Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Digital Signal Processing Laboratory (EE39203)

Autumn, 2022-23

Experiment 1 **Discrete and Continuous Time Signals**

Slot: Date:

Student Name: Roll No.:

Grading Rubric

	Tick the best applicable per row			Points
	Below Expectation	Lacking in Some	Meets all Expectation	
Completeness of the report				
Organization of the report (5 pts) *With cover sheet, answers are in the same order as questions in the lab, copies of the questions are included in report, prepared in LaTeX*				
Quality of figures (5 pts) *Correctly labelled with title, x-axis, y-axis, and name(s)*				
Understanding of continuous and discrete-time signals (15 pts) *Matlab figures, questions*				
Ability to compute integral manually and in Matlab (30 pts) *Manual computation, Matlab figures, Matlab codes, questions*				
Ability to define and display functions (1D and 2D) (30 pts) *Matlab figures, Matlab codes, questions*				
Understanding of sampling (15 pts) *Matlab figures, questions*				
			TOTAL (100 pts)	

Total Points (100): TA Name: TA Initials:

Digital Signal Processing Laboratory (EE39203) Experiment 1: Discrete and Continuous-Time Signals

Name: Sujay Vivek
Roll Number: 22EE30029

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1. Learning Objective

This lab aims to demonstrate the characteristics of continuous and discrete-time signals using digital computers and the MATLAB software environment. A continuous-time signal takes on a value at every point in time, whereas a discrete-time signal is only defined at integer values of the "time" variable. However, while discrete-time signals can be easily stored and processed on a computer, storing the values of a continuous-time signal for all points along a segment of the real line is impossible. In this experiment, we will illustrate that continuous-time signals can be processed by first approximating them as discrete-time signals through sampling.

3. Continuous-Time Vs. Discrete-Time

3.1 Displaying Continuous-Time Vs. Discrete-Time

To plot discrete-time signals as dots in a Cartesian coordinate system, the MATLAB `stem` command is used. Multiple plots on a single figure can be managed using the `subplot` command. Continuous-time signals can be approximated by computing their values at closely spaced points in time and plotting these with connecting lines using the `plot` function in MATLAB.

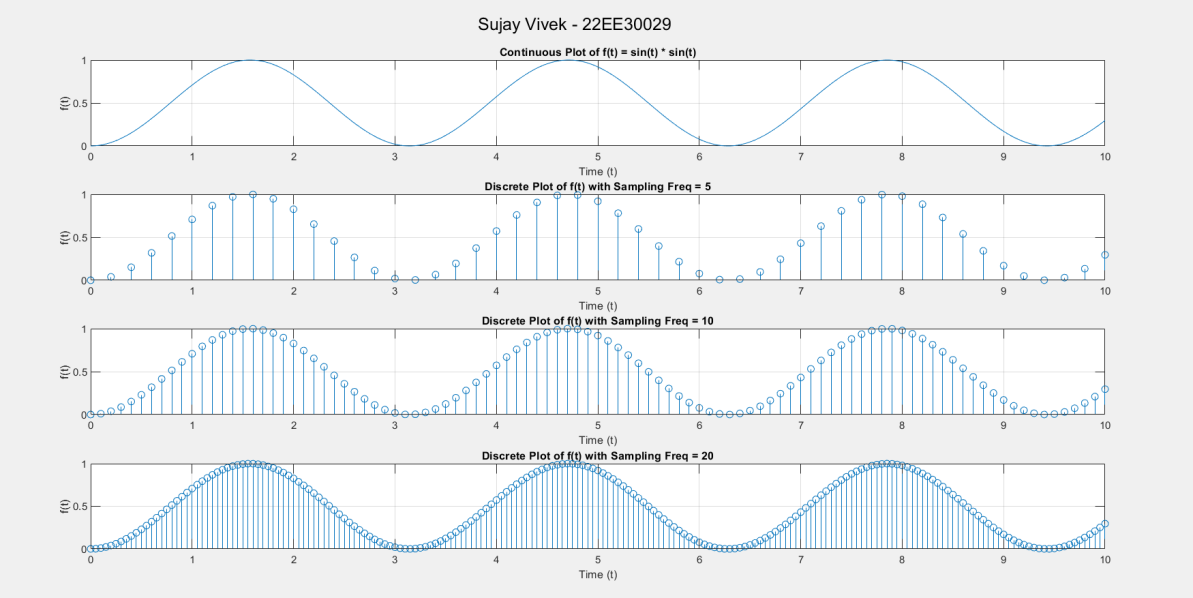
MATLAB Code

```
% Defining the function

f = @(t) sin(t).*sin(t);

% Creating Sampling Frequencies
Fs1 = 5;
Fs2 = 10;
Fs3 = 20;

% Creating Discrete Time values
t_continuous = linspace(0,10,1000);

t_discrete1 = 0:1/Fs1:10;    % Sampling Freq = 5
t_discrete2 = 0:1/Fs2:10;    % Sampling Freq = 10
t_discrete3 = 0:1/Fs3:10;    % Sampling Freq = 20

% Calculating the function values
y_cont = f(t_continuous);
y_disc1 = f(t_discrete1);
y_disc2 = f(t_discrete2);
y_disc3 = f(t_discrete3);

% Create the subplot for the continuous plot
subplot(4, 1, 1);
plot(t_continuous, y_cont);
title('Continuous Plot of f(t) = sin(t) * sin(t)');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

% Create the subplot for the discrete plot with Fs1
subplot(4, 1, 2);
stem(t_discrete1, y_disc1);
title('Discrete Plot of f(t) with Sampling Freq =
   5');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

