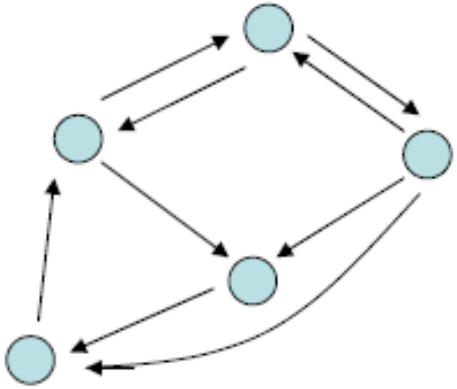
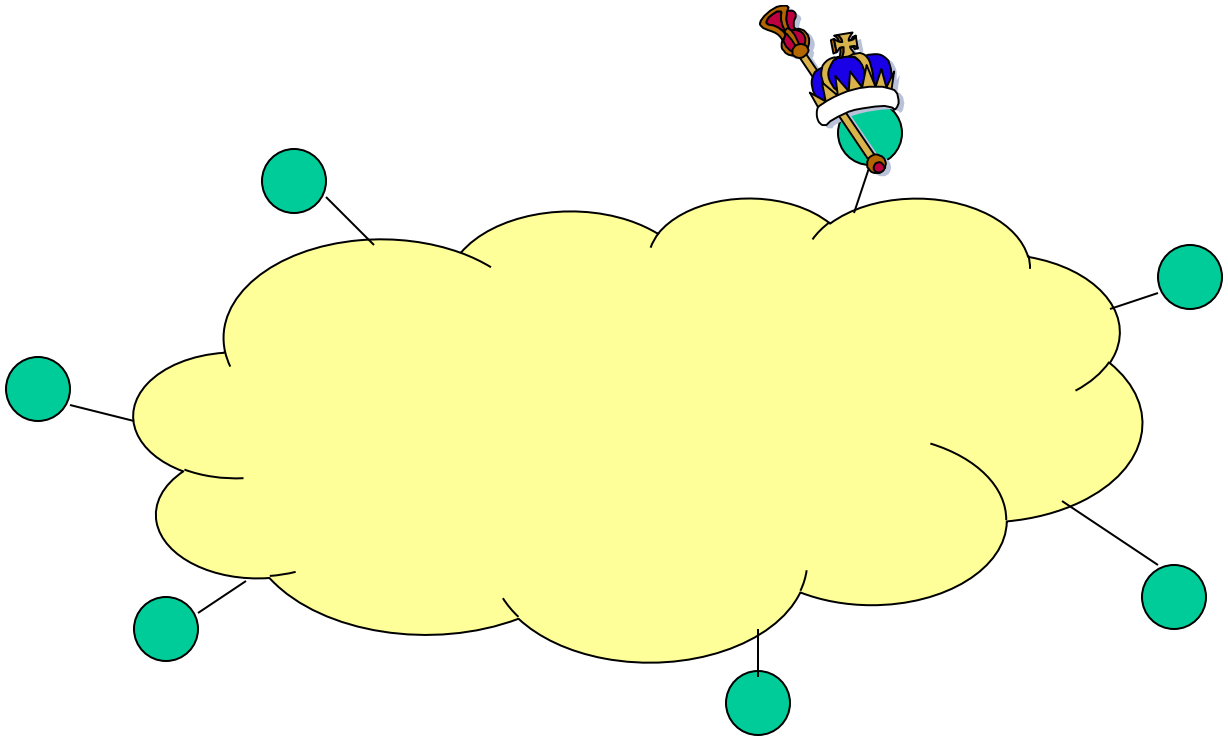
Leader election

- Network of processes.
- Want to distinguish exactly one, as the “leader”.
- Eventually, exactly one process should output “leader” (set special status variable to “leader”).



