CS 715: Advanced Topics in Algorithmic Game Theory and Mechanism Design

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Lecture 1

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Agenda

- Motivation
- Course Contents and Organization

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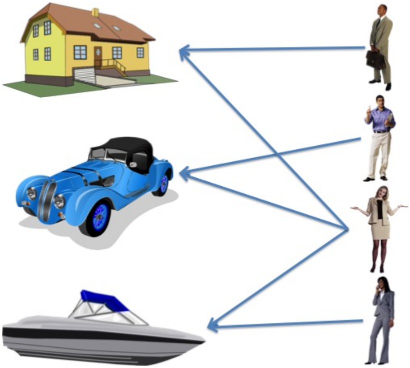
- Course Contents and Organization
- What is Game Theory
    - Elements of a Game
    - Extensive Form Games and Strategic Form Games
    - Dominant Strategy Equilibrium
    - Two Player Zero Sum Game

Example 1: Auctions (1)

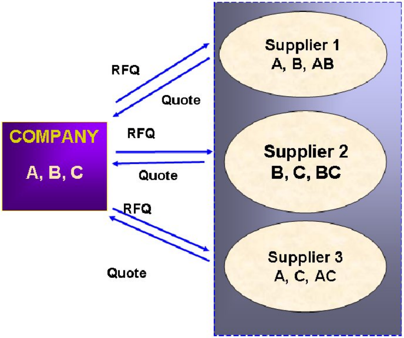
You are leaving the country permanently



- Different prospective buyers have interests in different combinations.
- Exact valuations are known to the buyers.
- You want to make maximum expected profit.

Example 1: Auctions (2)

Your company needs to procure large volumes of products A,B and C.



- 3 Suppliers, can supply different bundles
- Exact costs are known to the suppliers.
- You want to minimize expected costs.

Example 2: FCC Spectrum Auctions

- *In designing auctions for spectrum licenses, the FCC is required by law to meet multiple goals and not focus simply on maximizing receipts. Those goals include ensuring efficient use of the spectrum, promoting economic opportunity and competition, avoiding excessive concentration of licenses, preventing the unjust enrichment of any party, and fostering the rapid deployment of new services, as well as recovering for the public a portion of the value of the spectrum.* [1]
- Multiple bidders, interested in different combinations
- Govt needs to ensure some provided in the areas where business opportunity is less
- At least some fraction of licenses to small players
- Govt cares about social welfare, that is let the licenses be in the hands who value it most

[1] Rose, Gregory F., and Mark Lloyd. "The Failure of FCC Spectrum Auctions." Center for American Progress 5 (2006).

Example 3:Online Auctions

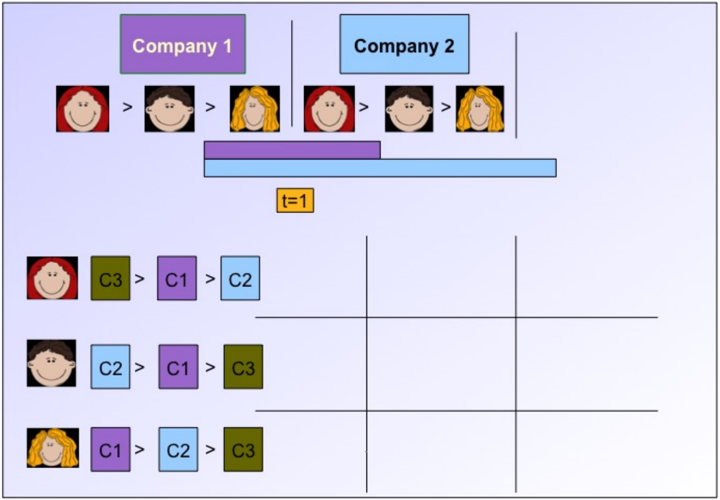
- Airline seat allocation, when agents arrive dynamically to book the tickets
- Hotel room bookings, prices to be varied according to demand
- Resource allocation such as computing power to dynamically arriving requests
- spicejet.com: Indian airline carrier offering flight ticket as low as INR 599 ($< \$10$) on low passenger flights

