On-line Mechanisms without Money

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CS715: Advanced Topics in Algorithmic Game Theory and Mechanism Design

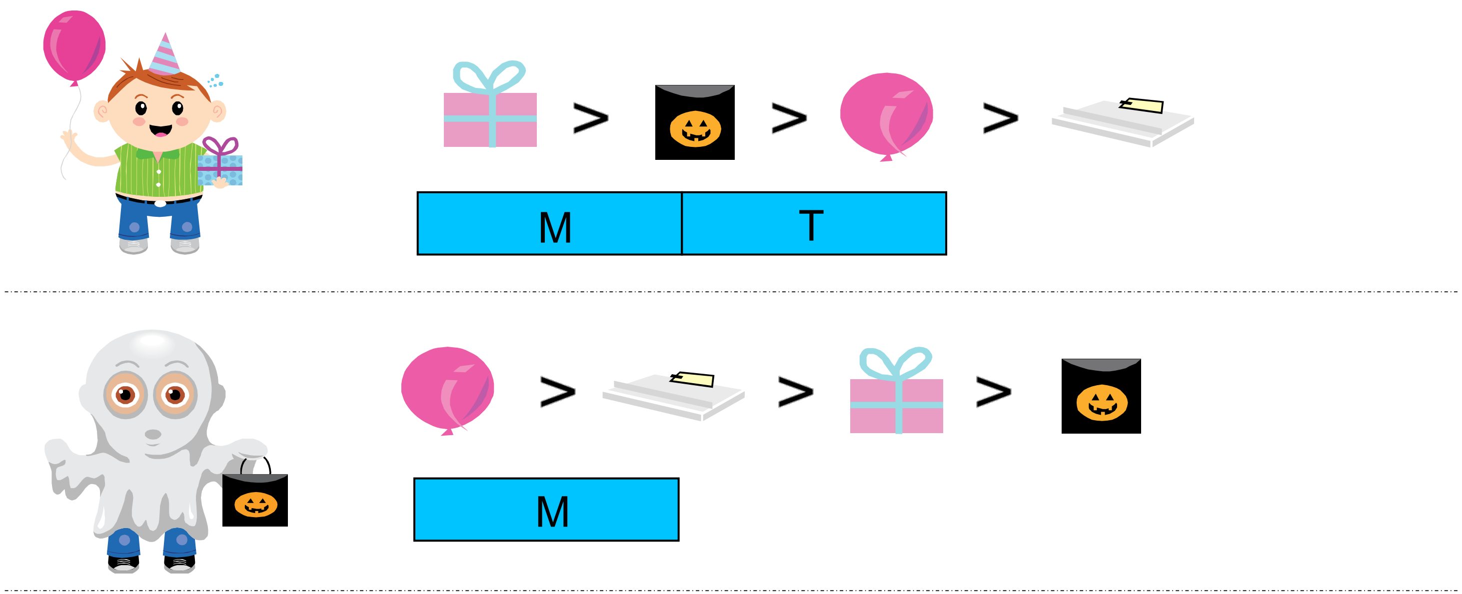
