Department of Electrical Engineering
Indian Institute of Technology Kharagpur

Digital Signal Processing Laboratory (EE39203)

Autumn, 2022-23

Experiment 5 **Digital Filter Design**

Slot: Date:

Student Name: Roll No.:

Grading Rubric

	Tick the best applicable per row			Points
	Below Expectation	Lacking in Some	Meets all Expectation	
Completeness of the report				
Organization of the report (5 pts) *With cover sheet, answers are in the same order as questions in the lab, copies of the questions are included in report, prepared in LaTeX*				
Quality of figures (5 pts) *Correctly labelled with title, x-axis, y-axis, and name(s)*				
Understanding and implementation of simple FIR filter (35 pts) *Difference eq., flow diagram, impulse response, plots of magnitude response, plots of original and filtered signals and their DTFT, matlab code, questions*				
Understanding and implementation of simple IIR filter (35 pts) *Difference eq., flow diagram, impulse response, plots of magnitude response, plots of original and filtered signals and their DTFT, matlab code, questions*				
Understanding parameters of lowpass filter design (20 pts) *Magnitude response plots with marked regions, questions.*				
			TOTAL (100 pts)	

Total Points (100): TA Name: TA Initials:

Digital Signal Processing Laboratory (EE39203)
Experiment 5
Digital Filter Design

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Introduction

Digital signal processing often requires altering the relative amplitudes of frequency components or removing undesired frequencies, a process known as filtering. Digital filters play a crucial role in various applications, such as audio systems, where they adjust bass and treble, or in systems performing interpolation and decimation on discrete-time signals. Filter design involves both time-domain and frequency-domain techniques. The Z-transform and Discrete-Time Fourier Transform (DTFT) are used to analyze filters in the frequency domain, while the implementation is typically in the time domain via difference equations. There are two main types of filters: Finite Impulse Response (FIR) and Infinite Impulse Response (IIR), distinguished by the nature of their impulse responses. A linear and time-invariant causal digital filter with input $x[n]$ and output $y[n]$ may be specified by its difference equation:

$$y[n] = \sum_{i=0}^{M} b_i x[n-i] - \sum_{k=1}^{N} a_k y[n-k] \tag{1}$$

The impulse response, $h[n]$, is the response of the filter to an input of $\delta[n]$, and is therefore the solution to the recursive difference equation:

$$h[n] = \sum_{i=0}^{M} b_i \delta[n-i] - \sum_{k=1}^{N} a_k h[n-k] \tag{2}$$

The transfer function of the filter in the Z-domain is given by:

$$H(z) \equiv \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{N-1} b_i z^{-i}}{1 + \sum_{k=1}^{M} a_k z^{-k}} \tag{3}$$

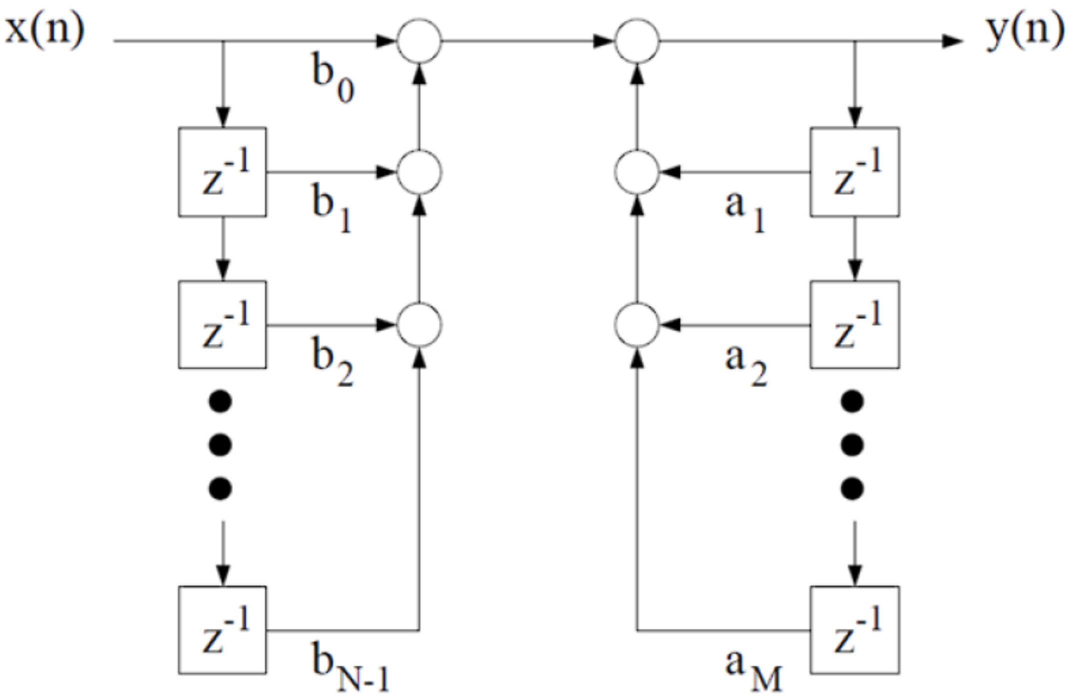

Figure 1: Direct form implementation for a discrete-time filter

1 Design of FIR filter

1.1 Notch filter

Lab Report:

Use MATLAB to compute and plot the magnitude of the filter's frequency response $|H_f(e^{j\omega})|$ as a function of ω on the interval $-\pi < \omega < \pi$, for the following three values of θ:

1. $\theta = \frac{\pi}{6}$
2. $\theta = \frac{\pi}{3}$
3. $\theta = \frac{\pi}{2}$

Put all three plots on the same figure using the `subplot` command.
Submit the difference equation, system diagram, and the analytical expression of the impulse response for the filter $H_f(z)$. Also submit the plot of the magnitude response for the three values of θ. Explain how the value of θ affects the magnitude of the filter's frequency response.

In order for the filter's impulse response to be real-valued, the two zeros must be complex conjugates of one another:

- **Zeros of the filter:**

$$z_1 = e^{j\theta}, \quad z_2 = e^{-j\theta}$$

where θ is the angle of z_1 relative to the positive real axis.