- Motivation: Leader can take charge of:
  - Communication
  - Coordinating data processing (e.g., in commit protocols)
  - Allocating resources
  - Scheduling tasks
  - Coordinating consensus protocols
  - …

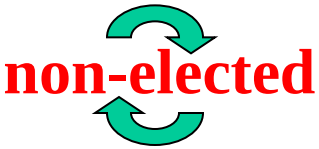
Leader Election: the idea



– **We study Leader Election in rings**
– illustrates techniques and principles
– good for lower bounds and impossibility results

The Problem

- Final states of processes partitioned in two classes:



- **Once entered a state, always in that state**

- **In every *admissible* execution, exactly one process (the *leader*) enters an elected state. All remaining enter a *non-elected* state**

Leader Election

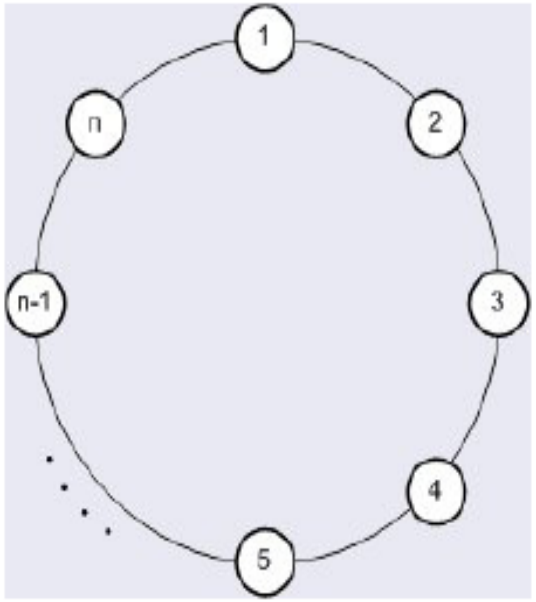
The election of a leader in a network requires the selection of a single, unique, process that will enter a state "leader" (or "elected") while all other processes enter the state "non-leader" (or "non-elected").

- The problem of leader election was formulated for the first time by LeLann (1977).
- The problem depicts the basic characteristics of a large family of problems encountered in real distributed systems.
- The problem has many variations.
- We start by considering the simple case where the network is a ring.

Ring Networks

- We assume that the network graph G is a ring consisting of n processes.
- Numbered $1 \ldots n$ in the clockwise direction.
- We often count $\mod n$, allowing 0 to be another name for process n, $n+1$ another name for process 1, ...
- The processes associated with the nodes of G do not know their indices, nor those of their neighbors.

Processes in a Ring



- We assume that message-generation and transition functions are defined in terms of local, relative names of their neighbors.

Problem Definition

An algorithm solves the problem of leader election if it meets the following specifications:

1. All halting states are split in two sub sets:
   1.1 all states that indicate the process as being "elected",
   1.2 all states that indicate the process as being "not-elected".
2. When a process reaches a halting state, the state-transition function only allows it to transit to states of the same subset.
3. In every execution of the algorithm
   - one and only one process is "elected",
   - all other processes are in "not-elected" state.

Variations of the problem

There are several variations of the problem:

- The ring can be either unidirectional or bidirectional.
- The number n of nodes may be either known or unknown to the processes.
- Processes may be identical or can be distinguished by each starting with a unique identifier (UID).
- It might be required that all not-elected processes eventually output the value "non-leader".
- It might be required that all non-elected processes eventually output the UID of the leader.
- We might wish to elect k leaders.
- ...

The LCR Algorithm

Algorithm LCR (informal)

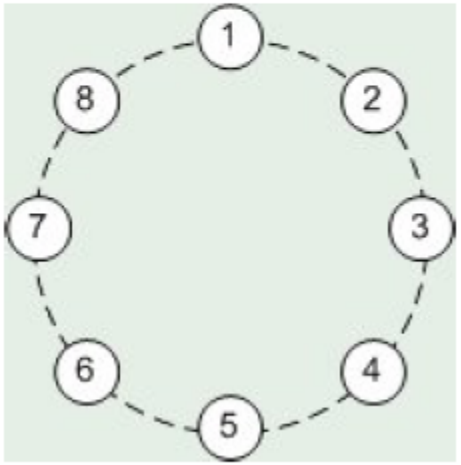
Each process sends its identifier around the ring. When a process receives an incoming identifier, it compares that identifier to its own. If the incoming identifier is greater than its own, it keeps passing the identifier; if it is less that its own, it discards the incoming identifier; if it is equal to its own, the process declares itself the leader.

