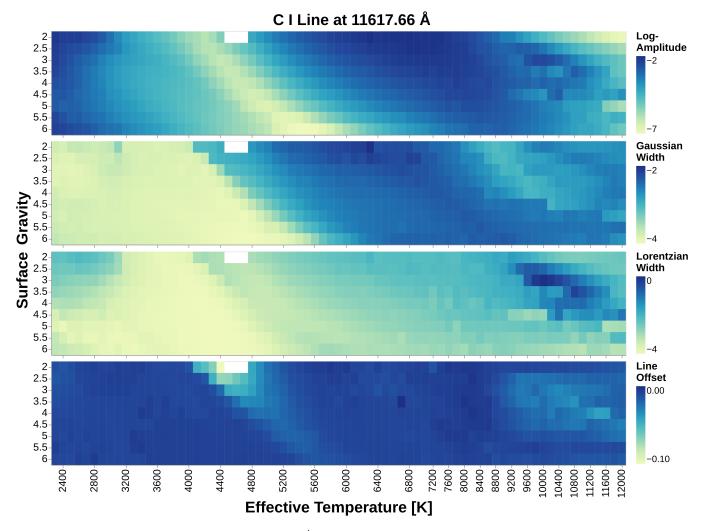


Figure 3. Heatmaps showing how $\ln(a)$, σ , γ , and $\mu' - \mu$ vary over the PHOENIX subset slice at solar metallicity of a C I spectral line, notable for being detected at nearly all subset grid points. Notice the missing chunk in the top left; this could perhaps be due to the line center shifting to the point that blase identified the chunk as a different line entirely (notice the line center offset increasing in magnitude approaching the chunk border). Since blase did not detect a spectral line here, we artificially populate those points with lines that have $\ln(a) = -1000$ during interpolation.

⁴⁴⁰ [Fe/H] by comparing a spectrum with our recon-⁴⁴¹ structions and solving the optimization problem. ⁴⁴² We elected to use Bayesian optimization for this com-⁴⁴³ ponent, specifically the gp_minimize function from the ⁴⁴⁴ scikit-optimize library (Head et al. 2018). This al-⁴⁴⁵ gorithm uses a Gaussian Process to model the objective ⁴⁴⁶ function, which in this case was the RMS (Root-Mean-⁴⁴⁷ Square) loss between the interpolated spectrum M and ⁴⁴⁸ the true spectrum D, defined as:

$$\mathcal{L} = \left\langle (\mathsf{M} - \mathsf{D})^2 \right\rangle^{1/2} \tag{4}$$

The optimizer was configured to first run 100 random valuations to seed the surrogate model, then run 20 more evaluations now guided by the surrogate model. A random evaluation is simply a randomly generated tuple of stellar parameters within the

454 parameter space of the PHOENIX generator, 455 which is then evaluated with the generator to 456 get a reconstruction and then compared with the 457 inference target to retrieve its RMS error. The 458 surrogate model is the approximation to the true 459 objective function that the optimizer constructs 460 based on the set of random evaluations (in this 461 case defined by a set of 100 points rather than 462 a fully continuous, infinitely resolved function) 463 Guided evaluations are different, they are not 464 randomly generated tuples of stellar parameters. 465 They use the surrogate model in order to select 466 new test points for the minimum of the objective 467 function. This totals to 120 evaluations, 100 to cre-468 ate the surrogate model, then 20 guided evaluations, 469 which was deemed sufficient for this study. Fine-tuning

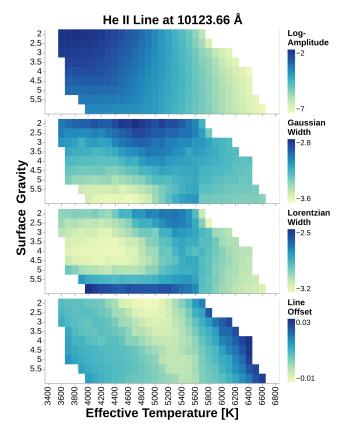


Figure 4. Heatmaps showing how $\ln(a)$, σ , γ , and $\mu' - \mu$ vary over the PHOENIX subset slice at solar metallicity of a He II spectral line. blase detects this line at a select chunk of grid points, leading to data only existing between $T_{\rm eff}$ 3600 K and 6600 K.