```

```
% Create the subplot for the discrete plot with Fs2
subplot(4, 1, 3);
stem(t_discrete2, y_disc2);
title('Discrete Plot of f(t) with Sampling Freq = 10');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

% Create the subplot for the discrete plot with Fs3
subplot(4, 1, 4);
stem(t_discrete3, y_disc3);
title('Discrete Plot of f(t) with Sampling Freq = 20');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

sgtitle('Sujay Vivek - 22EE30029');
```

Plots

Plotting with the help of MATLAB:



(a) Continous and Discrete Plots of f(t) using different Sampling Frequencies

Observation:

Looking at the above generated waveforms of different sampling frequencies, we can easily comment that the Continuous Time Plot is accurate as we have computed large number of values at closely spaced points in time.

The Discrete time waveform however has less number of computed values and hence there is less accurate compared to the continuous time plot. One can observe that as the Sampling Frequency reduces, the number of computed points on the waveform also reduces.

3.2 Vector Index vs. Time

In MATLAB, vector indices start from 1 and cannot be negative or zero. When sampling a continuous-time signal like $x(t)$, it's common to store these samples in a vector, often using the same variable name (e.g., $x[n]$). However, it's important to differentiate between the vector index **x[n]** and the function **x(t)**, as they are distinct concepts despite potentially sharing the same name.

MATLAB Code

```
1
2  % Creating a function to make things easier
3  function a = create_disc(gap)
4      a = 0:gap:10;
5  end
6
7  % Define the new function to be plotted
8  f = @(t) cos(t).*cos(t);
9
10 % Generate discrete time points with different
       sampling intervals
11 t_disc1 = create_disc(0.1);
12 t_disc2 = create_disc(0.5);
13 t_disc3 = create_disc(0.9);
14
15 % Calculate the function values at the discrete
       time points
16 y_disc1 = f(t_disc1);
17 y_disc2 = f(t_disc2);
18 y_disc3 = f(t_disc3);
19
20 % Create the first subplot with sampling interval
       0.1
21 subplot(3, 1, 1);
22 stem(t_disc1, y_disc1);
23 title('Discrete Plot of f(t) = cos(t) * cos(t) with
       Sampling Interval = 0.1');
24 xlabel('Time (t)');
25 ylabel('f(t)');
26 grid on;
27
```

```
% Create the second subplot with sampling interval
    0.5
subplot(3, 1, 2);
stem(t_disc2, y_disc2);
title('Discrete Plot of f(t) = cos(t) * cos(t) with
    Sampling Interval = 0.5');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

% Create the third subplot with sampling interval
    0.9
subplot(3, 1, 3);
stem(t_disc3, y_disc3);
title('Discrete Plot of f(t) = cos(t) * cos(t) with
    Sampling Interval = 0.9');
xlabel('Time (t)');
ylabel('f(t)');
grid on;

% Add a supertitle for the figure
sgtitle('Sujay Vivek - 22EE30029');
```