Lecture 6

Agenda

- Motivating Examples and Mechanism Design
- Mechanism Design without Money
  - House Allocation
- On-line Mechanisms Without Money
  - One-sided Markets (Dynamic House Allocation)
- Conclusion

Motivating Examples

- Dynamic allocations of goods
  - clean up task on wiki, science collaborators to perform useful task for society
- Dynamic re-allocation of goods
  - University dorm assignments, office space reassignments
- Dynamic Matchings
  - Campus recruitments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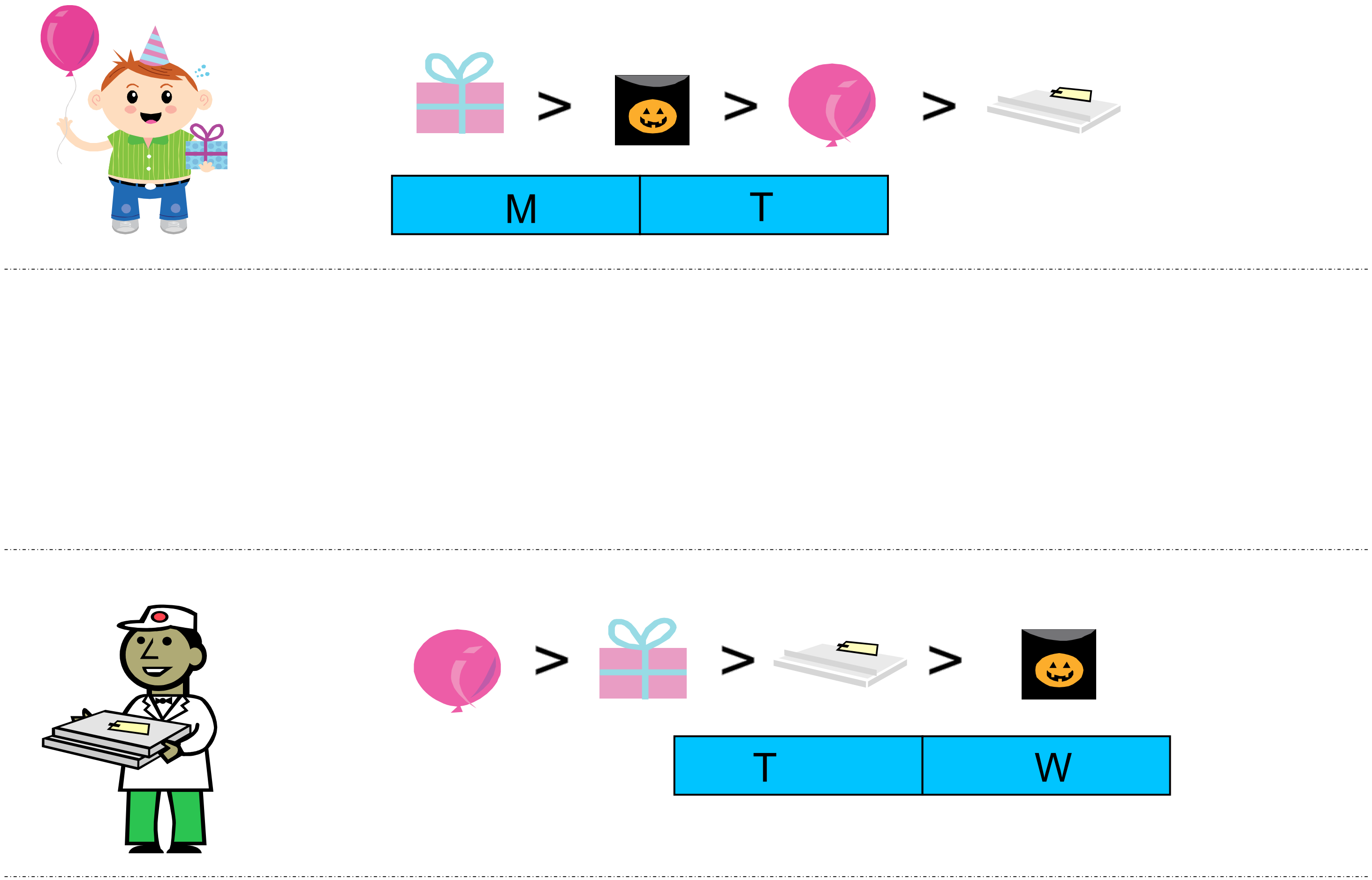