Example 4: House Allocation

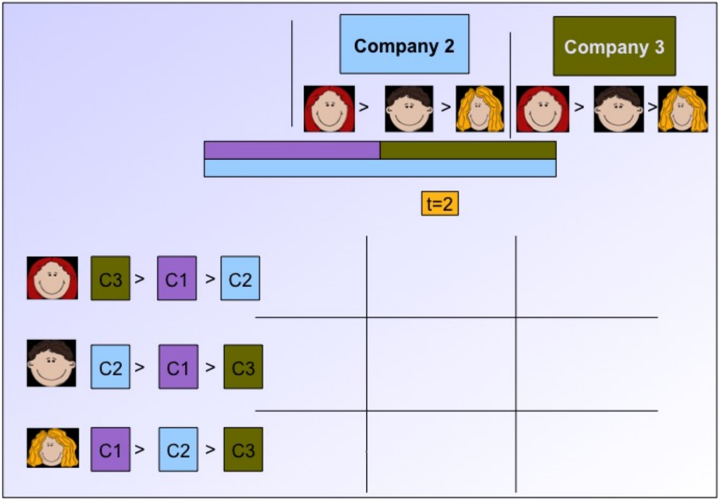
- $A_1 - A_5$ 5 friends got 5 dorm $R_1 - R_5$ respectively. However prefer other rooms over what they got.

A1	R5	>	R2	>	R1	>	R3	>	R4
A2	R1	>	R2	>	R3	>	R4	>	R5
A3	R1	>	R4	>	R3	>	R2	>	R5
A4	R1	>	R3	>	R2	>	R4	>	R5
A5	R2	>	R1	>	R3	>	R4	>	R5

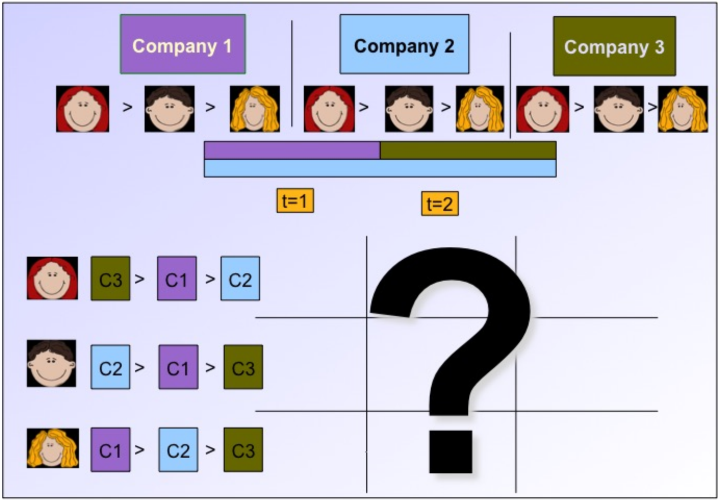
Example 5: Campus Recruitment



Example 5: Campus Recruitment



Example 5: Campus Recruitment



Challenges in All the above Examples

- Decisions depend upon the reported preferences/bids which are private to the agents
- The agents involved are into marketplace for making profits
  - Will not report true bids or preferences if its not in their interests

Goal

- Design a game such that it is in best interest of every agent to act according to designed protocol, report bids/preferences truthfully
- Achieve some system wide goal

- This calls for Mechanism Design Theory

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What is Mechanism Design Theory

- Mechanism design: an important game theoretic tool in microeconomics.
- Addresses: the problem of aggregating the announced preferences of multiple agents into a collective decision
- The agents are strategic

Mechanism Design

Mechanism Design is the art of designing rules of a game to achieve a specific outcome in presence of *multiple self-interested agents*, each with *private information* about their preferences

Economics and Computer Science

- Economics for computer science
  - design of autonomous systems, resource sharing
  - crowdsourcing, e-commerce, social networking
  - online advertising, prediction markets, online education forum, intelligent transportation
- Computer Science for Economics
  - Algorithms for equilibrium computation
  - Need efficient algorithms to implement economics markets such as auctions
  - Underlying problems are HARD
  - There is always tension between incentive constraints and approximations

Course Outcomes

You Learn:

- Quick Introduction to Game Theory and Mechanism Design
- Challenges in implementing ***good*** mechanisms
- Bayesian Optimal Mechanism Design (Revenue Maximization, Example 1)
- Optimal Combinatorial Auctions and Multi-unit Auctions (Welfare Maximization) (Example 2)
- Dynamic Auctions (aka online auctions) (Example 3)
- Mechanism with out money (Example 4,5)

Grading

Initial 16 Classes, followed by presentations.
Each student is expected to present a paper[2] and write a detailed report.