Plots

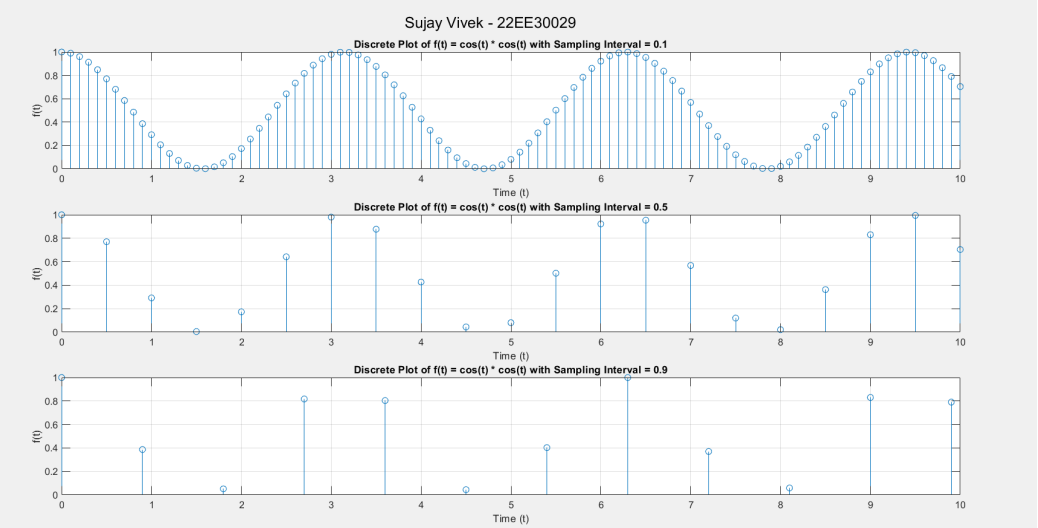


Figure 2: Plotting f(t) at different Sampling Intervals

Observation:

Observing each of the 3 graphs above, we can notice how Sampling Frequency affects the Generated Waveform of the function f(t). The Waveforms depict the use of Sampling Time Intervals 0.1, 0.5 and 0.9 top to bottom. And we observe that as the Sampling Time Intervals increase, the number of discrete points computed and displayed on the Waveform decreases, and hence the accuracy also decreases.

3.2 Graphing a Sine Wave for a particular Range

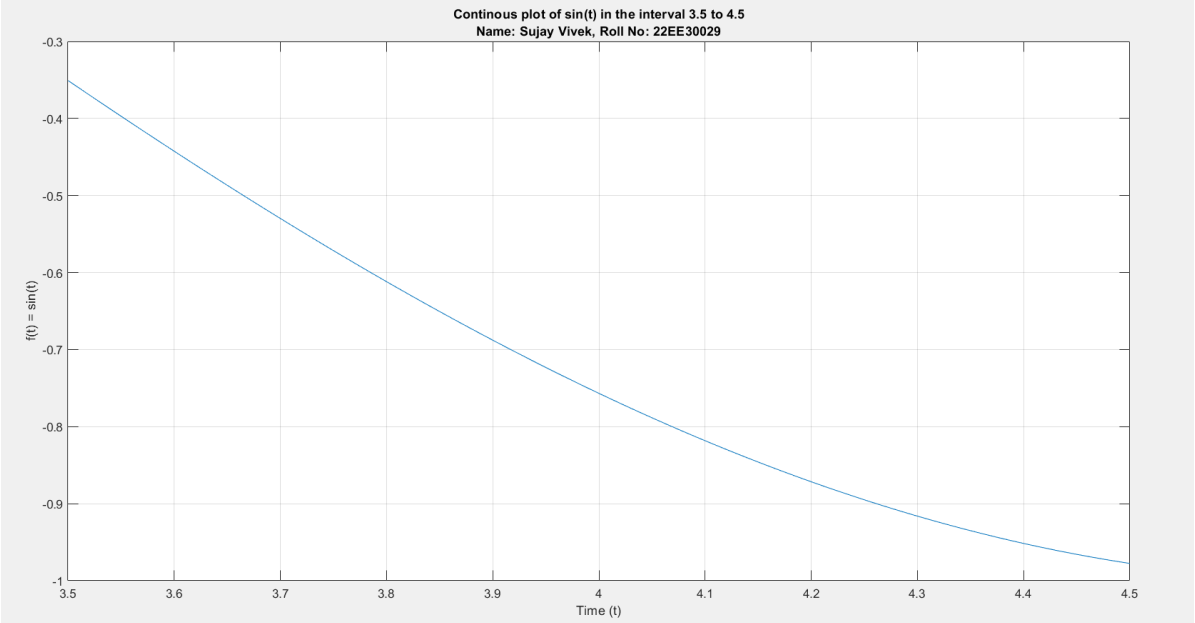
Writing Matlab command(s) that would print the graph of sin(t) for the values of t on the interval [3.5, 4.5] for an appropriate increment of t.