- **Transfer function:**

$$H_f(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1})$$

Substituting $z_1 = e^{j\theta}$ and $z_2 = e^{-j\theta}$, we get:

$$H_f(z) = \left(1 - e^{j\theta} z^{-1}\right)\left(1 - e^{-j\theta} z^{-1}\right)$$

Expanding:

$$H_f(z) = 1 - 2\cos(\theta) z^{-1} + z^{-2}$$

- **Z-Domain Representation:** The transfer function represents the ratio of the output $Y(z)$ to the input $X(z)$:

$$H_f(z) = \frac{Y(z)}{X(z)} = 1 - 2\cos(\theta) z^{-1} + z^{-2}$$

Multiplying both sides by $X(z)$:

$$Y(z) = \left(1 - 2\cos(\theta) z^{-1} + z^{-2}\right) X(z)$$

- **Difference Equation:** Taking the inverse Z-transform gives the difference equation:

$$y[n] = x[n] - 2\cos(\theta) x[n-1] + x[n-2]$$

- **Impulse Response:** The impulse response $h[n]$ is obtained by applying the unit impulse $\delta[n]$:

$$h[n] = \delta[n] - 2\cos(\theta)\delta[n-1] + \delta[n-2]$$

1.2 System Diagram

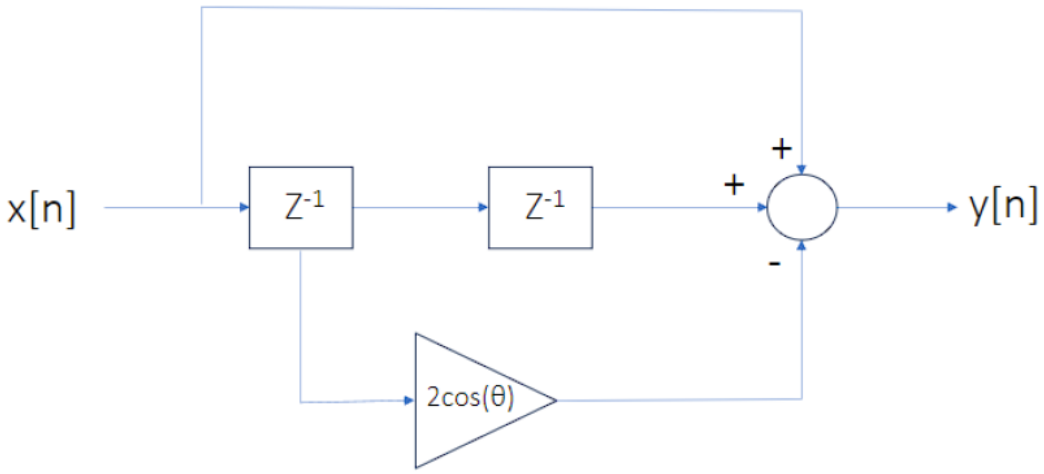

Figure 2: System Diagram

1.3 Magnitude response:

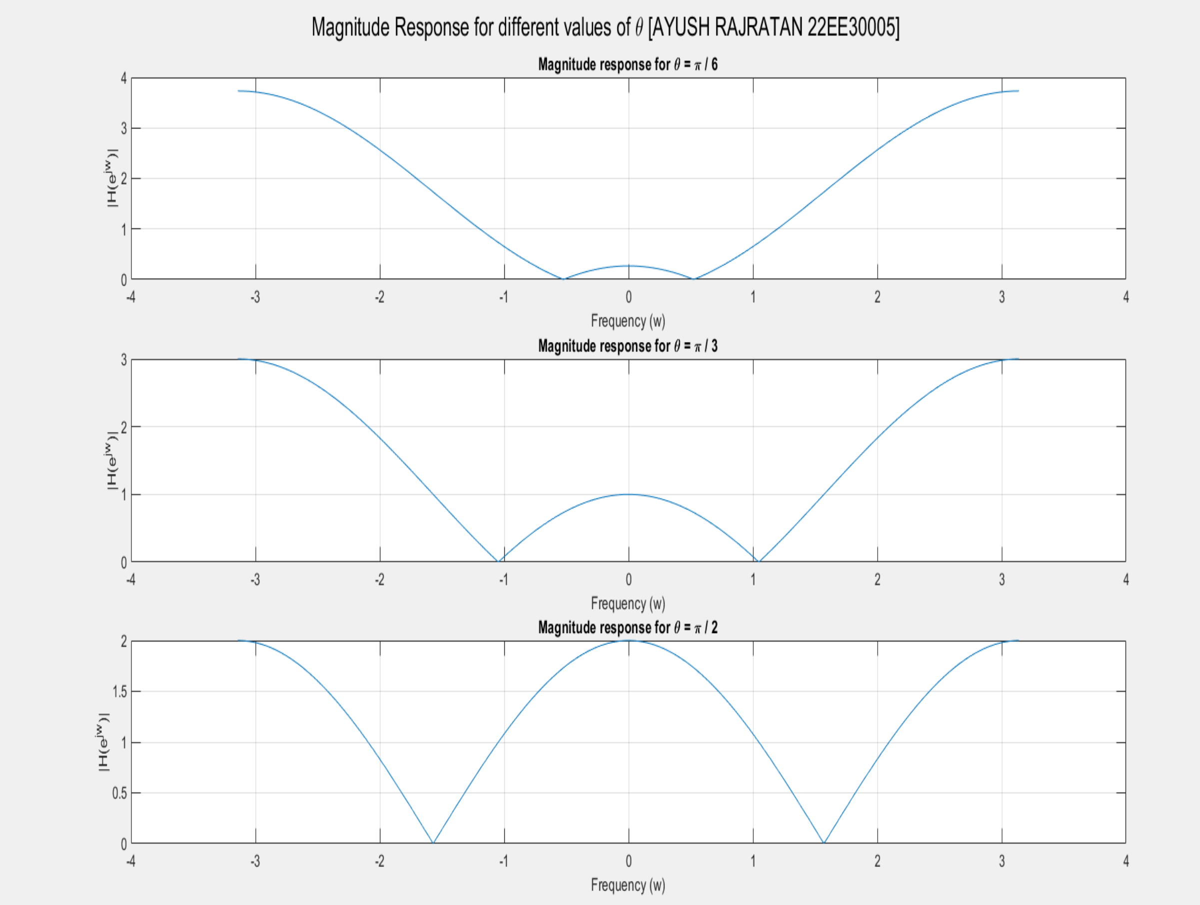

Figure 3: Magnitude Response for different values of theta

1.4 Discussion:

- **Magnitude response formula:**

$$|H_f(z)| = |z - z_1| \cdot |z - z_2|$$

  or equivalently:

$$|H(e^{i\omega})| = |e^{i\omega} - e^{i\theta}| \cdot |e^{i\omega} - e^{-i\theta}|$$

- **Magnitude response characteristics:**
  - Magnitude response becomes zero at $\omega = \pm\theta$.
  - A notch appears at these frequencies where $e^{i\omega}$ aligns with a zero of the transfer function.

- **Impact of θ:**
  - As ω approaches the zeros, magnitude decreases sharply.

- As ω moves away, magnitude increases.
- Reducing θ brings the zeros closer, making the notch narrower.

- **Filter design:**
  - Adjusting θ helps control the filter's frequency response.
  - Useful for attenuating specific frequencies (e.g., noise or interference removal).

Experiment: Removing Sinusoidal Interference

In this experiment, the filter $H_f(z)$ will be applied to remove unwanted sinusoidal interference from a speech signal. Download the audio signal 'nspeech1.mat' and the M-file 'DTFT.m'. Load 'nspeech1.mat' in Matlab using the command 'load nspeech1'. For both the original and filtered signals, submit:

- Time domain plot for 101 samples.
- Magnitude of the DTFT for 1001 samples.

Include the FIR filter function code. Discuss how the signal's frequency content changed and identify if the filter is lowpass, highpass, bandpass, or bandstop. Comment on the impact of filtering on audio quality.

