

Problem set 4

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Linear algebra

1. Real business cycles:

Consider the following macroeconomic model for real business cycles taken from Dejong and Dave (2011) Chapter 3:

Suppose that we have a representative household whose goal is to maximize their utility from consumption and leisure over time:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

where E_0 is the expected value at time 0, β is a parameter reflecting the household's discount rate (time preferences), c_t is the household's consumption at time t , and l_t is the household's leisure time at time t .

For this problem to work, we have to introduce four constraints. The first constraint is that the household produces goods using a Cobb-Douglas production function. That is,

$$y_t = z_t k_t^\alpha n_t^{1-\alpha}$$

where y_t represents the GDP at time t , z_t represents and exogenous (i.e., cannot be optimized or changed) technology stock at time t , k_t represents the gross capital stock at time t , and n_t represents the amount of labor supplied at time t . The second constraint is that total time spent working and at leisure should add to some fixed constant. Without loss of generality we can define the total to be one (1):

$$1 = n_t + l_t$$

The third constraint is that GDP can be spent in two ways: consuming and investing in future capital:

$$y_t = c_t + i_t$$

where i_t is the amount of investment at time t .

$$k_{t+1} = i_t + (1 - \delta)k_t$$

The last thing we need to define to make the problem solvable is the utility function. For this we are going to use a constant elasticity of substitution (CES) form. That is,

$$u(c, l) = \frac{(c^\phi l^{1-\phi})^{1-\rho}}{1-\rho}$$

where ρ and ϕ are parameters representing the trade off between happiness from consuming versus leisure in the model.

Your job is solve this consumer's problem using calculus by maximizing the total utility subject to the constraints defined above. In the end, you should obtain two equations. The first equation will reveal the consumer's inner-temporal choice between consumption and leisure. The second equation will reveal the consumer's intertemporal choice between consuming today and consuming tomorrow. If you are struggling, the answer is in Dejong and Dave (2011) Chapter 3.

Programming

2. Ridge regression:

Write some code which perform L_2 regularization. That is, solve OLS with a constraint that the L_2 norm of the betas is less than some threshold value.

$$\min_{\beta} \sum_{i=1}^n e_i^2 \text{ subject to } \sum_{i=1}^r \beta_i^2 \leq T$$