- Decentralized, Uniform algorithm.
- Uses only unidirectional communication.
- Uses only comparison operations on the UIDs.
- Only the leader performs an output.

Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
  - Processes are indexed from 1 to 8 clockwise.

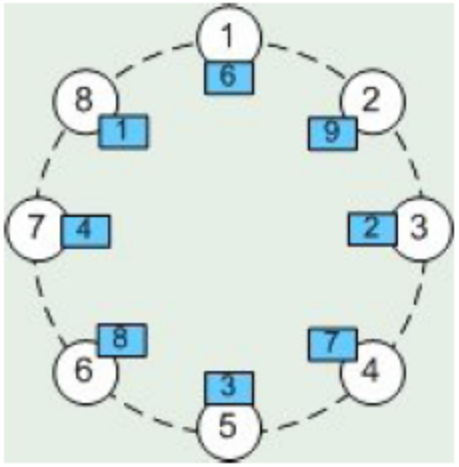
Synchronous Ring



Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
  - Processes are indexed from 1 to 8 clockwise.
- All processes have UIDs
  - Do now know the UIDs of the other processes.

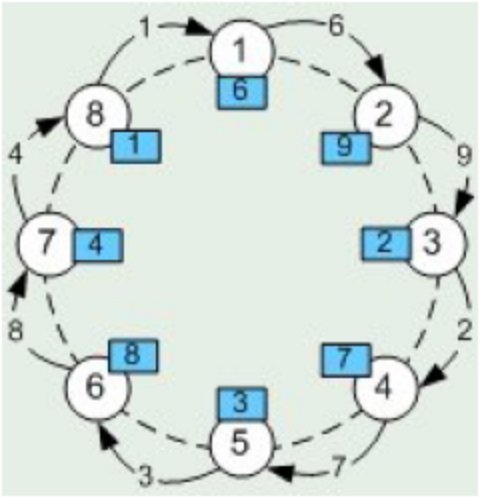
Synchronous Ring



Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
  - Processes are indexed from 1 to 8 clockwise.
- All processes have UIDs
  - Do now know the UIDs of the other processes.
- First round

Synchronous Ring



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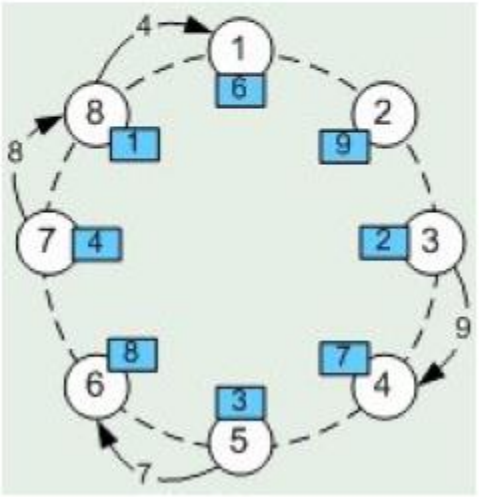
  - Do now know the UIDs of the other processes.
- First round
- Second round

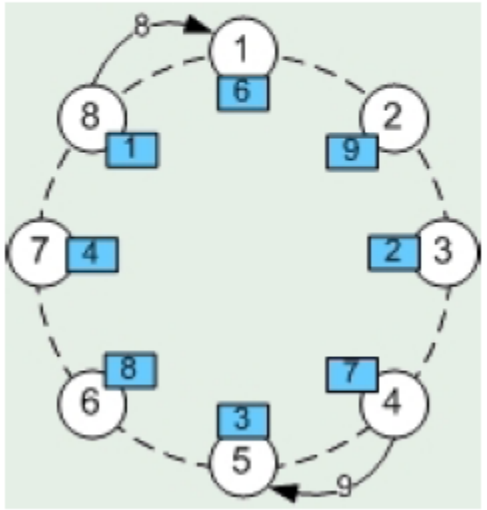
Synchronous Ring



Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
  - Processes are indexed from 1 to 8 clockwise.
- All processes have UIDs
  - Do now know the UIDs of the other processes.
- First round
- Second round
- Next rounds