470 these numbers is possible, but simply not warranted for 471 a proof-of concept method. Tweaking these numbers 472 could lead to a more accurate inference, however 473 it would incur additional computational cost as 474 well. One inference takes on average 200 seconds, us-475 ing the computer with specifications in Table 3. Inference algorithms that output more in-477 formation and produce posterior distributions 478 such as starfish (Czekala et al. 2015) or 479 MINESweeper (Cargile et al. 2020) take on the 480 order of an hour to complete an inference on 481 supercomputers. astroNN (Leung & Bovy 2019) 482 is a more direct comparison, and represents an 483 extremely powerful inference tool that performs 484 inference thousands of times faster thanks to its 485 neural network architecture.

4.2. Inference Accuracy

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To test the performance of the inference algorithm, we used the PHOENIX subset itself. This is possible for two reasons: first, the PHOENIX generator has not truly memorized the PHOENIX subset

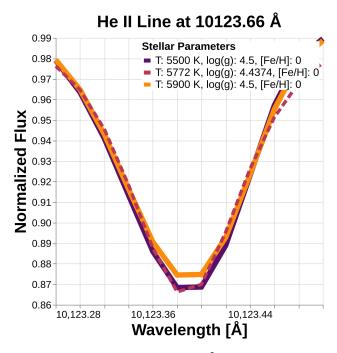


Figure 5. Plot of the 10123.66 Å He II spectral line shown at PHOENIX grid points in the neighborhood of the Sun's stellar parameters (solid lines), as well as a reconstruction at solar parameters with the PHOENIX generator (dotted line).

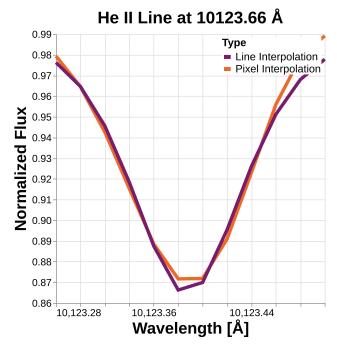


Figure 6. Comparison between the PHOENIX generator's line interpolation and a standard pixel interpolation of the 10123.66 Å He II spectral line. Notice that line interpolation results in more symmetrical line wings due to the use of Voigt profiles.

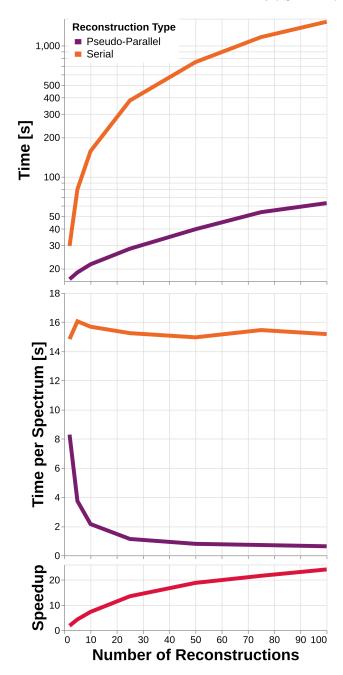


Figure 7. Plots showing the time taken to reconstruct varying numbers of spectra using the PHOENIX generator (lower is better). We can see that the time taken per spectrum for the serial implementation hovers around 15 seconds within run-to-run variance, while the pseudo-parallel implementation continually decreases in time taken per spectrum as the number of inputs increases. The speedup factor (higher is better) increases as more spectra are generated, which is also a desirable outcome.

⁴⁹² due to its Voigt profile basis and the ignoring of ⁴⁹³ weaker features, and secondly, random samples ⁴⁹⁴ within the parameter space are extremely un-

CPU	AMD Ryzen 9 7950X3D (16c/32t, 5.7 GHz boost)
RAM	32 GB
GPU	Nvidia GeForce RTX 4080 SUPER [16GB]

Table 3. Specifications of the machine used for general testing and for generating visualizations. This machine is owned by Sujay Shankar.

likely to be PHOENIX grid points themselves,avoiding any overlap.