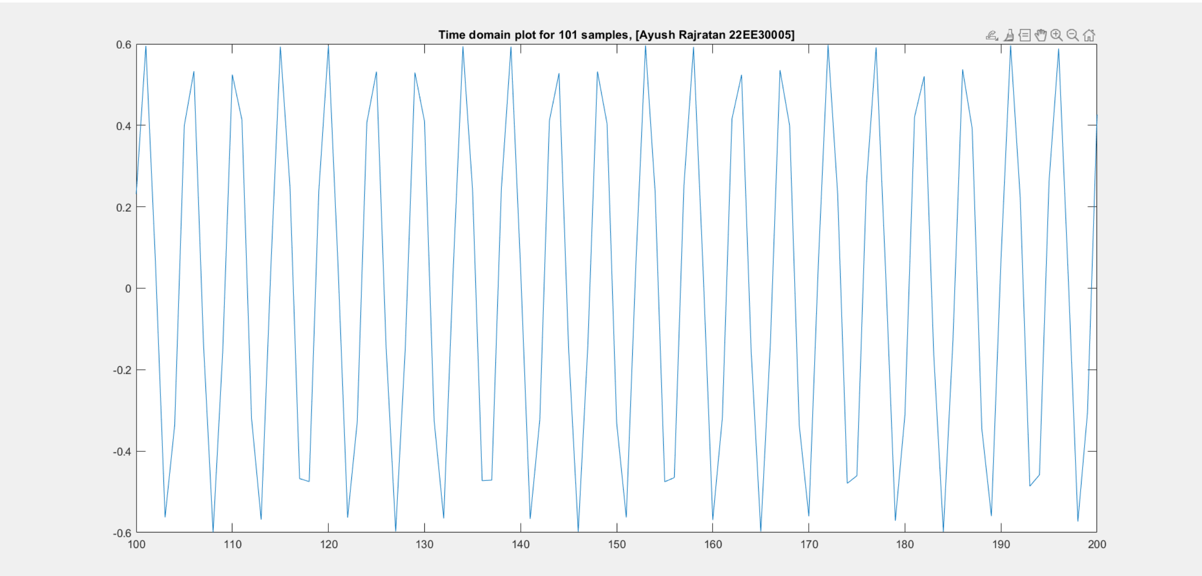

Figure 4: Time Domain plot of the signal for 101 samples

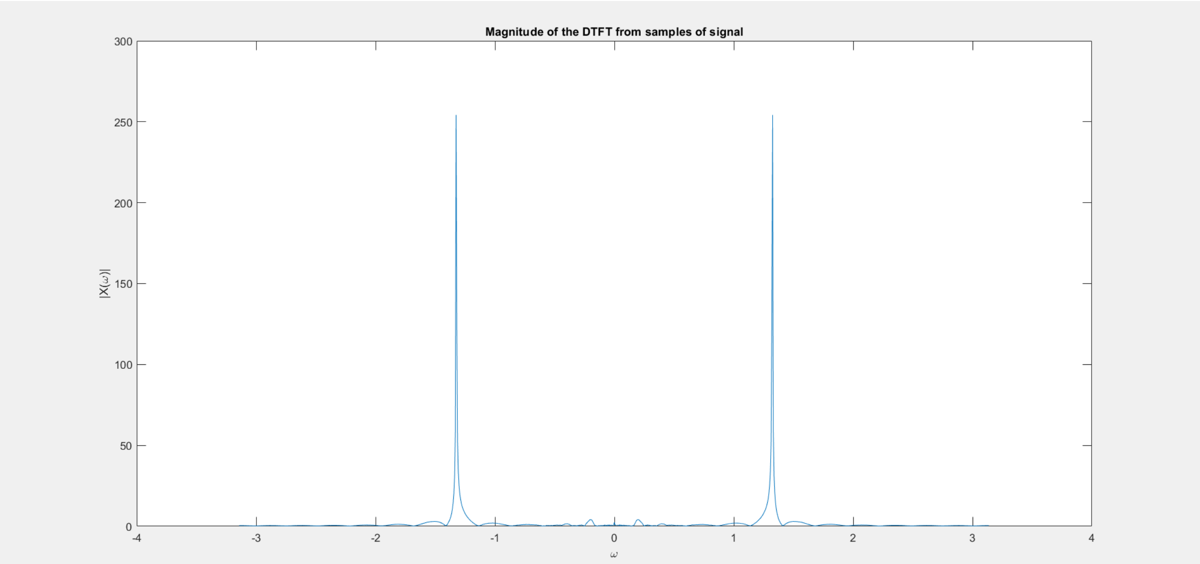

Figure 5: Magnitude of DTFT from samples of signal

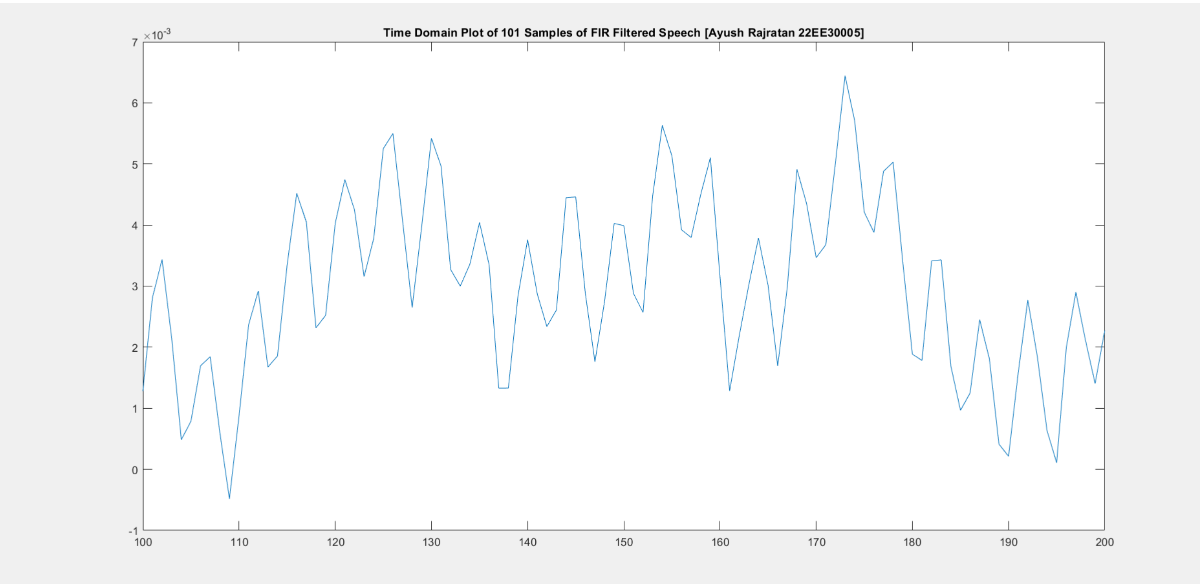

Figure 6: Time Domain Plot of 101 Samples of FIR Filtered Speech

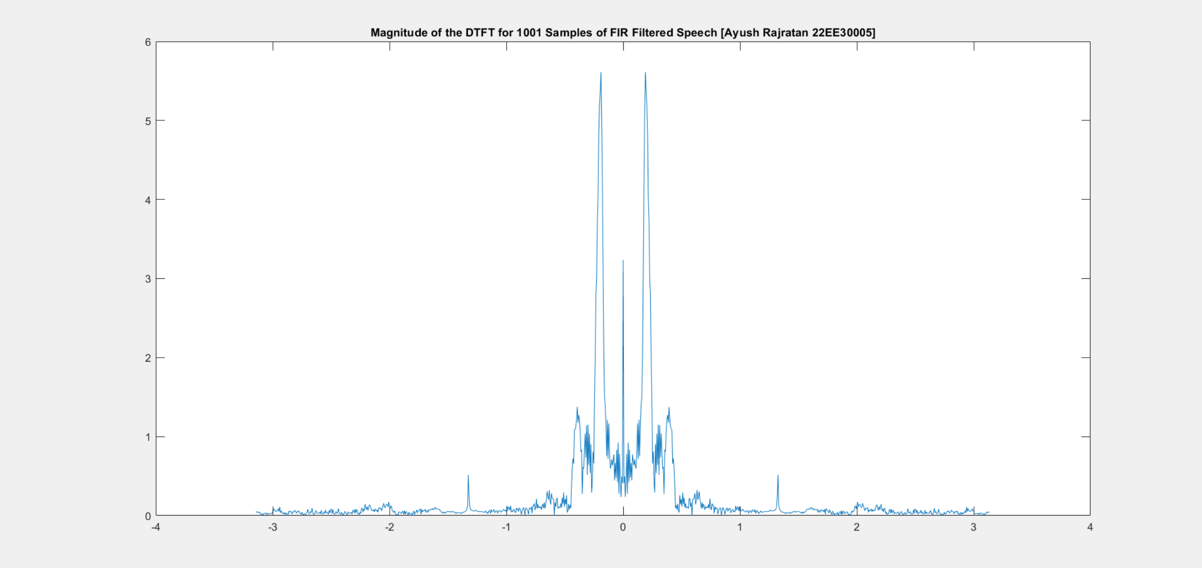

Figure 7: Magnitude of the DTFT for 1001 Samples of FIR Filtered Speech

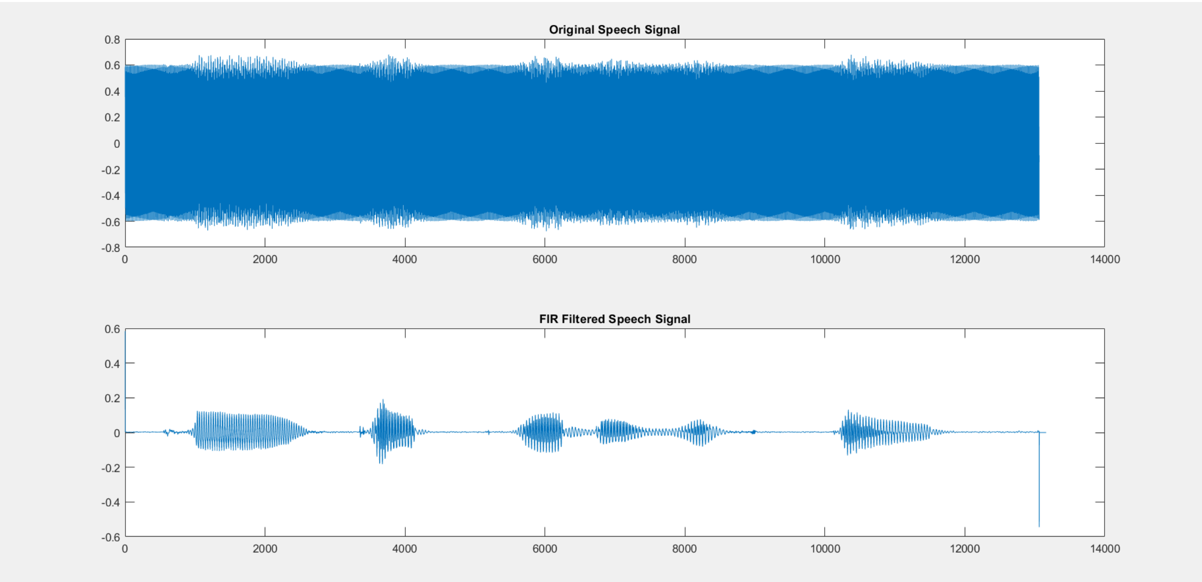

Figure 8: Original speech signal and FIR filtered signal

1.5 Discussion

- **Filter used:** Bandstop filter

$$H_f(z) = 1 - 2\cos(\theta)z^{-1} + z^{-2}$$

- **Objective:** Remove targeted sinusoidal interference, specifically at $\theta = \pm 1.32536$.

- **Filter effects:**
  - Interfering frequency components (beeping sound) were eliminated.
  - Dominant frequency peaks causing high-pitched noise were reduced in the DTFT.
- **Outcome:**
  - Low-frequency elements of the speech were amplified.
  - High-frequency noise was suppressed, enhancing speech clarity.
- **Result:**
  - The beeping sound was effectively removed.
  - The speech signal became clearer, and the spoken message (”Please get rid of this beep”) was easily understandable.
- **Conclusion:**
  - The bandstop filter successfully improved the audio quality by removing high-frequency noise while preserving important low-frequency speech components.

2 Design of a simple IIR Filter

Lab Report:

Submit the following:

1. The difference equation for the filter $H_i(z)$.
2. The system diagram representing the filter structure.
3. The analytical expression of the impulse response for the filter.
4. The plot of the magnitude of the frequency response for each value of r.
5. A detailed explanation of how the value of r affects the magnitude of the frequency response.

Difference Equation:
