Problem set 3

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Linear algebra

1. General Least Squares:

Suppose we are estimating a linear model:

$$y = \mathbf{X}\beta + e$$

However instead of the $Var(e) = \sigma^2\mathbf{I}_n$, we know that $Var(e) = \Sigma$. Derive the optimal OLS estimator given this condition. That is, minimize $e'\Sigma^{-1}e$ and obtain the optimal estimator. It may be helpful to take the square root Σ, i.e., $\Sigma = \Sigma^{1/2}\Sigma^{1/2}$.

Programming

2. OLS estimation of AR(1):

Consider the following AR(1) model:

$$x_t = \alpha + \rho x_{t-1} + e_t$$

Generate data from this model. Then estimate the model using ordinary least squares. The estimated $\hat{\rho}$ that you will find will be biased. Write a simulation to study this bias. See if you can find the functional form of the bias via simulation.