Sujay Jakka

Homework 5

Problem 1

1. The set of strings over the alphabet $\Sigma = \{a, b\}$ with more a's then b's.

R_1: $S \rightarrow XaX$
R_2: $X \rightarrow aXb \mid bXa \mid XX \mid a \mid \varepsilon$

2. The complement of the language $\{a^n b^n \mid n \geq 0\}$

R_1: $S \rightarrow Xa \mid bX \mid aSb$
R_2: $X \rightarrow aX \mid bX \mid \varepsilon$

Problem 2

$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$
$A_1 = \{a^i b^j c^k \mid i = j \text{ where } i, j, k \geq 0\}$
$A_2 = \{a^i b^j c^k \mid j = k \text{ where } i, j, k \geq 0\}$

CFG of Language A, Union of languages A_1 and A_2

R_1: $S \rightarrow MX \mid YN$
R_2: $M \rightarrow aMb \mid \varepsilon$
R_3: $N \rightarrow bNc \mid \varepsilon$
R_4: $X \rightarrow cX \mid \varepsilon$
R_5: $Y \rightarrow aX \mid \varepsilon$

This grammar is ambiguous because it is the union of languages A_1 and A_2 which has an intersection that is of all strings $a^i b^j c^k$ where i, j, and k are all equal. This means if MX is chosen it can lead to this as well as if YN is chosen in rule 1.

(Problem 2 (continued))

For example two different left most derivations lead to the same string abc.

① $S \Rightarrow MX \Rightarrow aMbX \Rightarrow a\varepsilon bX \Rightarrow a\varepsilon bcX \Rightarrow a\varepsilon bc\varepsilon = abc$

② $S \Rightarrow YN \Rightarrow axN \Rightarrow a\varepsilon N \Rightarrow a\varepsilon bNc \Rightarrow a\varepsilon b\varepsilon N = abc$

Problem 3

R_1: $A \rightarrow BAB \mid B \mid \varepsilon$
R_2: $B \rightarrow 00 \mid \varepsilon$

Step 0 create a new start variable

R_1: $S \rightarrow A$
R_2: $A \rightarrow BAB \mid B \mid \varepsilon$
R_3: $B \rightarrow 00 \mid \varepsilon$

Step 1 remove ε-productions

R_1: $S \rightarrow A \mid \varepsilon$
R_2: $A \rightarrow BAB \mid B \mid AB \mid BA \mid A \mid BB$
R_3: $B \rightarrow 00$

Step 2 remove Unit Production in R_2

R_2: $A \rightarrow BAB \mid 00 \mid AB \mid BA \mid BB$

Step 3 remove Unit Production in R_1

R_1: $S \rightarrow BAB \mid 00 \mid AB \mid BA \mid BB \mid \varepsilon$

R_1: $S \rightarrow BAB \mid 00 \mid AB \mid BA \mid BB \mid \varepsilon$
R_2: $A \rightarrow BAB \mid 00 \mid AB \mid BA \mid BB$
R_3: $B \rightarrow 00$

Step 4, move all terminals to unit productions where RHS is one terminal

R_1: $S \rightarrow BAB \mid XX \mid AB \mid BA \mid BB \mid \varepsilon$
R_2: $A \rightarrow BAB \mid XX \mid AB \mid BA \mid BB$
R_3: $B \rightarrow XX$
R_4: $X \rightarrow 0$

Step 5 create two variable RHS, by replacing long production rules

R_1: $S \rightarrow YB \mid XX \mid AB \mid BA \mid BB \mid \varepsilon$
R_2: $B \rightarrow YB \mid XX \mid AB \mid BA \mid BB$
R_3: $B \rightarrow XX$
R_4: $X \rightarrow 0$
R_5: $Y \rightarrow BA$