MATLAB Code

```
%Now trying to print the sin curve within a given interval
 f = @(t) sin(t);
   a = 3.5;
   b = 4.5;

%Creating the time values
t_c = linspace(a,b,1000);

y3 = f(t_c);
%Getting the output in y3 for each of the time values
plot(t_c, y3);

title(["Continous plot of sin(t) in the interval 3.5 to 4.5","Name: Sujay Vivek, Roll No: 22EE30029"]);

xlabel('Time (t)');
ylabel('f(t) = sin(t)');
grid on;
```

Plots

Plots generated by MATLAB:



2.3 Analytical Calculation

Computing the following integrals manually:

$$x_1(t) = \int_0^{2\pi} \sin^2(7t)\, dt$$

$$x_2(t) = \int_0^{1} e^t\, dt$$

Answer:

$$
\begin{aligned}
x_1(t) &= \int_0^{2\pi} \sin^2(7t)\,dt \\
&= \int_0^{2\pi} \frac{1-\cos(14t)}{2}\,dt \\
&= \frac{1}{2}\int_0^{2\pi} (1-\cos(14t))\,dt \\
&= \frac{1}{2}\left[\int_0^{2\pi} 1\,dt - \int_0^{2\pi} \cos(14t)\,dt\right] \\
&= \frac{1}{2}\left[t\Big|_0^{2\pi} - \frac{\sin(14t)}{14}\Big|_0^{2\pi}\right] \\
&= \frac{1}{2}\left[2\pi - 0\right] \\
&= \pi
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
x_2(t) &= \int_0^{1} e^t\,dt \\
&= \left[e^t\right]_0^1 \\
&= e^1 - e^0 \\
&= e - 1
\end{aligned}
\tag{2}
$$

3.4 Numerical Computation of Continuous-Time Signals

In this section, we will numerically compute integrals using the Riemann method. The Riemann integral approximates the area under a curve by breaking the region into many rectangles and summing their areas. Each rectangle is chosen to have the same width t, and the height of each rectangle is the value of the function at the start of the rectangle's interval. We will create MATLAB functions to approximate the integral of $\mathbf{sin^2(7t)}$ over $[0, 2\pi]$ and $\mathbf{exp(t)}$ over $[0, 1]$, using varying numbers of rectangles. The function will avoid loops and use the `sum` command.

We will also plot how the approximation improves with more rectangles.

MATLAB Code

```
function I = integ1(N)
    t = linspace(0,2*pi,N+1);
    fx = sin(7*t).*sin(7*t);
    t_dif = 2*pi/N;
    I = t_dif*sum(fx(1:end-1));
end

function J = integ2(N)
    t_val = linspace(0,1,N+1);
    fx = exp(t_val);
    t_diff = 1/N;
    J = t_diff*sum(fx(1:end-1));
end

N_vals = 1:100;
I_vals = zeros(size(N_vals));
J_vals = zeros(size(N_vals));

for i=1:length(N_vals)
    I_vals(i) = integ1(i);
    J_vals(i) = integ2(i);
end


% Create the subplot for the continuous plot
subplot(2, 1, 1); % 2 rows, 1 column, 1nd plot in
   the subplot
```

```
27 plot(I_vals);
28 title('Integral 1 plotting');
29 xlabel('N');
30 ylabel('f(t) = sin(7t)*sin(7t)');
31 grid on;
32
33 % Create the subplot for the continuous plot
34 subplot(2, 1, 2); % 2 rows, 1 column, 2nd plot in
     the subplot
35 plot(J_vals);
36 title('Integral 2 plotting');
37 xlabel('N');
38 ylabel('f(t) = exp(t)');
39 grid on;
40
41 sgtitle('Sujay Vivek - 22EE30029');
```

Plots

Include plots of the numerical integration results here.

