Problem set 2

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Linear algebra

1. Partition matrices:

Consider using a partition matrix in OLS. That is, consider $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2]$ and

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

where $\mathbf{X}_1$ is $n \times r_1$, $\mathbf{X}_1$ is $n \times r_2$, β_1 is $r_1 \times 1$, β_2 is $r_2 \times 1$, and $r_1 + r_2 = r$, the overall number of columns in $\mathbf{X}$.

When we take the inverse of a 2×2 partition matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} (\mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{D})^{-1} & -(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} \\ -(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{C}\mathbf{A}^{-1} & (\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1} \end{bmatrix}$$

Find the equations for $\hat{\beta}_1$ and $\hat{\beta}_2$ using the OLS estimator $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'y$

Programming

2. White estimator:

Extend the linear model code we wrote in class to implement a White corrected OLS estimator. The robust variance covariance matrix for the White estimator is

$$\widehat{\mathrm{Var}}(\hat{\beta}) = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\hat{\mathbf{\Omega}}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}$$

where $\hat{\mathbf{\Omega}} = \mathrm{diag}(\hat{e}^2)$. You can either create a new inheriting class or you can add the functionality as an option in the least squares code that we wrote.