We know that in typical observational settings, a coarse estimate for $T_{\rm eff}$ tends to be fairly well-constrained from ancillary data such as photometric color index. So when testing the inference algorithm, we limited its search space to only include $T_{\rm eff}$ which lay within 500 K of the true value on either side. $\log(g)$ and [Fe/H] were allowed to vary freely. The test sample consisted of 21 $T_{\rm eff}$ values: [3000, 11000] K in steps of 400 K, $5 \log(g)$ values: [2, 6] in steps of 0.5, and $2 [{\rm Fe/H}]$ values: [-0.5, 0] in steps of 0.5, totaling 210 unique spectra in the test set. Two test passes were run, meaning that 420 inference tests were run in total.

In Figure 8, we show the residuals of the inferred stellar parameters with respect to the true values. We see that there is a distinction between the algorithm's behavior above and below $T_{\rm eff}=7000~{\rm K}$, so we define low and high temperature regimes accordingly. We know that as $T_{\rm eff}$ increases, we detect less lines with blase, as shown in Figure 2, and lines tend to spread out, possibly resulting in artifacts from our wing cut approximation. Any number of physical or computational factors could explain this behavior, although the apparent sharpness of the transition leads us to postulate that it could be due to PHOENIX grid $T_{\rm eff}$ sampling changing from 100 K to 200 K at this point.

For $T_{\rm eff}$ < 7000 K, the inferred $T_{\rm eff}$ differed from the true value by an average of 93 K, $\log(g)$ 0.24, and [Fe/H] 0.056 (standard deviations of 139 K, 0.40, and 0.15, respectively). For $T_{\rm eff} \geq$ 7000 K, the inferred $T_{\rm eff}$ differed from the true value by an average of 347 K, $\log(g)$ 0.26, and [Fe/H] 0.16 (standard deviations of 293 K, 0.32, and 0.23, respectively). We see that blase3D is more effective and generally more self-consistent when predicting stellar parameters of lower temperature spectra.

In the low-temperature regime, our median $T_{\rm eff}$ residual is about 10 K, about half that of MINESweeper (Cargile et al. 2020) and astronn

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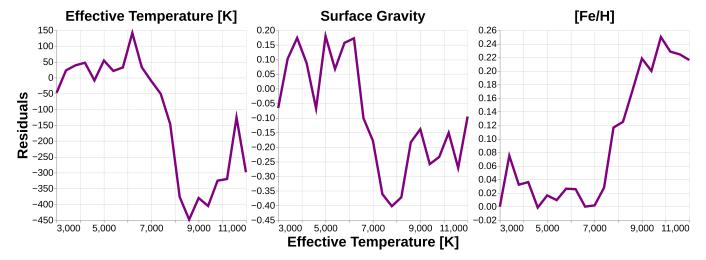


Figure 8. Residual plots for the GP minimization inference algorithm, all plotted with respect to $T_{\rm eff}$ (values aggregated across $\log(g)$ and [Fe/H]). Notice the interesting change in behavior of all residuals at around $T_{\rm eff} = 7000$ K.

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540 (Leung & Bovy 2019), but about ten times larger than ASPCAP (García Pérez et al. 2016). Median log(g) and [Fe/H] residuals are 0 dex, which are comparable to all three aforementioned methods, however blase3D is overall less consistent as the standard deviation of the resid-546 uals are about an order of magnitude larger. 547 At high temperatures, the median $T_{\rm eff}$ residual changes drastically to about -434 K, which 549 is significantly worse than the other methods. $\log(g)$ becomes a few times less accurate, and 551 both $\log(g)$ and [Fe/H] have larger standard deviations, all leading to large inconsistencies and worse performance in this temperature regime. blase3D at this stage does not quite reach the level of contemporary methods that can deal with more realistic data and cover much larger parameter spaces, but the fact that, in its best case, it can perform to within an order of magnitude using a drastically simpler implementation, sparks promise for future improvements.

