Sujay Jakka

Homework 6

Problem 1

$A = \{0^n 1 0^{2n} 1 0^{3n} \mid n \geq 0\}$

Assume A is a CFL, then there exists a pumping constant p such that the pumping lemma holds for a $s \in A$ that is $|s| \geq p$.

Case 1: VXY contains all 0's

$S = \underbrace{0^p 1 0^p}_{u} \underbrace{0^p}_{vxy} \underbrace{1 0^{3p}}_{z}$ $\quad u = 0^p 1 0^p, v = 0^p, x = \varepsilon, y = \varepsilon, z = 10^{3p}$

If v is pumped down to 0 then S would become $S = 0^p 1 0^p 1 0^{3p}$, this string is not a part of the language A. The division where VXY contains all 0's cannot be pumped.

Case 2: VXY contains a 1.

$S = \underbrace{0^p}_{u} \underbrace{1}_{vxy} \underbrace{0^{2p} 1 0^{3p}}_{z}$ $\quad u = 0^p, v = 1, x = \varepsilon, y = \varepsilon, z = 0^{2p} 1 0^{3p}$

If V is pumped down to 0 then S would become $S = 0^p 0^{2p} 1 0^{3p}$, this string is not a part of the language A. This division where VXY contains a 1 cannot be pumped.

The string S cannot be pumped in either of the divisions of the string therefore the language A is not context free as it does not satisfy the pumping lemma.

Problem 2

$B = \{0^n 0^{2n} 1 0^{3n} \mid n \geq 0\}$

Yes, this language is context-free.

$S \to 000S000 \mid 1$

Problem 3

Let C be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0's and 1's. Show that C is not context free.

$C = \{ww^R \mid w \in \{0,1\}^* \text{ where } \#0\text{'s} = \#1\text{'s}\}$

Assume C is a CFL, then there exists a pumping constant P such that the pumping lemma holds for a $S \in C$ that is $|S| \geq P$.

$S = 1^P 0^P 0^P 1^P \qquad |S| \geq P$

Case 1: VXY is all 0's or all 1's

$S = \underbrace{1^P}_{u} \underbrace{0^P}_{vxy} \underbrace{0^P 1^P}_{z} \qquad u = 1^P, v = 0^P, x = \varepsilon, y = \varepsilon, z = 0^P 1^P$

If V is pumped down to 0 then S would become $S = 1^P 0^P 1^P$, this string is not part of the language C as it does not have a equal number of 0's and 1's. The division where VXY contains all 0's or all 1's cannot be pumped.

Case 2: VXY contains both 0's and 1's

$S = \underbrace{1^{P/2}}_{u}\underbrace{1^{P/2}0^{P/2}}_{vxy}\underbrace{0^{P/2}0^{P}1^{P}}_{z}$ $u = 1^{P/2}$, $v = 1^{P/2}0^{P/2}$, $x = \varepsilon$, $y = \varepsilon$, $z = 0^{P/2}0^{P}1^{P}$

If V is pumped down to 0 then S would become $1^{P/2}0^{3P/2}1^{P}$, this string is not part of the language C as it does not contain a equal number of 0's and 1's. It is also not a palindrome. The division where Vxy contains both 0's and 1's cannot be pumped.

The String S cannot be pumped for either of the divisions of the string therefore the language C is not context free as it does not satisfy the pumping lemma.

Problem 4

Let $\Sigma = \{1, 2, 3, 4\}$ and $D = \{w \in \Sigma^* \mid$ in w, the number of 1s equal the number of 2s, and the number of 3's equal the number of 4s$\}$. Show that D is not context free.

$S = 1^P 3^P 2^P 4^P$ $|S| \geq p$

Case 1: VXY contains only 1's, only 2's, only 3's, or only 4's.

$S = \underbrace{1^P}_{u}\underbrace{3^P}_{vxy}\underbrace{2^P 4^P}_{z}$ $u = 1^P$, $v = 3^P$, $x = \varepsilon$, $y = \varepsilon$, $z = 2^P 4^P$

If V is pumped down to 0 then S would become $S = 1^P 2^P 4^P$, this string is not part of the language D as there are not an equal number of 3's and 4's. This division where VXY contains only all 1's, all 2's, all 3's, or all 4's cannot be pumped.

Case 2: vxy contains two types of symbols (eg. 1 and 3 or 3 and 2)

$S = 1^p 3^{p/2} 3^{p/2} 2^{p/2} 2^{p/2} 4^p$ $\quad u = 1^p 3^{p/2}, v = 3^{p/2} 2^{p/2}, x = \varepsilon, y = \varepsilon, z = 2^{p/2} 4^p$

$\underbrace{1^p 3^{p/2}}_{u} \underbrace{3^{p/2} 2^{p/2}}_{vxy} \underbrace{2^{p/2} 4^p}_{z}$

If v is pumped down to 0 then S would become
$S = 1^p 3^{p/2} 2^{p/2} 4^p$, this string does not contain an equal number of 1s and 2s and also does not contain an equal number of 3's and 4's. This division where vxy contains two type of symbols cannot be pumped.

The string S cannot be pumped in either of the divisions of the string therefore the language D is not context free as it does not satisfy the pumping lemma.