Given transfer function:

$$H_i(z) = \frac{1-r}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$$

Rearranging, we have:

$$\frac{Y(z)}{X(z)} = \frac{1-r}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$$

Multiplying both sides:

$$(1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2})Y(z) = (1 - r)X(z)$$

Taking the inverse z-transform of both sides, we get the difference equation:

$$y[n] - 2r\cos(\theta)y[n-1] + r^2 y[n-2] = (1-r)x[n]$$

Thus, the required difference equation is:

$$y[n] = 2r\cos(\theta)y[n-1] - r^2 y[n-2] + (1-r)x[n]$$

Analytical Expression for the impulse response of $H_i(z)$:
The transfer function is given by:

$$H_i(z) = \frac{1-r}{1 - 2r\cos(\theta)z^{-1} + r^2 z^{-2}}$$

We begin by factorizing the denominator as:

$$H_i(z) = \frac{1-r}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}$$

Next, we express $H_i(z)$ in terms of partial fractions:

$$H_i(z) = \frac{A}{1 - re^{j\theta}z^{-1}} + \frac{B}{1 - re^{-j\theta}z^{-1}}$$

where A and B are constants to be determined. To find A and B, we multiply both sides by the denominator $(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})$ and then solve for A and B.
After simplification, we obtain:

$$1 - r = A(1 - re^{-j\theta}z^{-1}) + B(1 - re^{j\theta}z^{-1})$$

By setting $z^{-1} = 0$, we get the equation $1 - r = A + B$, which we solve for A and B.
Now, applying the inverse z-transform of $\frac{1}{1-az^{-1}}$, which is:

$$\mathcal{Z}^{-1}\left\{\frac{1}{1 - az^{-1}}\right\} = a^n u[n]$$

we obtain the impulse response in the time domain:

$$h[n] = A(re^{j\theta})^n u[n] + B(re^{-j\theta})^n u[n]$$

Since A and B are complex conjugates, we can combine these terms using Euler's formula:

$$h[n] = \frac{1-r}{2j\sin(\theta)}\left((re^{j\theta})^n - (re^{-j\theta})^n\right)u[n]$$