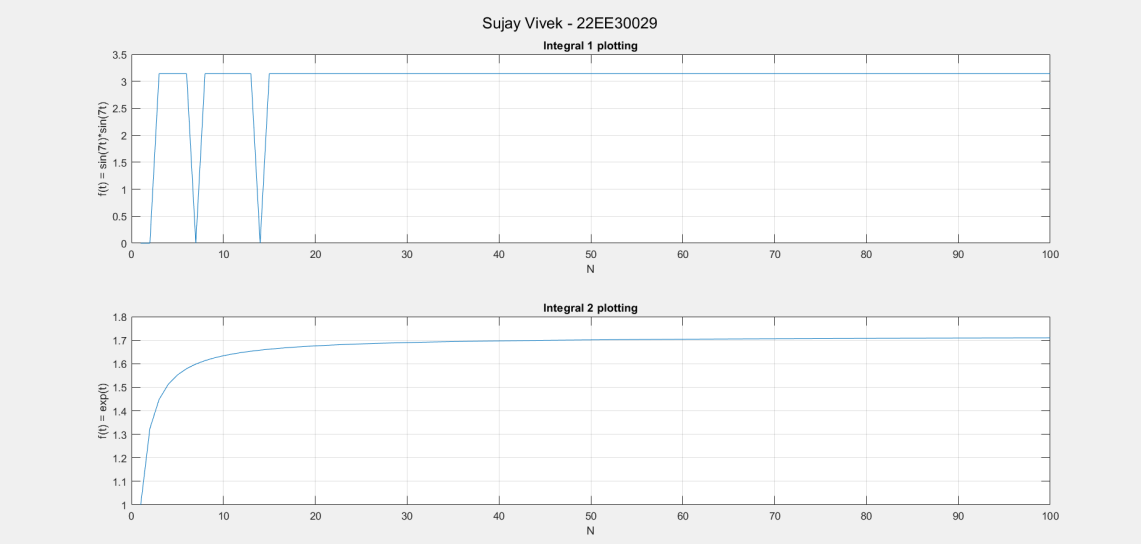


Figure 3: Reinmann Integration

Here I(7) = 4.9×10^{-30} and I(14) = 1.6×10^{-29} which is almost = 0

Reason: When using a method like the rectangular approximation, the interval is divided into N rectangles. I(7) uses 7 rectangles, and I(14) uses 14 rectangles. Obviously, as the number of rectangle increases, we can recreate the function in a much accurate way and hence, we receive more accurate answers as we increase the number of rectangles.

4. Special Functions

4.1 Plotting Continuous-Time Functions

Plotting the following continuous-time functions over the given respective intervals.

$$x_3(t) = \frac{\sin(\pi t)}{\pi t}, \quad t \in [-10, 10]$$

$$x_4(t) = \text{rect}(t), \quad t \in [-1, 2]$$

MATLAB Code

```
g = @(t) sin(pi*t)./(pi*t);  %Function 1

h = @(t) (abs(t)<=0.5);   %Function 2

gx_vals = linspace(-10,10,1000); %creating time
   intervals
gy_vals = g(gx_vals); %Storing all ouput values in
   1 variable

hx_vals = linspace(-1,2,100);
hy_vals = h(hx_vals); %Storing all the points.

% Create the subplot for the continuous plot
subplot(2, 1, 1); % 2 rows, 1 column, 1nd plot in
   the subplot
plot(gx_vals,gy_vals);
title('function1 plot');
xlabel('time');
ylabel('f(t) = sin(pi*t)/pi*t');
grid on;

% Create the subplot for the continuous plot
subplot(2, 1, 2); % 2 rows, 1 column, 2nd plot in
   the subplot
plot(hx_vals, hy_vals);
title('function2 plot');
xlabel('time');
```

```
24 ylabel('f(t) = rect(t)');
25 grid on;
26
27 sgtitle('Name: Sujay Vivek, Roll No: 22EE30029');
```