- Scribe: 10 points
- Presentation: 35 points
- Final report: 20 points
- Oral Exam: 35 points

[2] FOCS, SODA STOC, WINE, ACM EC

Introduction to Game Theory

Game Theory

- Analysis of conflict
- According to Myerson

Definition (Game Theory)

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers.

- Analyses and Predicts the behaviour of strategic agents (players) with conflicting interests
- Suggests the strategies to play

Prisoner's Dilemma

Prisoner's Dilemma stated differently

- Two friends (A,B) caught for cheating in exams
- Invigilator takes both of them to two different rooms
- Offers A: If you be confess cheating and B does not, I will let you go with only 1 point deduction and deduct 10 points from B.
- If both of you confess, I deduct 5 points from each
- Similarly if you dont confess, but he confesses, I will deduct 10 points from you
- In the absence of nobody confessing, I will deduct 2 points each

What is Game?

Elements of a game[3]:

- **Players**: the agents playing the game.
- **Actions**: that change the state of the game.
- **States** of the game.
- **Knowledge** (beliefs) of the state and actions.
- **Outcome** of the players actions, in particular payoffs for each player.
- **Payoff or Utility** That each player derives from the outcome
- **Assumption**: every player acts rationally so as to maximize its own payoff.
- Information about game is common knowledge

[3] Credits Prof Boi Faltings

Strategies

- Strategy: is an algorithm or rule by which each player chooses an action.

  Today's Talk
- Pure strategy: for each state (or believed state), the action is chosen in a deterministic way.
- $\forall i \in N$, S_i be the space of strategies available to player i
- $S = S_1 \times S_2 \times \ldots S_n$ is space of strategy profiles that all the players can play.
- we write $s = (s_i, s_{-i})$, where s_{-i} is strategy followed by all the players except player i.

Extensive Form Games

Recall Prisoner's Dilemma

- 2 players A and B.
- Actions: player A and B choose between C and NC.
- States: Initial + 2 states for actions of player A + 4 states for each combination of actions (2×2).
- Knowledge: A and B both know the game and rules. However both do not know the others choice.
- Outcomes: 4 possible outcomes
- Utilities: depend on the outcome

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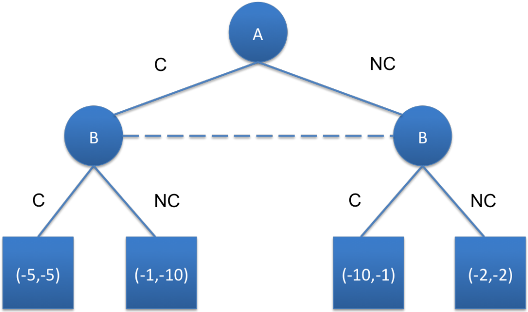
Extensive Form Games



- Dotted line indicates Player B does not know in which state she is.
- Such sets are called as Information sets.
- If all information sets are singleton, then its called a game with **perfect information**

Classification of Games

- Co-operative vs Non Co-operative

  joint action of groups vs individual actions

- Perfect information vs Imperfect information

- Extensive form vs Strategic Form vs Characteristic Form

- Complete Information vs Incomplete Information

  Some (or all) agents have private information when the game begins

- Rational vs Bounded Rational (players)

  Chess!!