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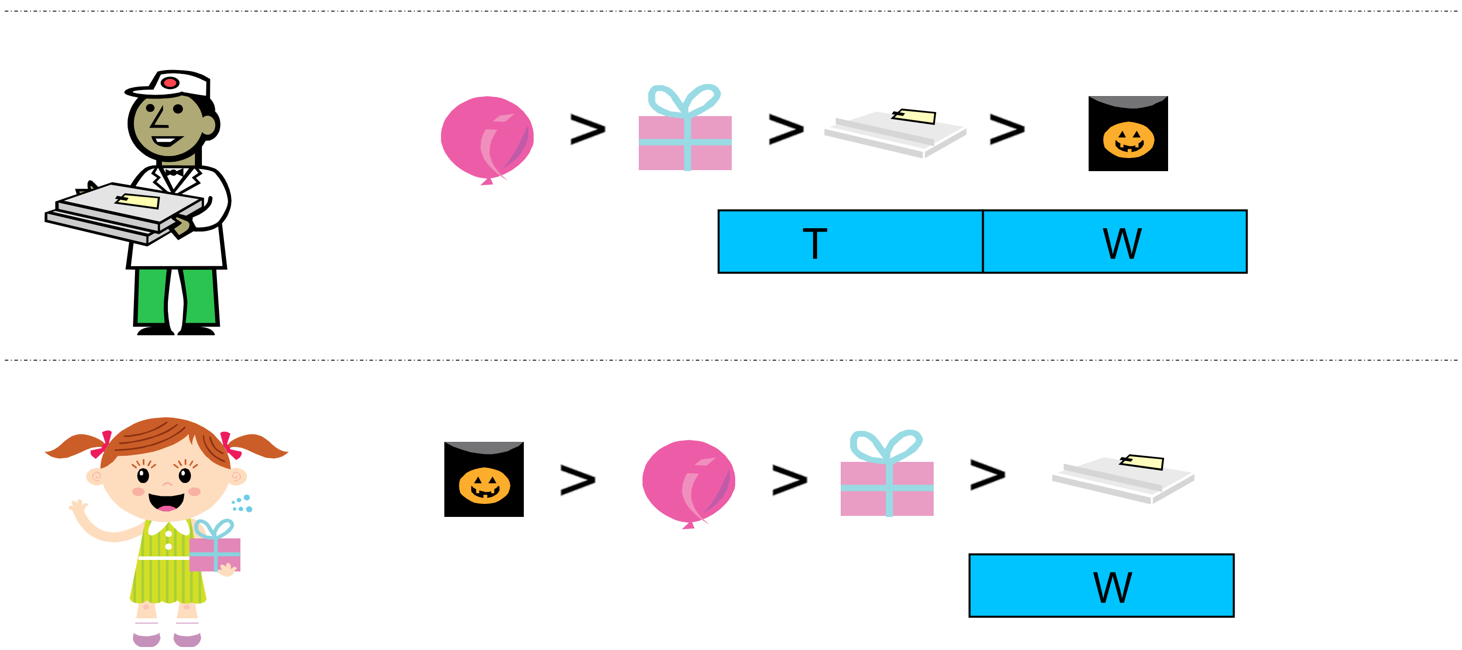
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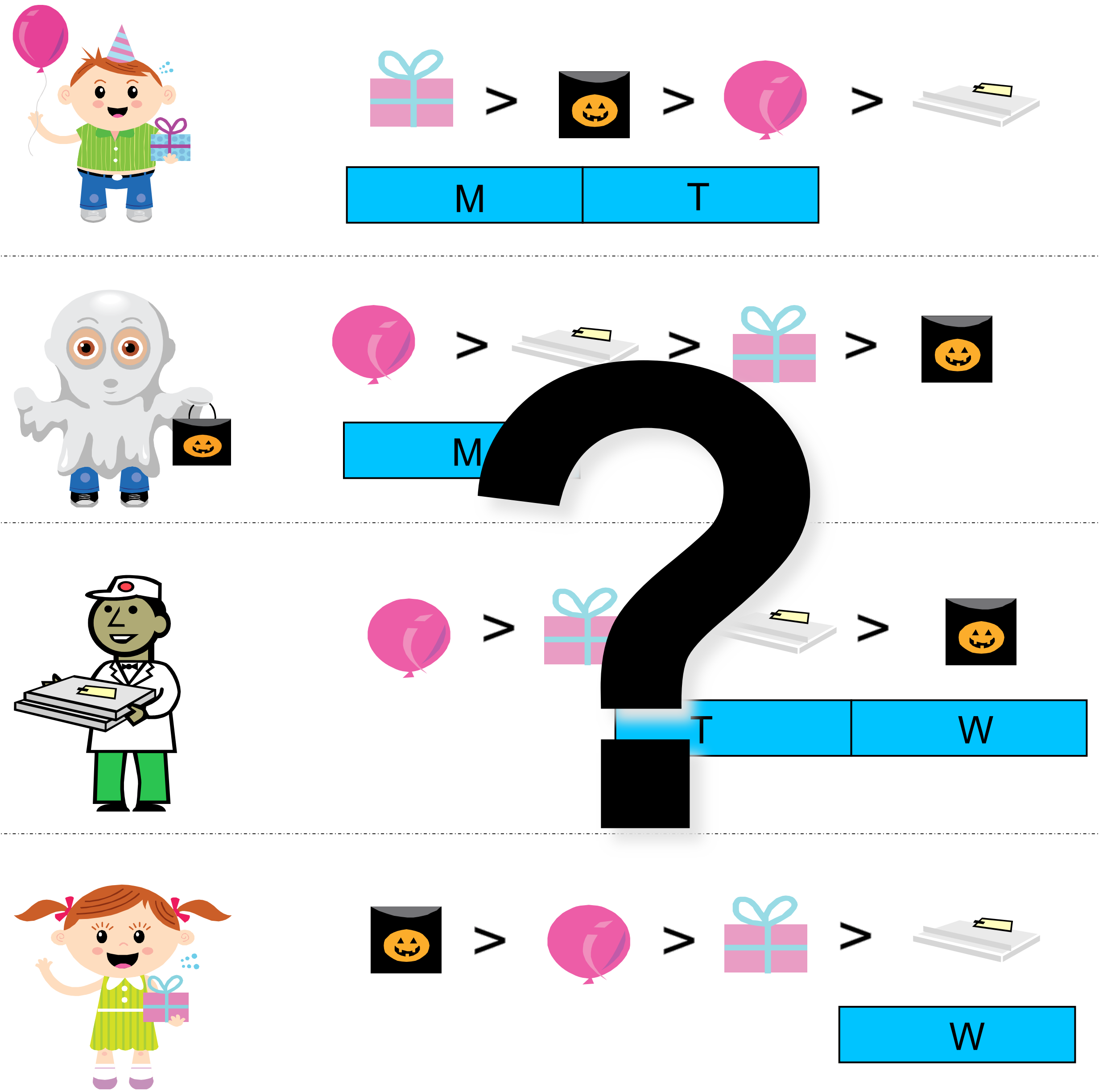
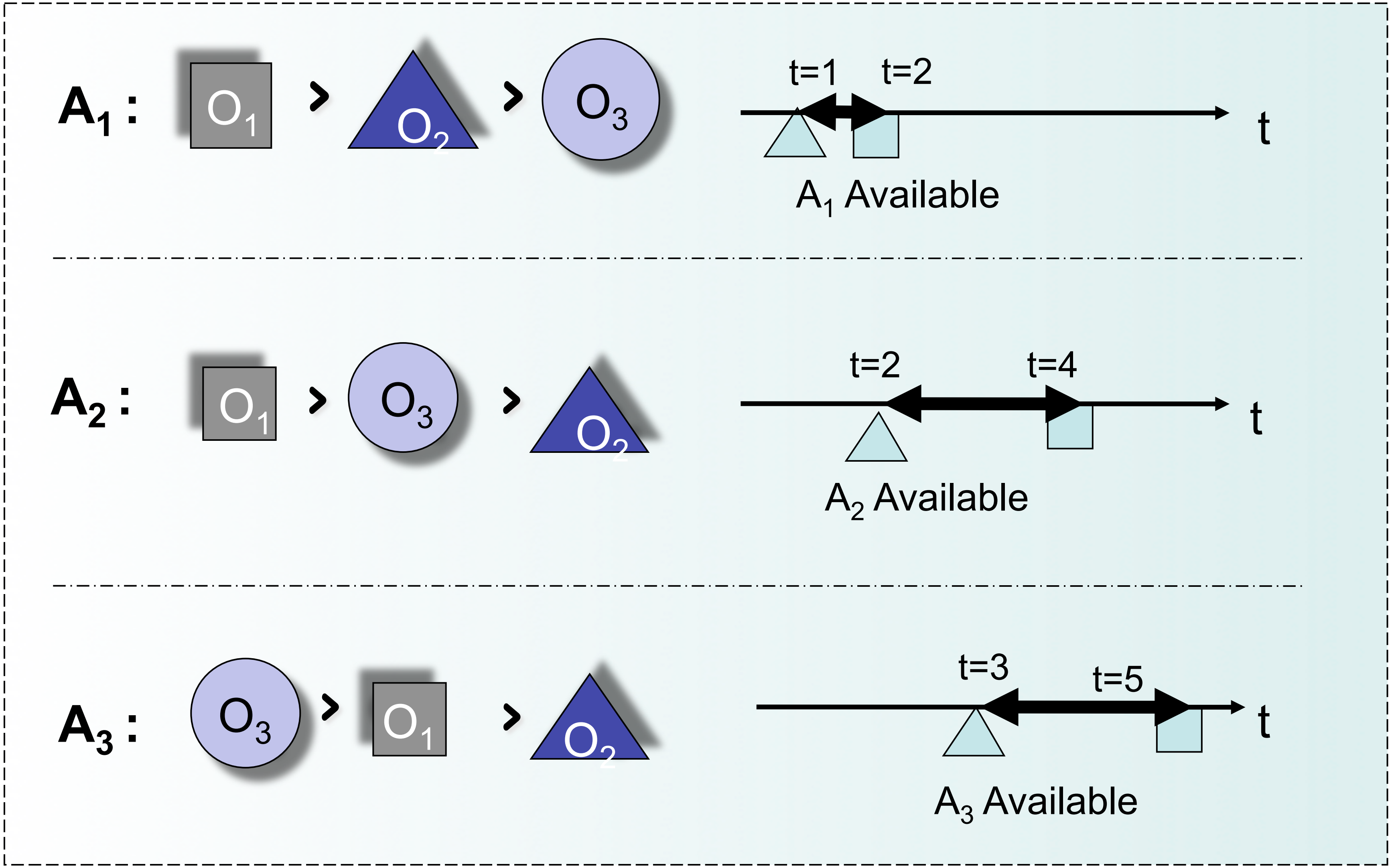