Using the identity $e^{jx} - e^{-jx} = 2j\sin(x)$, the expression simplifies to:

$$h[n] = \frac{(1-r)r^n \sin((n+1)\theta)}{\sin(\theta)}u[n]$$

Thus, the final expression for the impulse response is:

$$h[n] = \frac{(1-r)r^n \sin((n+1)\theta)}{\sin(\theta)} u[n]$$

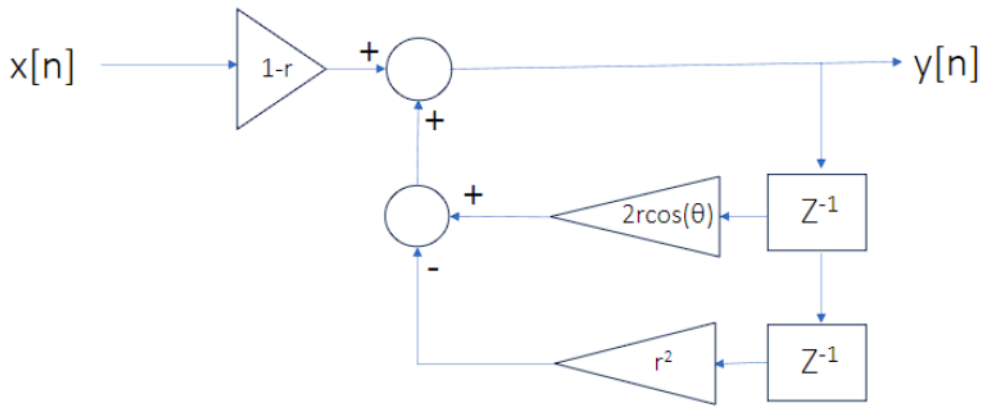

Figure 9: System diagram

2.1 Magnitude Response for Different Values of r

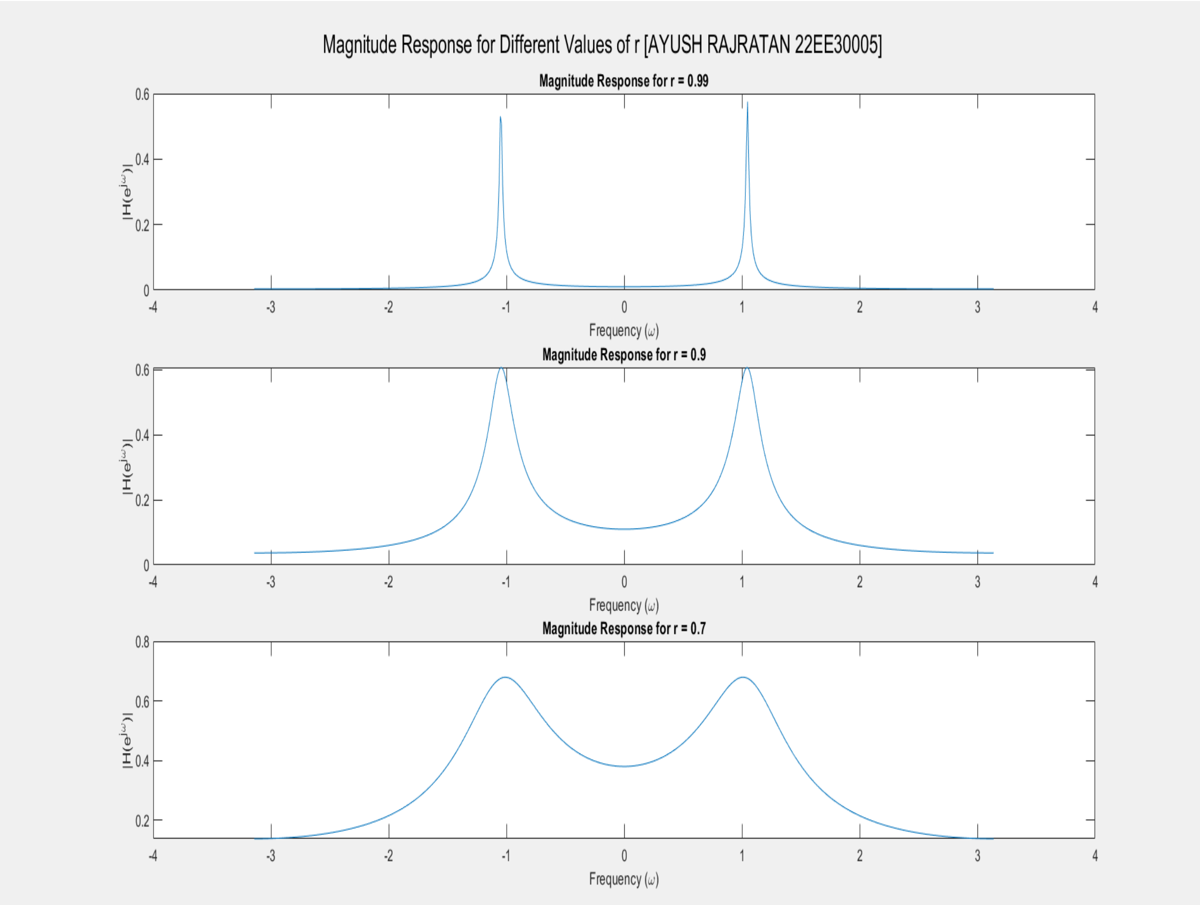

Figure 10: Magnitude Response for Different Values of r

2.2 Discussion

- **Parameter r:** Controls the position of the poles in the z-plane.
- **Effect of r approaching 1:**
  - Poles move closer to the unit circle.
  - The magnitude response has a sharper peak near $\omega = \theta$.
  - Bandwidth becomes narrower around the frequency θ.
- **Effect of small r:**
  - Poles are farther from the unit circle.
  - The magnitude response has a wider bandwidth and a less pronounced peak.
- **Increasing r:**

- Filter becomes more selective.
- Frequencies farther from θ are attenuated more.
- Frequencies near θ pass with minimal attenuation.

- **Magnitude response behavior:**
  - Magnitude response is inversely related to the distance of the poles from the unit circle.
  - As r increases, the peak becomes more pronounced at $\omega = \theta$.

- **Practical considerations:**
  - r can be tuned to emphasize or attenuate specific frequencies.
  - r too close to 1 may cause instability or ringing in the filter's output.

Lab Report:

For both the PCM signal and the filtered output, submit the following:

1. The time domain plot of the signal for 101 points.
2. The plot of the magnitude of the DTFT computed from 1001 samples of the signal.
3. The plot of the magnitude of the DTFT for ω in the range $[\theta - 0.02, \theta + 0.02]$.