Strategic Form Games (Normal form Games)

N : Set of players
$N = \{1, 2, \ldots, n\}$

S_1 : Strategies available to player 1
S_2 : Strategies available to player 2
$\vdots$
S_n : Strategies available to player n
$S = S_1 \times S_2 \times \ldots \times S_n$
Strategy space of all the players

$U_1 : S \to \mathbb{R}$
$U_2 : S \to \mathbb{R}$
$\vdots$
$U_n : S \to \mathbb{R}$

Utility or Payoff Functions

- This is also known as **matrix form** games

Example: Prisoner's Dilemma

	No Confess **NC**	Confess **C**
No Confess **NC**	**- 2, - 2**	**- 10, - 1**
Confess **C**	**-1, - 10**	**- 5, - 5**

Example: Prisoner's Dilemma

	No Confess NC	Confess C
No Confess NC	- 2, - 2	- 10, - 1
Confess C	-1, - 10	- 5, - 5

http://www.youtube.com/watch?v=S0qjK3TWZE8

Dominant Strategies

Strongly Dominated Strategy

- Given a game $\Gamma = < N, (S_i), (U_i) >$, a strategy $s_i \in S_i$ is said to be strongly dominated if there exists another strategy $s_i' \in S_i$ such that

$$U_i(s_i, s_{-i}) < U_i(s_i', s_{-i}) \forall s_{-i} \in S_{-i}$$

In such a case, we say strategy s_i' strongly dominates strategy s_i.

Strongly Dominant Strategy

- A strategy $s_i^* \in S_i$ is said to be a *strongly dominant strategy*, for player i if it strongly dominates every other strategy $s_i \in S_i$. That is, $\forall s_i \neq s_i$,

$$U_i(s_i, s_{-i}) < U_i(s_i^*, s_i) \forall s_i \in S_i$$

Dominant Strategies Equilibria

Weakly Dominant Strategy

- Given a game $\Gamma =< N, (S_i), (U_i) >$, a strategy $s_i \in S_i$ is said to be weakly dominant if there exists another strategy $s_i' \in S_i$ such that That is, $\forall s_i \neq s_i$,

$$U_i(s_i, s_{-i}) \leq U_i(s_i^*, s_i) \forall s_i \in S_i$$

with strict inequality for at least one s_{-i}. In such a case, we say strategy s_i^* is weakly dominant strategy s_i.

Strongly (Weakly) Dominant Strategy Equilibrium A profile of strategies $(s_1^*, s_2^*, \ldots, s_n^*)$ is called a *strongly dominant strategy equilibrium* of the game $\Gamma =< N, (S_i), (U_i) >$ if $\forall i = 1, 2, \ldots, n$, the strategy s_i^* is a strongly dominating strategy for player i.

Example: Prisoner's Dilemna

	No Confess NC	Confess C
No Confess NC	- 2, - 2	- 10, - 1
Confess C	-1, - 10	- 5, - 5

Example: Prisoner's Dilemna

	No Confess NC	Confess C
No Confess NC	- 2, - 2	- 10, - 1
Confess C	-1, - 10	- 5, - 5

- Possible strategies: (NC,NC), (C,NC), (NC,C) and (C,C)
- C is best response if other player is playing C
- Note: C is best response even other player is playing NC
- C is dominant strategy for both the players
- (C,C) is Dominant Strategy Equilibrium

Example: Tragedy of Commons

Garrett Hardin: dilemma occurring in the situation when multiple agents act rationally in self-interest and ultimately deplete a shared limited resource

Image Credits: Wikipedia

Example: Tragedy of Commons

Garrett Hardin: dilemma occurring in the situation when multiple agents act rationally in self-interest and ultimately deplete a shared limited resource

Image Credits: Wikipedia

- Each farmer either can allow his cow to graze or does not keep a cow
- If he allows, say he receives benefit of unit 1
- However, damage to environment is 5 for each cow.
- Total damage equally shared by all the farmers

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Tragedy of Commons Contd...

N	Set of farmers $=\{1,2,\ldots,n\}$
S_i	$\{0,1\}$ $\forall i$ Strategy for each farmer
$U_i(s_1, s_2, \ldots, s_n)$	$s_i - \frac{5(s_1+s_2\ldots+s_n)}{n}$

Tragedy of Commons Contd...