5. DISCUSSION

5.1. Scientific Applications

This study's paradigm of spectral inference can enable scientists to adopt self-consistent model grids and analyze their spectral line behavior in an interpretable fashion, which has been fairly uncommon practice thus far. Notably, this system tracks the shifting of spectral lines as a function of stellar parameters, which traditionally has been uncommon for algorithms to assess systematically. We balance rigidity in the Voigt profile basis and flexibility in line parameter tunability, resulting in an interpretable system.

To summarize, we bring to scientists the ability to understand exactly what spectral properties we consider
mathematically and how we expect those properties to
behave on an incredibly detailed level (the 4 line parameters of all spectral lines and their corresponding interpolating manifolds). We treat the PHOENIX precomputed model grid as the ground truth, constructing
the PHOENIX generator that then interfaces with
our inference algorithm.

5.2. Technical Considerations

To reiterate, the manifold fitting steps are not end-584 to-end autodifferentiable. As seen in Figure 1, these steps rely on scipy, which is not equipped with autodiff. 586 Without autodifferentiability, the ability to "machine 587 learn" is significantly reduced compared to a hypothet-588 ical monolithic JAX or PyTorch system. We see three reasons for for developing a non-autodifferentiable sys-590 tem. First, the familiar scipy-based system will serve 591 as an easy entry point for most practitioners who are 592 unfamiliar with autodiff, and may still benefit from and 593 modify the code without expert ML knowledge. Second, 594 this non-autodiff version serves as an initial benchmark 595 against which an inevitable autodifferentiable version of 596 blase may be compared. Finally, the inventory of fa-597 miliar interpolation algorithms have not yet been ported 598 to PyTorch or JAX, since machine learning or Gaussian 599 Process fitting schemes are generally preferred within 600 the ML community.

The interpolation scheme presented here represents a proof of concept, showing that leveraging the mapping between synthetic spectral lines and their inputs can yield a semi-empirical basis for data-model comparisons. There are numerous design considerations that could be

606 improved upon with future work. These include but are 607 not limited to the following:

- Limited PHOENIX Subset: The PHOENIX subset used in this study did not include the full PHOENIX grid, which expands the [Fe/H] range to [-4.0, 1.0] dex and the $\log(g)$ range to [0, 6], and also includes the alpha element abundance parameter, which we elected to fix at 0 for this study. In addition to the actual stellar parameters, we also took a subset of the PHOENIX wavelength range, with the full [500, 55000] Å wavelength range also being left to future work. With this, users would be able to fit a greater variety of stellar spectra in many different wavelength regimes.
- Strict Wavelength Range: Currently, the generator only supports inference on spectra whose wavelength limits are either equal to it, or encompass that of the generator and have been truncated to match. However, when the spectrum in question has a smaller wavelength range than the generator, currently there is no functionality to truncate the generator. This would require externally indexing the generator's individual interpolators by line center position and selectively evaluating those to eliminate wasteful computation. This takes burden off the users to truncate their data to the PHOENIX generators, making use simpler.
- Single Model Grid: The PHOENIX grid is not the only model grid of synthetic spectra available, and it does not apply to all types of stars. Future work would extend the reach of this study's algorithm to encompass other model grids such as the Sonora series of substellar models (Marley et al. 2021; Karalidi et al. 2021; Morley et al. 2024; Mukherjee et al. 2024), ATLAS (Kurucz 2005), and coolTLUSTY (Lacy & Burrows 2023), reaching practitioners studying various types of stars and spectra. Future blase versions will be able to have an option for the user to input which model grid they would like to base the inference on, and to get even more advanced, perhaps even have the ability to intelligently determine which model grid to use automatically.
- Memorization vs. Generalization: The current design of the algorithm constructs manifolds using interpolation. This means that performance is good at points close to PHOENIX subset grid points, but is highly dependent on the type of interpolation used. As interpolators require memorization of the data, advanced interpolation be-