Synchronous Ring



Example Execution

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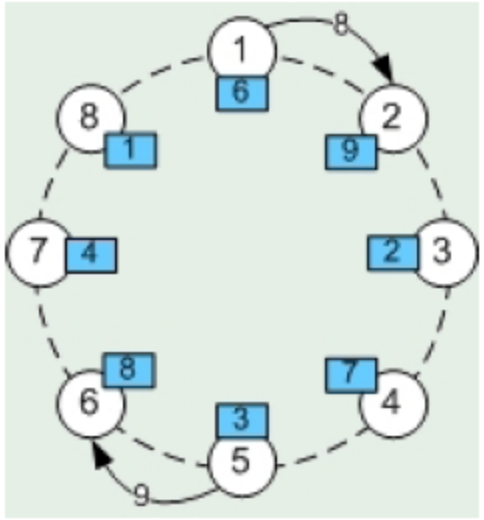
- First round
- Second round
- Next rounds

Synchronous Ring



Example Execution

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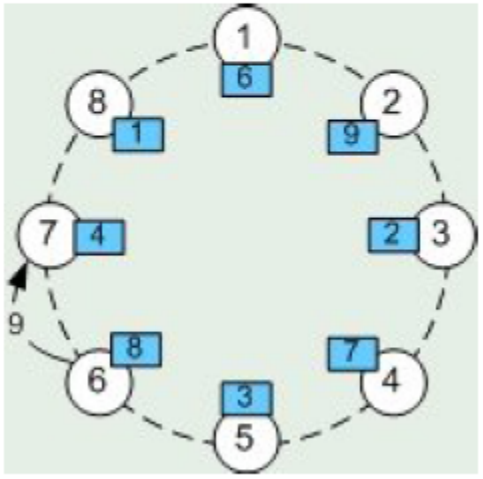
- First round
- Second round
- Next rounds

Synchronous Ring



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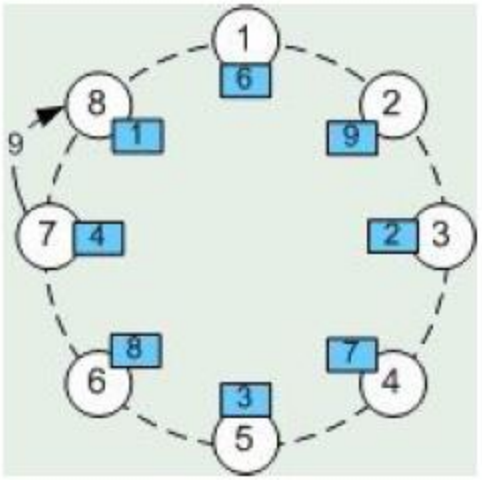
- First round
- Second round
- Next rounds

Synchronous Ring



Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
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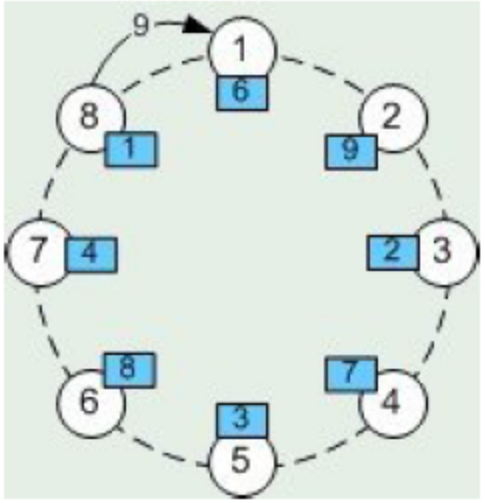
- All processes have UIDs
  - Do now know the UIDs of the other processes.
- First round
- Second round
- Next rounds