Plots

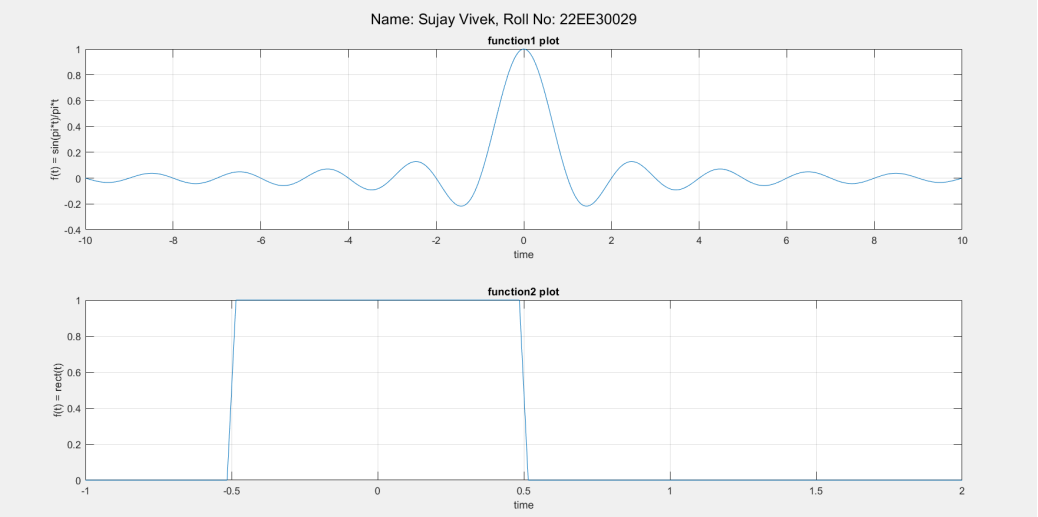


Figure 4: Continuous-time plots of g(t) *and* h(t)

4.2 Plotting Discrete-Time Functions

Plotting the following discrete-time functions over the given respective intervals.

$$x_1[n] = a^n \left(u[n] - u[n-10]\right) \quad \text{for } n \in [-20, 20]$$

$$x_2[n] = \cos(\omega n)\, a^n u[n] \quad \text{for } \omega = \frac{\pi}{4}, \text{ and } n \in [-1, 10]$$

MATLAB Code

```
% Discrete-time function x1[n] for different values

% Range of n
n = -20:20;


u = (n>=0) - (n>=10);

a_values = [0.8, 1.0, 1.5]; %different a values

figure;
orient('tall') %As given the the question for preventing overcrowding of subplots

for i = 1:length(a_values)
    a = a_values(i);
    x1 = a.^n .*u; %Compute x1[n]
    subplot(3,1,i);
    stem(n,x1,'filled');
    title(['Discrete-Time Function x_1[n] for a= ' num2str(a)]);
    grid on;

    sgtitle('Sujay Vivek - 22EE30029');
end


%Discrete-time function x2[n] for omega = pi/4 and different values of a
n2 = -1:10;
```

```
28
29 u2 = (n2 >= 0);
30
31 omega = pi/4;
32 figure;
33 orient('tall');
34
35 for i = 1:length(a_values)
36     a = a_values(i);
37     x2 = cos(omega * n2) .* (a .^n2) .*u2;
38     subplot(3,1,i);
39     stem(n2,x2,'filled');
40     xlabel('n');
41     ylabel(['x2[n] for a =', num2str(a)]);
42     title(['Discrete-Time Function x_2[n] for a =',
          num2str(a)]);
43     grid on;
44
45     sgtitle('Sujay Vivek - 22EE30029');
46 end
```