N	Set of farmers $=\{1,2,\ldots,n\}$
S_i	$\{0,1\}$ $\forall i$ Strategy for each farmer
$U_i(s_1, s_2, \ldots, s_n)$	$s_i - \frac{5(s_1+s_2\ldots+s_n)}{n}$

- If $n > 5$, each farmer keeps cow
- If $n < 5$, each farmer prefers not to keep cow
- Suppose, Government puts environment tax of 5 to those who keep cow
- $U_i(s_1, s_2, \ldots, s_n) = s_i - 5 * s_i - \frac{5(s_1+s_2\ldots+s_n)}{n}$
- Each farmer prefers not to keep cow

Does Dominant Strategy Equilibrium Always Guaranteed?

Matching Pennies Game

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

- Two Players: Start 1 CHF
- Keep Head or Tail Up
- If match, player 1 gets both the coins. Else, player 2 gets both the coins

No Dominant Strategy for any of the player.
Observe, in this game, gain of one player is loss to other.
This leads to zero-sum games

Two Player Zero Sum Games

- Gain of one player $=$ Loss to the Other
  Total sum of utilities $= 0$.
- $\Gamma = < \{1, 2\}, S_1, S_2, U_1, -U_1 >$. Also called as Matrix Games.
- We can represent the game by a single $m \times n$ matrix. For Example, game $\Gamma^Z =$:

1	2	1
0	-1	2
-1	0	-2

- First player as *row player* and other as *column player*.

Equilibrium in Two player Zero Sum Games

- Saddle Point: Given a matrix A, a saddle point is a_{ij} if

$$a_{ij} \leq a_{il} \ \forall l = 1, 2, \dots, n;$$

$$a_{ij} \geq a_{kj} \ \forall k = 1, 2, \dots, m;$$

- If such saddle point exists, it is called an equilibrium.
  The strategy that achieves this is mini-max strategy.
- Let $u_R = \max_i \min_j a_{ij}$ and $u_C = \min_j \max_i a_{ij}$. If saddle point exists, $u_R = u_C$
- If saddle point exists, then for row player: she is maximizing her min assured gain.
  For column player: she is minimizing her worst loss (same as maximizing her min assured gain).
- If row player is playing i, the column player cannot reduce her loss by deviating j. Same holds true for row player too.

Equilibrium Continued

Game Γ^Z:

1	2	1
0	-1	2
-1	0	-2

$u_R = 1, u_C = 1$. Hence (1,1) is equilibrium.

Matching Pennies

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

$u_R = -1, u_C = 1$. No pure strategy equilibrium.

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Mixed Strategies

- In matching pennies game, row player tosses a coin and if H, then play H, else T.
- Similarly column player plays her action.
- Row player expected payoff = Pr(H,H) - Pr(H,T) - Pr(T,H) + Pr(T,T) = 0
- Column Player expected utility = -Pr(H,H) + Pr(H,T) + Pr(T,H) - Pr(T,T)
- Such randomization over across is called as mixed strategy

Mixed Strategies

- Say for player i, there are i_k actions, $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$.
- She decides to play these actions with probabilities $p_{i_1}, p_{i_2}, \ldots, p_{i_k}$ with $p_{i_1} + p_{i_2} + \ldots + p_{i_k} = 1$
- $S_i = i_k$ dimensional simplex
- For player i, expected payoff $=$
$$\sum_{s_{-i} \in S_{-i}} p_{i_1} * p(s_{-i}) * U(a_{i_1}, s_{-i}) + p_{i_2} * p(s_{-i}) * U(a_{i_2}, s_{-i}) + \ldots + p_{i_k} * p(s_{-i}) * U(a_{i_k}, s_{-i})$$
- This leads to Utility Theory

Further Reading

- Game Theory by Roger Myerson. Harvard University press, 2013.
- gametheory.net
- Prof Y Narahari: http://lcm.csa.iisc.ernet.in/gametheory/lecture.html
- Y Narahari. Game Theory and Mechanism Design. World Scientific Publishing Company, 2014.
- Algorithmic Game Theory, edited by Noam Nisan, Tim Roughgarden, Eva Tardos and Vijay Vazerani.