Also hand in the code for the IIR filter filtering function. Comment on how the signal looks and sounds before and after filtering. How would you expect changes in r to change the filtered output? Would a value of $r = 0.9999999$ be effective for this application? Why might such a value for r be ill-advised?

In the context of filtering the given PCM signal, the parameter θ is calculated based on the modulation frequency and sampling rate. With the signal modulated at 3146 Hz and sampled at 8000 Hz, the angular frequency θ can be determined as:

$$\theta = \frac{3146}{8000} \times 2\pi \approx 2.47086$$

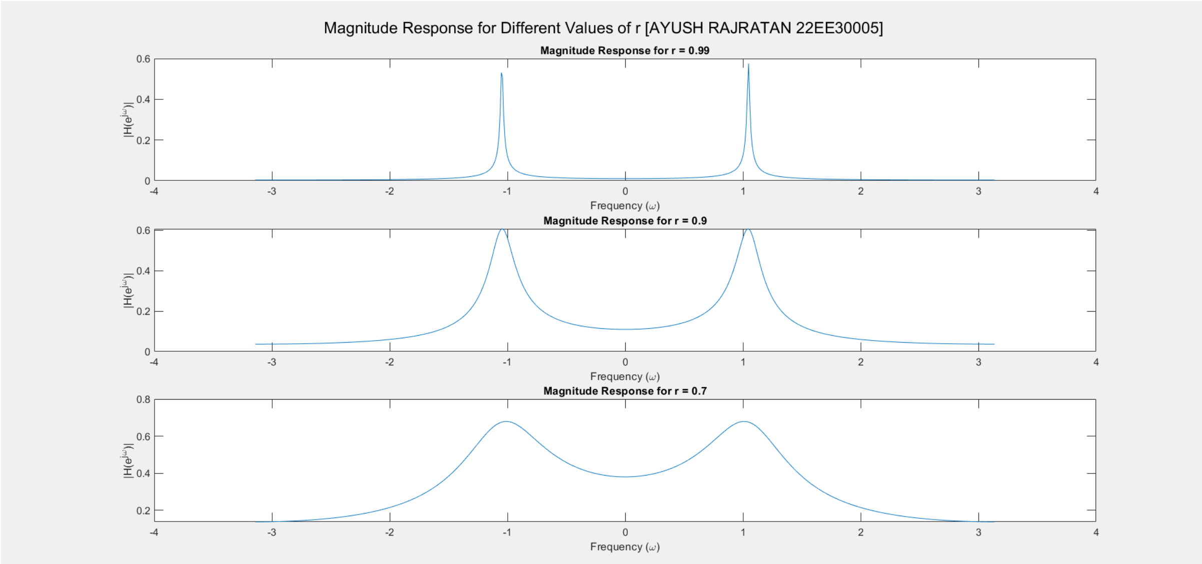

Figure 11: Plots for the original audio signal

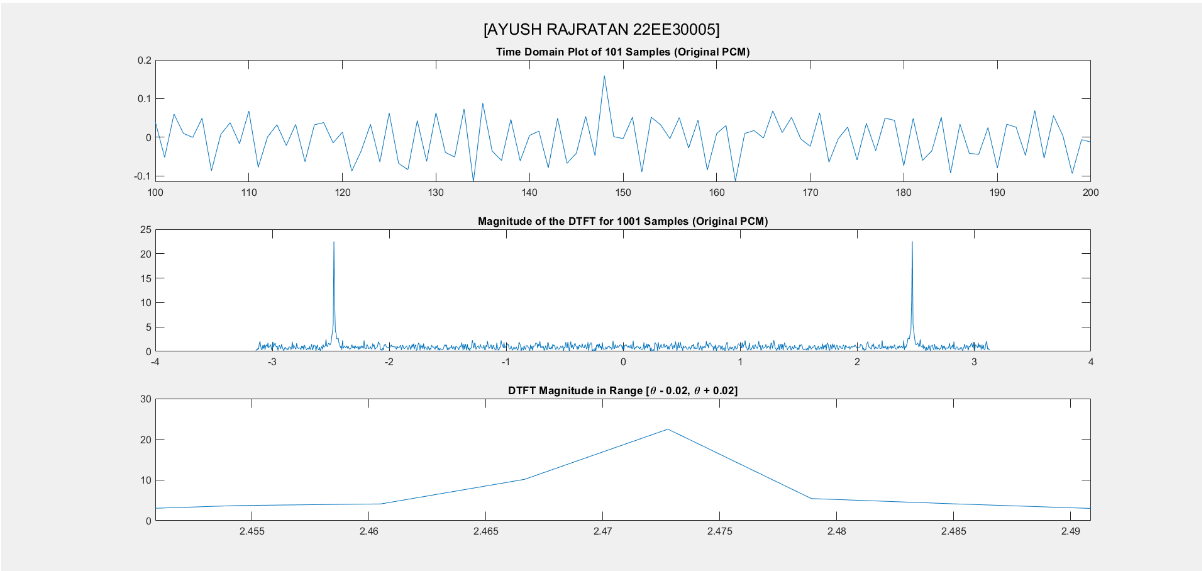

Figure 12: Plots for the filtered audio signal

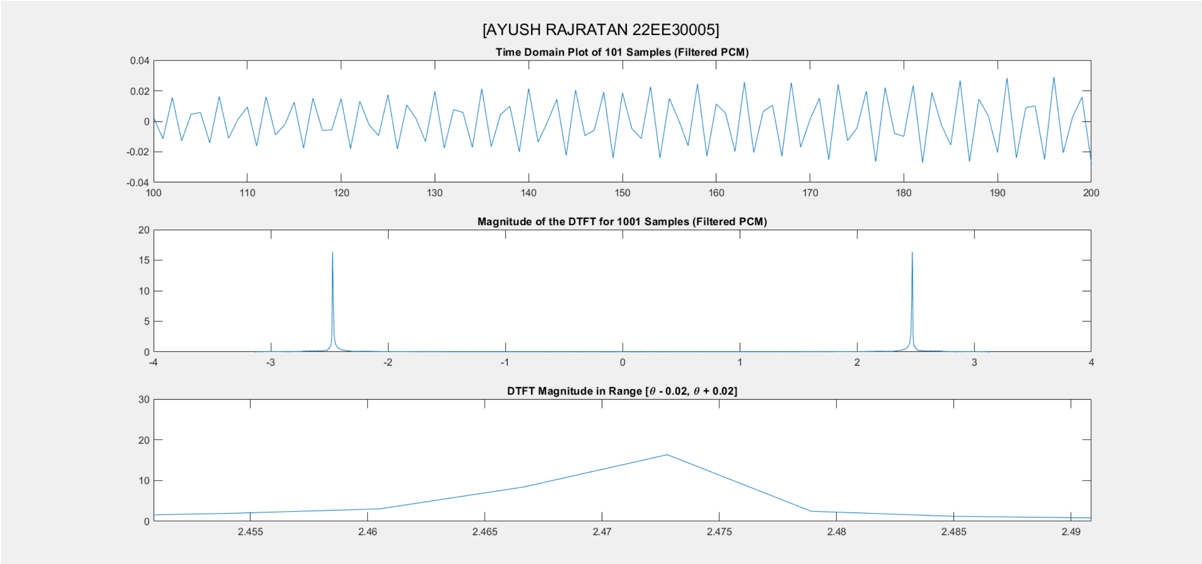

Figure 13: Original signal v/s Filtered signal

2.3 Discussion

- **Time-Domain and Frequency-Domain Analysis:**
  - Applying the IIR filter to the PCM signal involves generating time-domain plots.
  - The Discrete-Time Fourier Transform (DTFT) is used to analyze the frequency content.
  - Filtering results in attenuation of unwanted frequencies near θ, leading to a cleaner signal in the audible range.
- **Effect of Changing Parameter r:**
  - Increasing r brings poles closer to the unit circle in the z-plane.
  - This sharpens the filter's response around θ, improving frequency selectivity.
  - However, higher values of r also increase the risk of instability, especially as r approaches 1.
- **Risks of a High r Value:**
  - Setting r to a very high value, such as 0.9999999, leads to an excessively narrow filter.
  - This could cause the filter to attenuate frequencies near θ too much.
  - Such high selectivity can introduce side effects, like ringing or distortion, due to the narrow bandwidth of the filter.
- **Optimal r Value:**

- A moderate value of r is preferred for balancing frequency selectivity and stability.