Plots

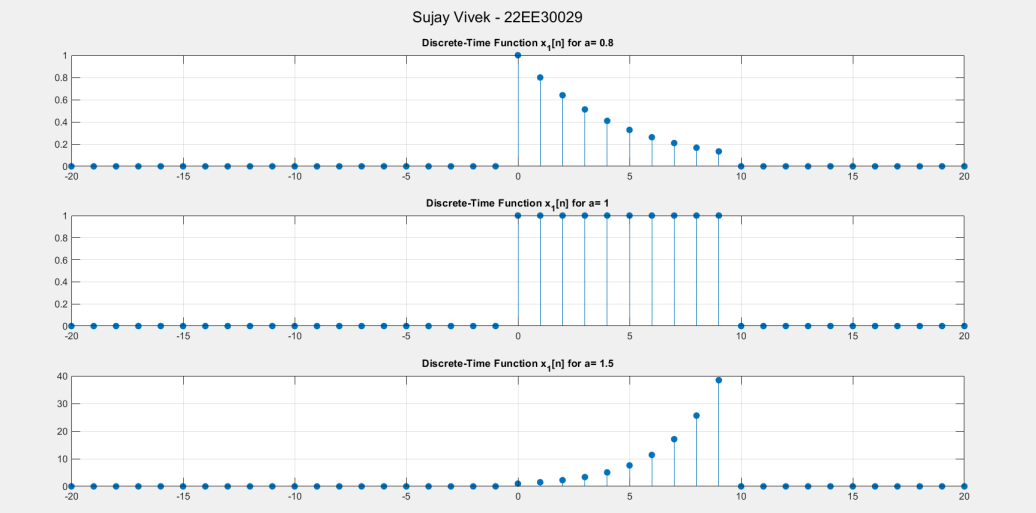


Figure 5: Discrete-time plot of x1[n]

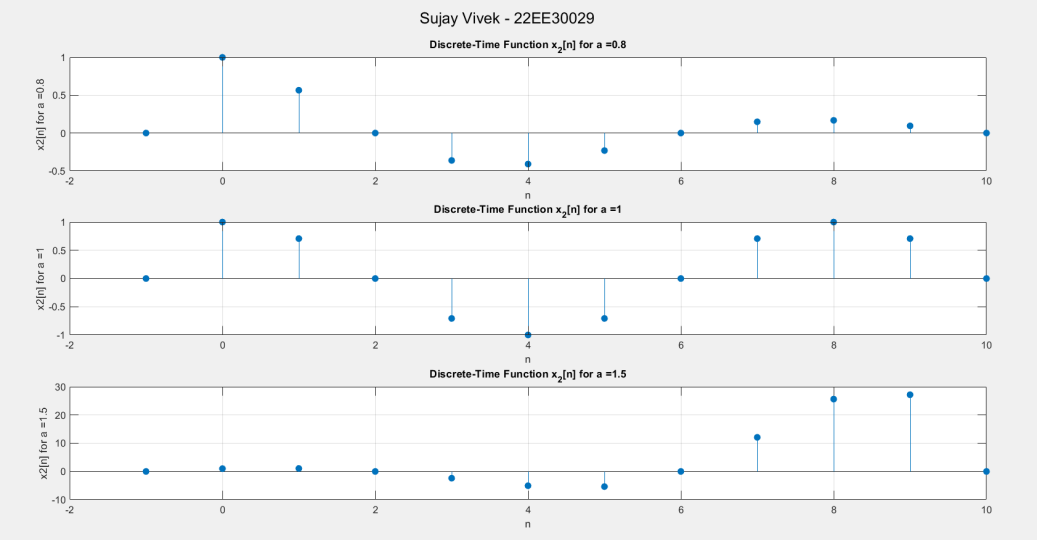


Figure 6: Discrete-time plot of x2[n]

4. Sampling

In this section, the concept of sampling, which involves converting a continuous-time signal into a discrete-time signal by taking samples at uniformly spaced intervals, is explored. The time between two consecutive samples is called the sampling period. For example, a sampling period of 0.1 seconds implies that the value of the signal is stored every 0.1 seconds. The signal $f(t) = \sin(2\pi t)$ is sampled with a period T_s, resulting in a discrete-time signal $x[n] = \sin(2\pi T_s n)$. The signal is then plotted for various values of T_s using the `stem` command, with all plots organized in a single figure via the `subplot` command.

