Homework #1

Problem 1

0.1f $\{n \mid n$ is an integer and $n = n+1\}$ - A empty set

0.1e $\{w \mid w$ is a string of 0s and 1s and w equals the reverse of $w\}$ - A set of all palindromes that consist of only 0s and 1s

0.6d Domain: $\{(1,6),(1,7),(1,8),(1,9),(1,10),(2,6),(2,7),(2,8),(2,9),(2,10),(3,6),(3,7),(3,8),(3,9),(3,10),(4,6),(4,7),(4,8),(4,9),(4,10),(5,6),(5,7),(5,8),(5,9),(5,10)\}$

Range: $\{6,7,8,9,10\}$

0.6e $g(4, f(4)) = g(4,7) = 8$

Problem 2

1. For all $n \in \mathbb{N}$: $5^n + 5 < 5^n + 1$

Proof: We will show the statement holds true by induction on n.

Base Case: We will show the statement is true for $n=1$.

$5^1 + 5 < 5^{1+1} =$

$5 + 5 < 5^2 =$

$10 < 25$ True ✓

Induction Hypothesis:

Assume the statement, $5^n + 5 < 5^n + 1$, holds for $n=k$. That is, $5^k + 5 < 5^{k+1}$. Where $k \in \mathbb{N}$.

Inductive Step: We want to show the statement is true for $n=k+1$ given the induction hypothesis. Which is, $5^k + 5 < 5^{k+1}$.

$$5^{k+1} + 5 < 5^{(k+1)+1}$$
$$5^k \cdot 5 + 5 < 5^{k+1} \cdot 5$$
$$5(5^k + 1) < 5^{k+1} \cdot 5$$
$$5^k + 1 < 5^{k+1}$$
$$5^k + 1 < \overbrace{5^k + 5 < 5^{k+1}}^{\leftarrow \text{Induction Hypothesis}}$$

This shows the the statement $5^n + 5 < 5^{n+1}$ holds true for $n=k+1$. $\blacksquare$

2. For all $n \in \mathbb{N}$: $\sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \left(\frac{n(n+1)}{2}\right)$

Proof: We will show the equality holds true by induction on n.

Base Case: We will show the equality holds true for $n=1$.

$$\sum_{i=1}^{1} (-1)^i i^2 = (-1)^1 (1)^2 = -1$$

$$(-1)^1 \left(\frac{1(1+1)}{2}\right) = (-1)\left(\frac{2}{2}\right) = -1 \qquad \text{True} \checkmark$$

Induction Hypothesis: Assume the equality holds for $n=k$. That is $\sum_{i=1}^{k} (-1)^i i^2 = (-1)^k \frac{k(k+1)}{2}$. Where $k \in \mathbb{N}$.

Inductive Step: We want to show the equality holds true for $n = k+1$ given the induction hypothesis. The induction hypothesis is $\sum_{i=1}^{k} (-1)^i i^2 = (-1)^k \frac{k(k+1)}{2}$.

$$\sum_{i=1}^{k+1} (-1)^i i^2 = (-1)^{k+1} \frac{(k+1)((k+1)+1)}{2}$$

$$\sum_{i=1}^{k} (-1)^i i^2 + (-1)^{k+1} (k+1)^2 =$$

$$\underbrace{(-1)^k \frac{k(k+1)}{2}}_{\text{Inductive Hypothesis}} + (-1)^{k+1}(k+1)^2 =$$

$$\frac{(-1)^k k(k+1) + 2((-1)^{k+1}(k+1)^2)}{2} =$$

$$\frac{(-1)^k (k+1)(k + 2(-1)(k+1))}{2} =$$

$$\frac{(-1)^k (k+1)(k - 2(k+1))}{2} =$$

$$\frac{(-1)^k (k+1)(k - 2k - 2)}{2} =$$

$$\frac{(-1)^k (k+1)(-k-2)}{2} =$$

$$\frac{(-1)^k (-1)(k+2)(k+1)}{2} =$$

$$\frac{(-1)^{k+1} (k+1)((k+1)+1)}{2} =$$

$$(-1)^{k+1} \frac{(k+1)((k+1)+1)}{2} = (-1)^{k+1} \frac{(k+1)((k+1)+1)}{2}$$

This shows the equality $\sum_{i=1}^{n} (-1)^i i^2 = (-1)^n \frac{n(n+1)}{2}$ holds true for $n = k+1$. $\blacksquare$