- comes extremely expensive in terms of disk utilization. Future work would involve constructing manifolds using more generalizable ML methods such as lasso or ridge regression, which would allow for much better generalization, high speed, and lower disk utilization at the expense of some accuracy.
- Extrinsic Absence: The current design of our algorithm does not account for extrinsic parameters that modify the appearance of spectra such as rotational broadening and Doppler shifting. Future work would need to develop ways to tune these extrinsic parameters alongside stellar parameters.
- Framework Overhead: As this algorithm is currently more proof of concept than practical, it uses convenience functions from various libraries, which naturally introduces some level of overhead and leaves performance on the table. Future work would involve writing custom functions expressly designed for blase, most likely a complete rewrite of the library from the ground up. This has the potential to greatly increase the speed of this algorithm, depending on how much overhead is avoided with a bespoke implementation.
- Pseudo-Interpretability: Our algorithm boasts interpretability by considering spectral lines as the objects of interest as opposed to the rather uninterpretable flux values of other approaches. However, this is only a step in the direction of interpretability. True interpretability would decompose a spectrum not into a set of spectral lines, but into a set of species component spectra, which requires a much more advanced understanding of different species and their behavior, as well as direct access to a radiative transfer code as opposed to an off-the-shelf model grid. This approach would also extend the inference from just stellar parameters defined by a grid to any set of parameters accounted for in the radiative transfer model, down to specific species abundances. So while we were able to identify the spectral lines used in our figures, it is not necessarily valuable to try to identify all 128,723 lines that we identify as unique with our algorithm. blase is agnostic to the identity of the line that it is optimizing. We study these lines as blase sees them (i.e. their four shape parameters), because for the purposes of this study, that is the only information that is useful. Having more interpretability would let scientists actually study certain species and their spectral lines.

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- The Continuum Black Box: Continuum normalization is a process that is not yet completely understood, and is currently done as a preprocessing step with a fairly simple algorithm. Future work would dive deeper into the science of continua and develop more advanced methods that can discern continua with greater accuracy and less modeling restrictions. This would increase accuracy for end users.
- One Voigt Fits All: The current assumption of blase is that every spectral line is a Voigt profile. This assumption is largely true, but there are situations where that is simply not enough. Future studies need to account for more advanced spectral line profiles and procedures to deal with phenomena such as ro-vibrational bands. This would increase accuracy for end users.

6. CONCLUSION

In this study, we have presented a proof-of-concept 724 725 algorithm that **deals with** a subset of the PHOENIX 726 spectral model grid (T_{eff} : [2300, 12000] K, $\log(g)$: [2, 6] dex, [Fe/H]: [-0.5, 0] dex, λ : [8038, 12849] Å). We create state dictionaries for all spectra in the 729 PHOENIX subset using blase, recasting spectra 730 into lists of Voigt profiles, and lossily compress the 731 **spectral data by a** factor of around 20. They are 732 available on Zenodo at https://zenodo.org/records/ 733 11246174 (Shankar 2024).

We create the PHOENIX generator and implement a 735 performant spectral reconstruction algorithm, enabling 736 anyone to create reconstructions of PHOENIX spectra with continuously valued stellar parameters.

We then introduce and test our GP minimization 739 algorithm to infer stellar parameters. Two test passes on 210 noise-free synthetic models ($T_{
m eff}$: 741 [3000, 11000] K, $\log(q)$: [2, 6] dex, [Fe/H]: [-0.5, 0] 742 dex) yield average absolute deviations from true values

743 of **93** K in T_{eff} , **0.24** dex in $\log(g)$, and **0.056** dex in ₇₄₄ [Fe/H] in low-temperature ($T_{\rm eff}$ < 7000) regimes. ₇₄₅ In high-temperature regimes ($T_{
m eff} \geq 7000$), the ₇₄₆ errors increase to 347 K in $T_{\rm eff}$, 0.26 dex in $\log(g)$, 747 and 0.16 dex in [Fe/H].

In its current state, our algorithm requires that the 749 spectra not contain noticeable Doppler shifting, rota-750 tional broadening, noise, or other confounding factors. 751 The methods discussed here represent a step down a 752 road not traveled in spectral inference, and have the 753 potential to become more advanced in the future by 754 fully utilizing the strengths of physics-informed machine 755 learning.

 \mathbf{All} figures in this paper are publicly 757 available on GitHub at https://github.com/ 758 Sujay-Shankar/blase3D under paper/paper2.

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