MATLAB Code

```
% Defining the sampling periods
ts_val = [1/10, 1/3, 1/2, 10/9];

% Define the ranges for n
n_ranges = {0:100, 0:30, 0:20, 0:9};

%Create a new figure
figure;

```

```
% Loop through each Ts values
for i = 1:length(ts_val)
    Ts = ts_val(i);
    n = n_ranges{i};

    xn   = sin(2*pi*Ts*n);
    subplot(2,2,i);
    stem(n, xn, 'filled');
    %Labelling the axes and title
    xlabel('n');
    ylabel(['x[n] for T_s =', num2str(Ts)]);
    title(['T_s =', num2str(Ts)]);

    axis([min(n), max(n), -1, 1]);
    grid on;
end

sgtitle('Discrete Time Signal x[n] = sin(2*piT_sn)
   for Different Sampling Periods');
```

Observations:

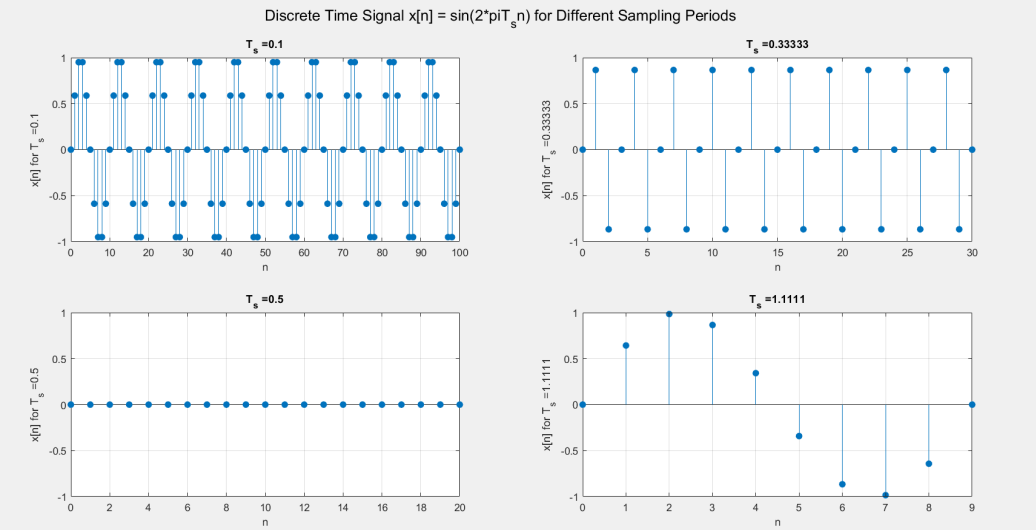
Ts = 1/10: The Discrete Time Signal is almost similar to the Continuous Time Signal since the Sample Interval is very small. The generated waveform is quite dense

Ts = 1/3: Larger Sampling period in the case of Ts = 1/3. Hence the waveform generated is a little less denser, though we can make out it is similar to the original continuous waveform.

Ts = 1/2: As the sampling period increases, the number of computed values decreases. The signal is more sparser and less dense.

Ts = 10/9: Here the Ts = 10/9, which is slightly a little more than 1. Hence the number of computed values is very less due to a large Time Interval that is being used for Sampling. Hence the waveform is less dense and very sparse.

Plots



Understanding

Sampling in digital communication is converting a continuous-time signal into a discrete-time signal. It can also be defined as the process of measuring the discrete instantaneous values of a continuous-time signal.

Digital signals are easier to store and have a higher chance of repressing noise. This makes sampling an important step in converting analog signals to digital signals with its primary purpose as representing analog signals in a discrete format.

Sampling plays an essential role in digital communication systems because it turns continuous analog signals into discrete digital data, allowing them to be processed, transmitted, stored, and manipulated efficiently in the digital world. Noise reduction, error detection and correction, compression and signal processing are all enabled by this conversion, which is crucial for modern communication systems. Digital representation provides for long-distance data transmission with reduced signal deterioration, as well as precise modulation, demodulation, and other signal processing processes, facilitating dependable communication and compatibility among diverse devices and platforms.