- This ensures effective filtering without introducing unwanted artifacts.

3 Lowpass Filter Design Parameters

Lab Report:

1. Submit the plots of the magnitude response for the two filters (not in decibels). On each of the plots, mark the passband, the transition band, and the stopband.
2. Submit the plots of the magnitude response in decibels for the two filters.
3. Explain how the filter size affects the stopband ripple. Why does it have this effect?
4. Comment on the quality of the filtered signals. Does the filter size have a noticeable effect on the audio quality?

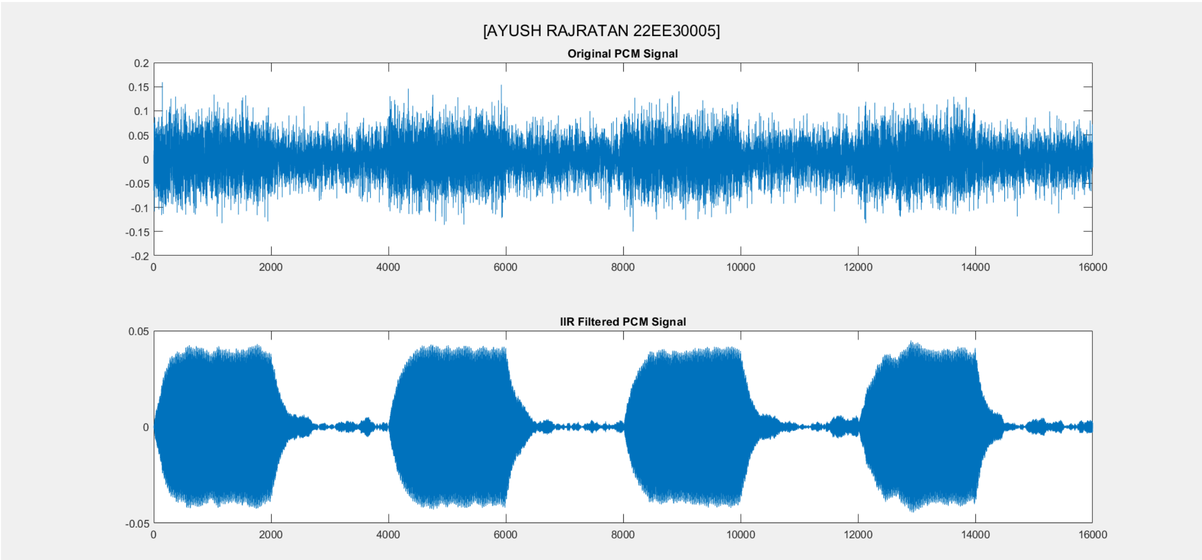

Figure 14: Magnitude response for the two filters (not in decibels)

Plot Markings:

- -1.8 to 1.8 - Passband
- -2.2 to -1.8 and 1.8 to 2.2 - Transition band
- -3 to -2.2 and 2.2 to 3 - Stopband

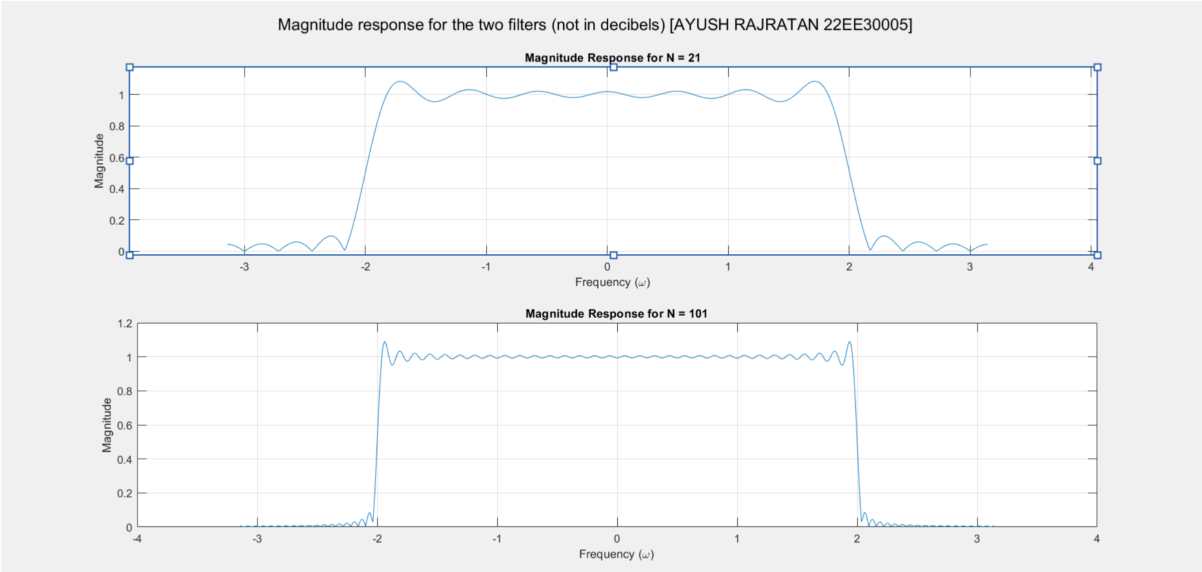

Figure 15: Magnitude response for the two filters (in decibels)

3.1 Effect of Filter Size on Stopband Ripple

- **Reduction in Stopband Ripple:**
  - Increasing the filter size significantly reduces the amplitude of ripples in the stopband.
  - This results in a smoother attenuation profile.
- **Better Approximation of Ideal Response:**
  - A larger filter can more accurately mimic the ideal response of a low-pass filter.
  - The transition band becomes narrower as filter size grows, allowing better attenuation of unwanted frequencies.
- **Improved Sinc Function Approximation:**
  - The reduction in stopband ripple can be attributed to the better approximation of the sinc function.
  - More filter coefficients allow for a more accurate representation of the desired frequency response.
- **Better Frequency Component Capture:**
  - With additional samples, the filter captures more frequency components, improving suppression of undesired frequencies.
- **Improvement in Signal Quality:**

- The reduction in ripple enhances the overall quality of the filtered signal.
- Unwanted noise is minimized, resulting in a cleaner output.