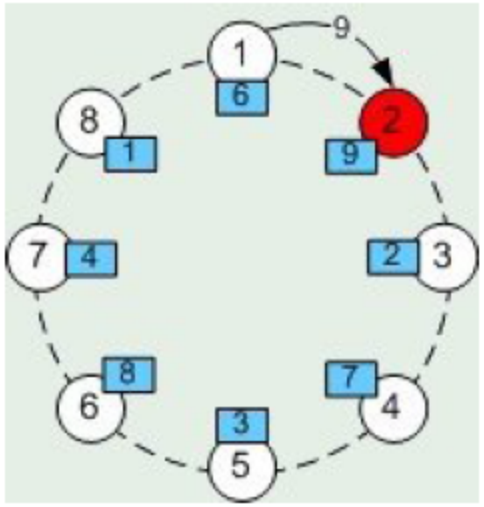
Synchronous Ring



Example Execution

- Let's assume a synchronous ring of $n = 8$ processes.
  - Processes are indexed from 1 to 8 clockwise.
- All processes have UIDs
  - Do now know the UIDs of the other processes.
- First round
- Second round
- Next rounds
- Leader election – process 2

Synchronous Ring



Algorithm Properties

Let i_{max} denote the index of the process with the maximum UID, and u_{max} its UID.

- Process i_{max} is elected leader at the end of round n.
- No other processes apart from i_{max} ends up in "elected" state.
- The time complexity is $\mathcal{O}(n)$
- The message complexity varies...
  - $\mathcal{O}(n^2)$ — worst case,
  - $\mathcal{O}(n)$ — best case,
  - $\mathcal{O}(n \log n)$ — average case.

Algorithm FloodMax

Every process maintains a record of the maximum UID it has seen so far (initially its own). At each round, each process propagates this maximum on all of its outgoing edges. After $diam(G)$ rounds, if the maximum value seen is the process's own UID, the process elects itself the leader; otherwise, it is a non-leader.

- Processes are not aware of the total number of processes (n).
- Processes are aware of the network diameter — $\delta = diam(G)$
- Comparison-based algorithm.

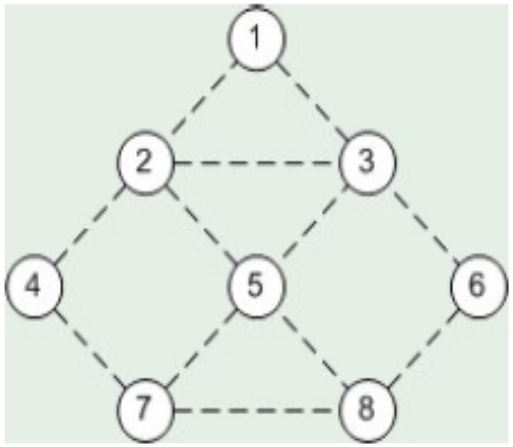
Pseudo-code for FloodMax

```
#DEFINE UID = <...>;
#DEFINE δ = <...>;
void main() {
   bool leader = false;
   int max_id = UID;
   for (int i = 0 ; i < δ; i++ ) {
      sendMessage(max_id);
      while (int new_msg = readMessage()) {
         if (new_msg > max_id)
            max_id = new_msg;
      }
   }
   if (max_id == UID)
     leader = true;
}
```

Example of Execution for FloodMax Algorithm

- Let a synchronous distributed system of $n = 8$ processes..
  - General network where $\delta = 3$
  - Processes are index $1 \ldots 8$

