Linear Algebra -

$P = X(X'X)^{-1}X' \quad M = I_n - P \quad y = X\beta + e \quad \hat{\beta} = (X'X)^{-1}X'y$

a) WTS : $P \cdot P = P$

$$P \cdot P = X(X'X)^{-1}X' \cdot X(X'X)^{-1} \cdot X'$$
$$= X(X'X)^{-1} \cdot I \cdot X'$$
$$= X(X'X)^{-1} \cdot X'$$
$$= P$$

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b) WTS : $M \cdot M = M$

$$M \cdot M = (I_n - P)(I_n - P)$$
$$= I_n - 2P + P \cdot P$$
$$= I_n - 2P + P$$
$$= I_n - P$$

c) WTS : $Py = \hat{y}$

$$y = X\beta + e$$
$$y = X\hat{\beta} + \hat{e}$$
$$\hat{y} = X\hat{\beta} = X(X'X)^{-1}X'y$$
$$= Py$$

d) WTS : $My = \hat{e}$

$$M = I_n - P$$
$$My = (I_n - P)y$$
$$= y - Py$$
$$= y - \hat{y}$$
$$= \hat{e}$$

e) WTS: $Py + My = y$

$\Rightarrow Py + (I_n - P)y$

$\Rightarrow Py + I_n y - Py$

$\Rightarrow y$

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f) WTS: $\hat{y} \perp \hat{e}$

$\hat{y} = Py \qquad \hat{e} = My$

$\hat{y} \cdot \hat{e} = Py \cdot My$

$= y^2 (P(I_n - P))$

$= y^2 (P I_n - P \cdot P)$

$= y^2 (P - P)$

$= y^2 (0)$

$= 0$

$\hat{y}$ & $\hat{e}$ are orthogonal