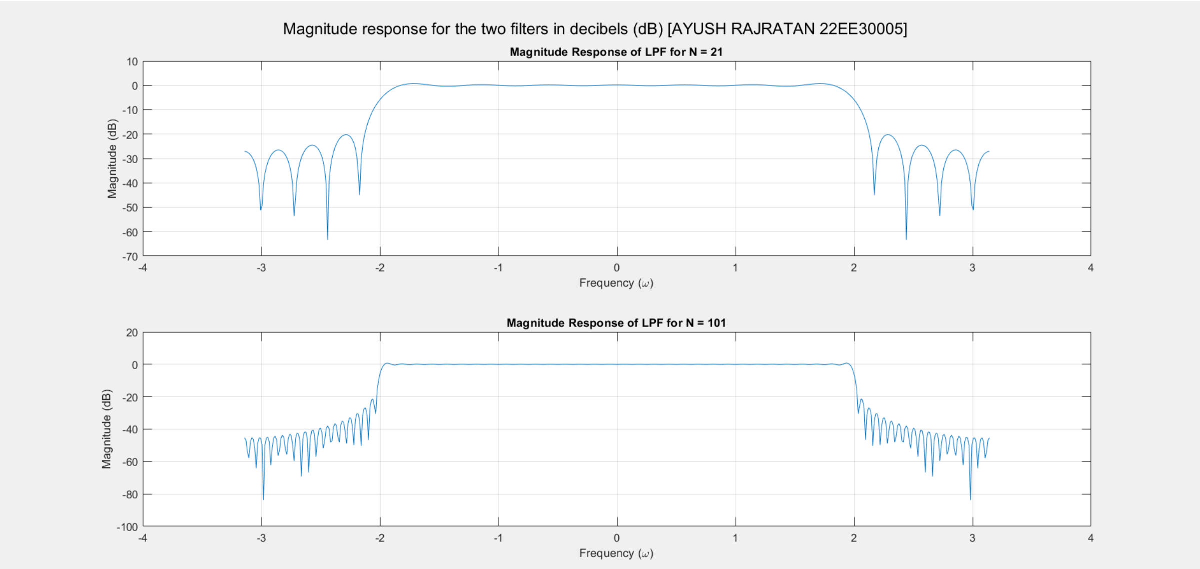

Figure 16: Comparison of filtered and unfiltered signal

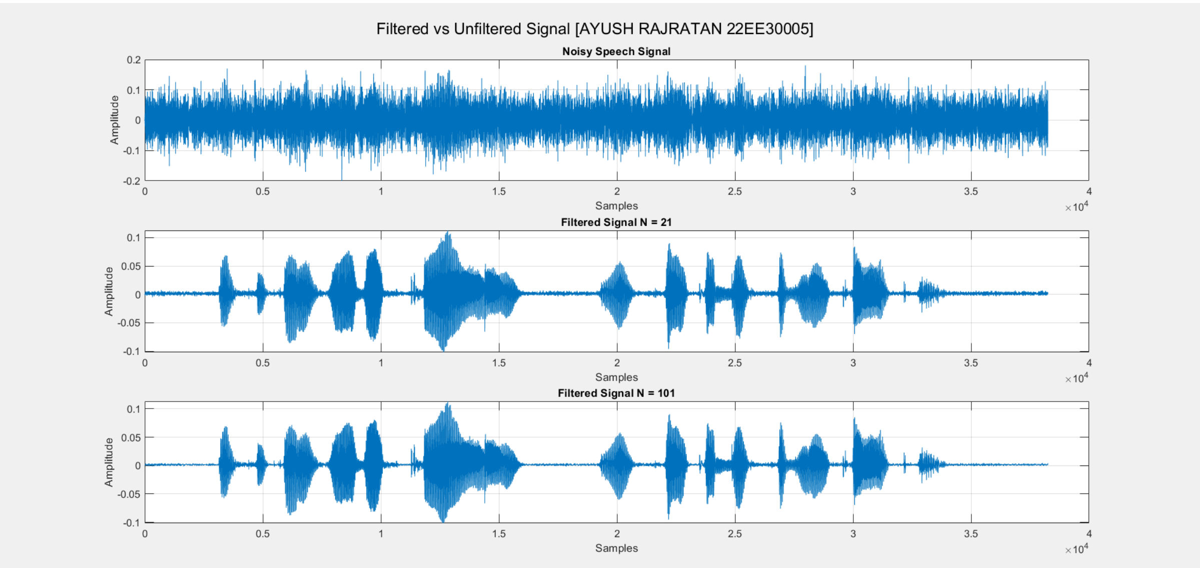

Figure 17: Unfiltered and filtered audio signal

3.2 Effect of Filter Size on Stopband Ripple

The size of a low-pass filter, specifically the number of taps or coefficients, significantly impacts the filter's characteristics, particularly in terms of stopband ripple. Stopband ripple refers to the fluctuations in the magnitude response of a filter within the stopband, which ideally should be attenuated to zero.

3.3 Relationship Between Filter Size and Stopband Ripple

Effects of Increasing Filter Size N:

- **Improved Stopband Attenuation:** Larger N results in better suppression of frequencies outside the passband.
- **Increased Oscillations:** Larger N may introduce oscillations at the edges of the stopband due to the sinc function's Fourier transform.
- **Narrower Transition Band:** The transition band between passband and stopband becomes narrower, allowing better isolation of desired frequencies, but may increase ripple.

Frequency Response:

$$|H(e^{j\omega})| = \begin{cases} 1 & \text{for } |\omega| < \omega_p \\ \text{Ripple} & \text{for } \omega_p < |\omega| < \omega_s \\ \text{Attenuation} & \text{for } \omega_s < |\omega| < \pi \end{cases}$$

Where: - ω_p is the passband edge frequency. - ω_s is the stopband edge frequency.

3.4 Quality of Filtered Signals

The primary goal of employing a low-pass filter in this context is to eliminate background noise, particularly high-frequency noise, from the audio signals. A cutoff frequency of 2 Hz was utilized for the filters applied to the audio signals.

3.5 Observations on Filtered Signals

Results of Filtering:

The filtering process demonstrates a significant improvement in audio clarity, as seen from the following observations:

- **Reduction in Background Noise:** Both filtered signals show substantial attenuation of background noise, leading to clearer speech.
- **Comparison Between Filter Sizes:**
  - For $N = 21$, residual noise persists, as the stopband does not fully attenuate high-frequency components.
  - For $N = 101$, high-frequency noise is more effectively reduced, resulting in a clearer and more distinct output.

- **Auditory Quality:** Larger filter sizes contribute to improved audio quality, with listeners reporting less high-frequency noise and a more pleasant listening experience.

Conclusion: Filter size is critical in determining audio signal quality. Larger filter sizes help minimize stopband ripple and enhance clarity, leading to superior auditory output.