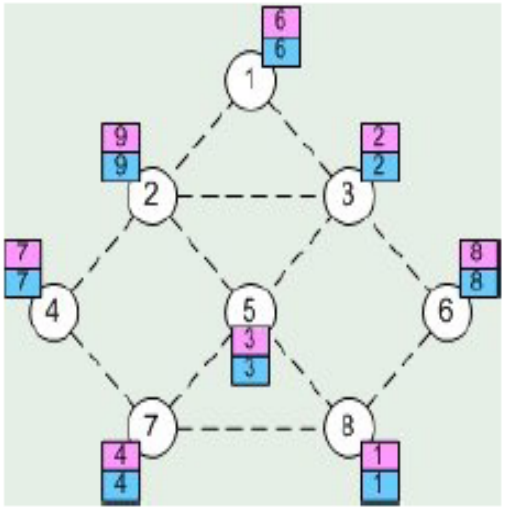
General Network



Example of Execution for FloodMax Algorithm

- Let a synchronous distributed system of $n = 8$ processes..
  - General network where $\delta = 3$
  - Processes are index $1 \ldots 8$
- The processes have UID.
  - Not aware of the UID of the other processes.

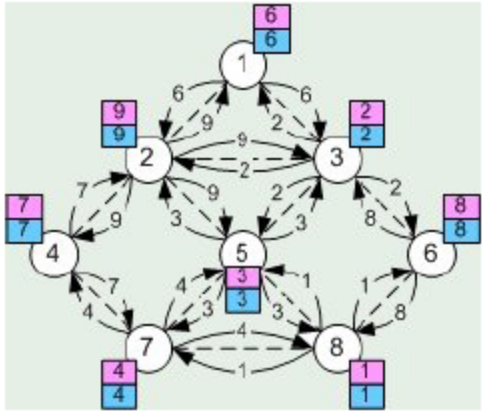
General Network



Example of Execution for FloodMax Algorithm

- Let a synchronous distributed system of $n = 8$ processes..
  - General network where $\delta = 3$
  - Processes are index $1 \ldots 8$
- The processes have UID.
  - Not aware of the UID of the other processes.
- First Round

General Network



Example of Execution for FloodMax Algorithm

- Let a synchronous distributed system of $n = 8$ processes..
  - General network where $\delta = 3$
  - Processes are index $1 \ldots 8$
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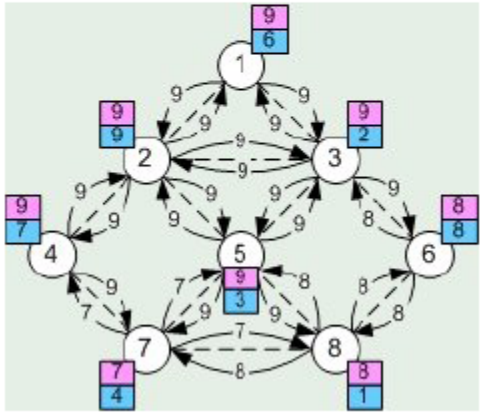
  - Not aware of the UID of the other processes.
- First Round
- Second Round

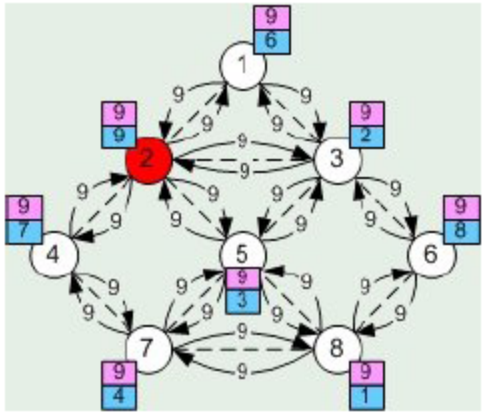
General Network



Example of Execution for FloodMax Algorithm

- Let a synchronous distributed system of $n = 8$ processes..
  - General network where $\delta = 3$
  - Processes are index $1 \ldots 8$
- The processes have UID.
  - Not aware of the UID of the other processes.
- First Round
- Second Round
- Leader Election

General Network



Properties of FloodMax Algorithm

Let n processes and m channels, where the process with the highest UID is i_{max}.

- Process i_{max} is elected leader at the end of round δ.
- No other process is in state "elected".
- Time complexity is $\mathcal{O}(diam(G))$.
- Message complexity $\mathcal{O}(diam(G) \cdot m)$.