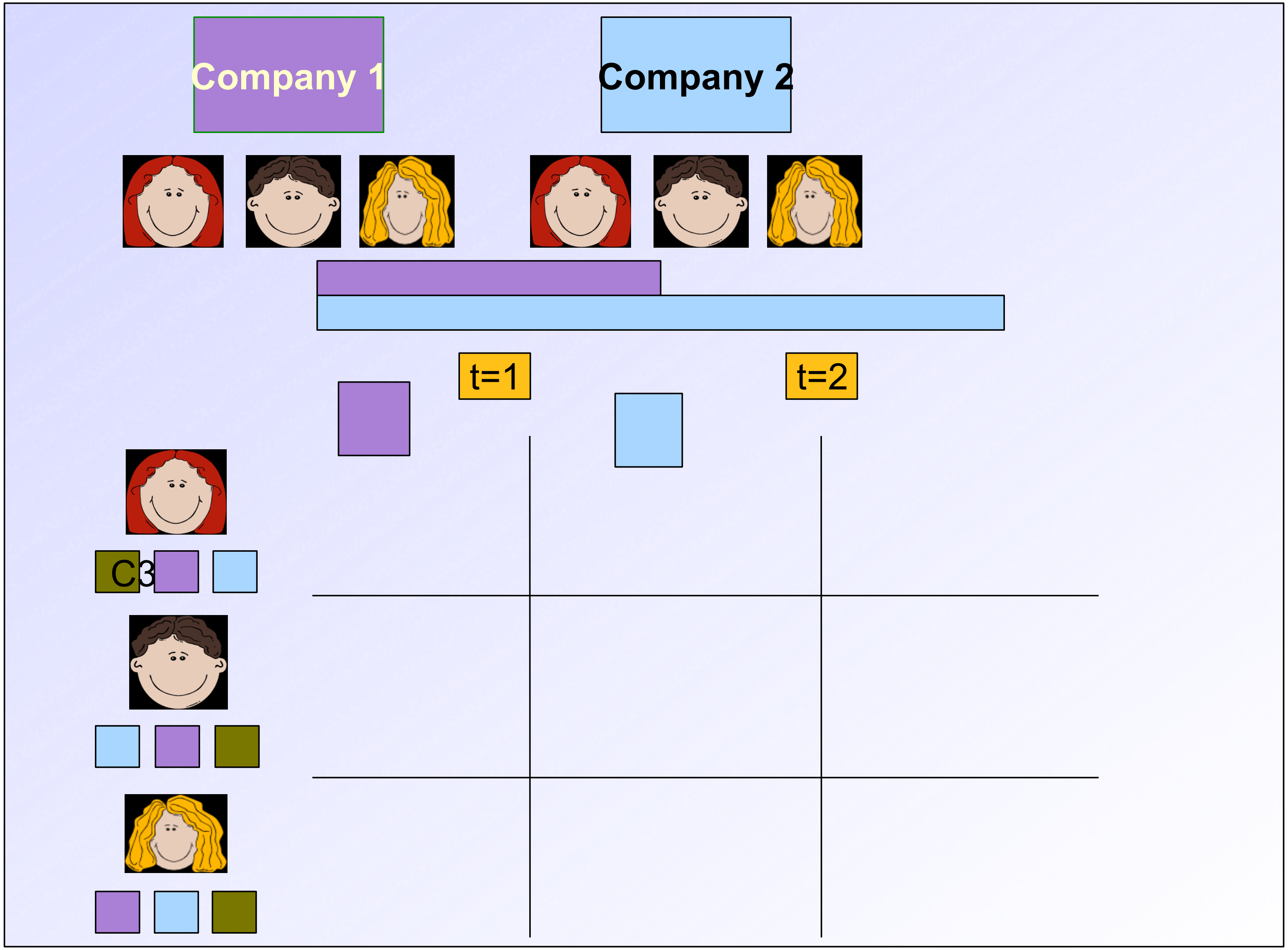


Online Mechanisms without Money

Dynamic allocation of goods





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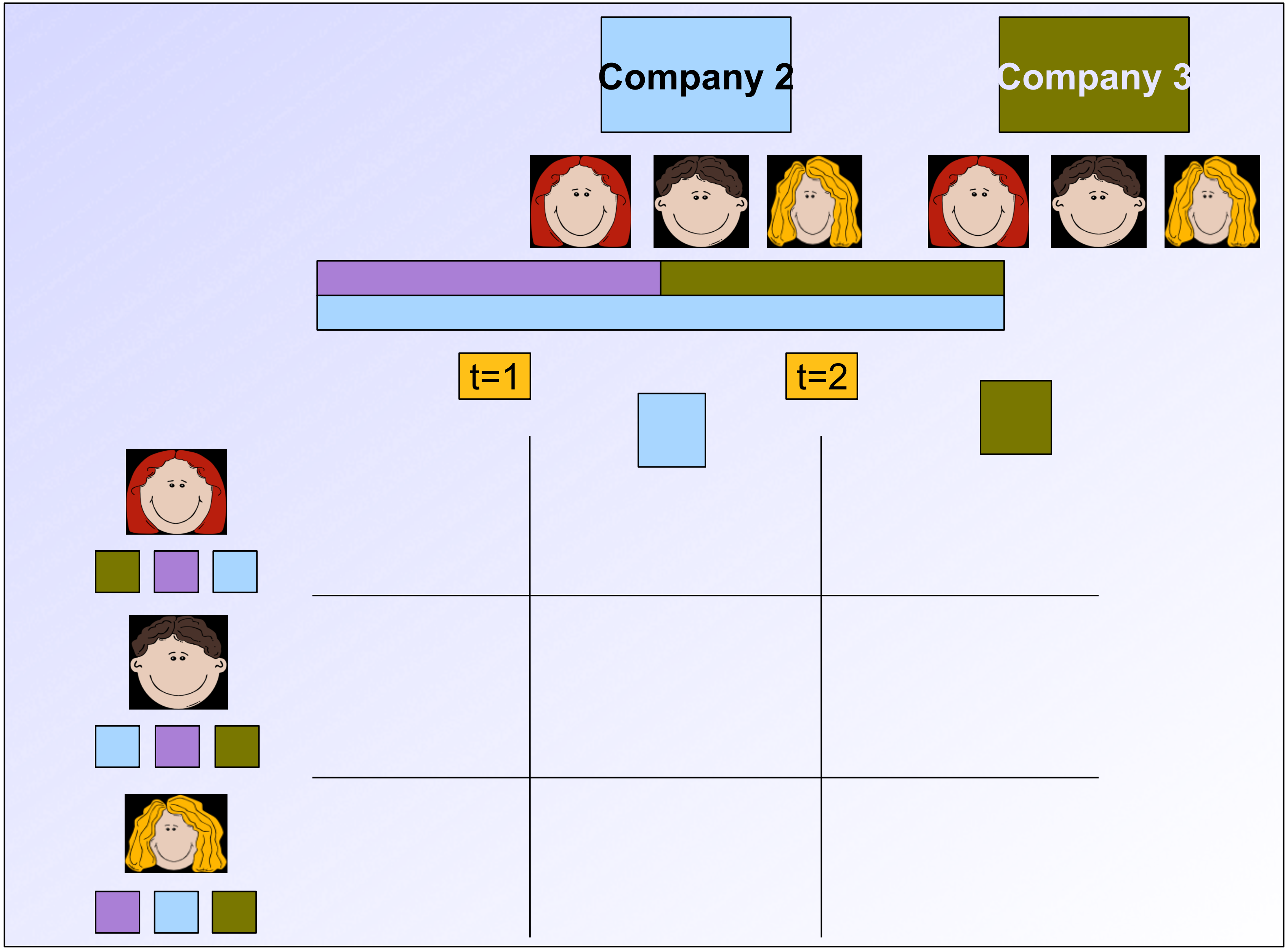
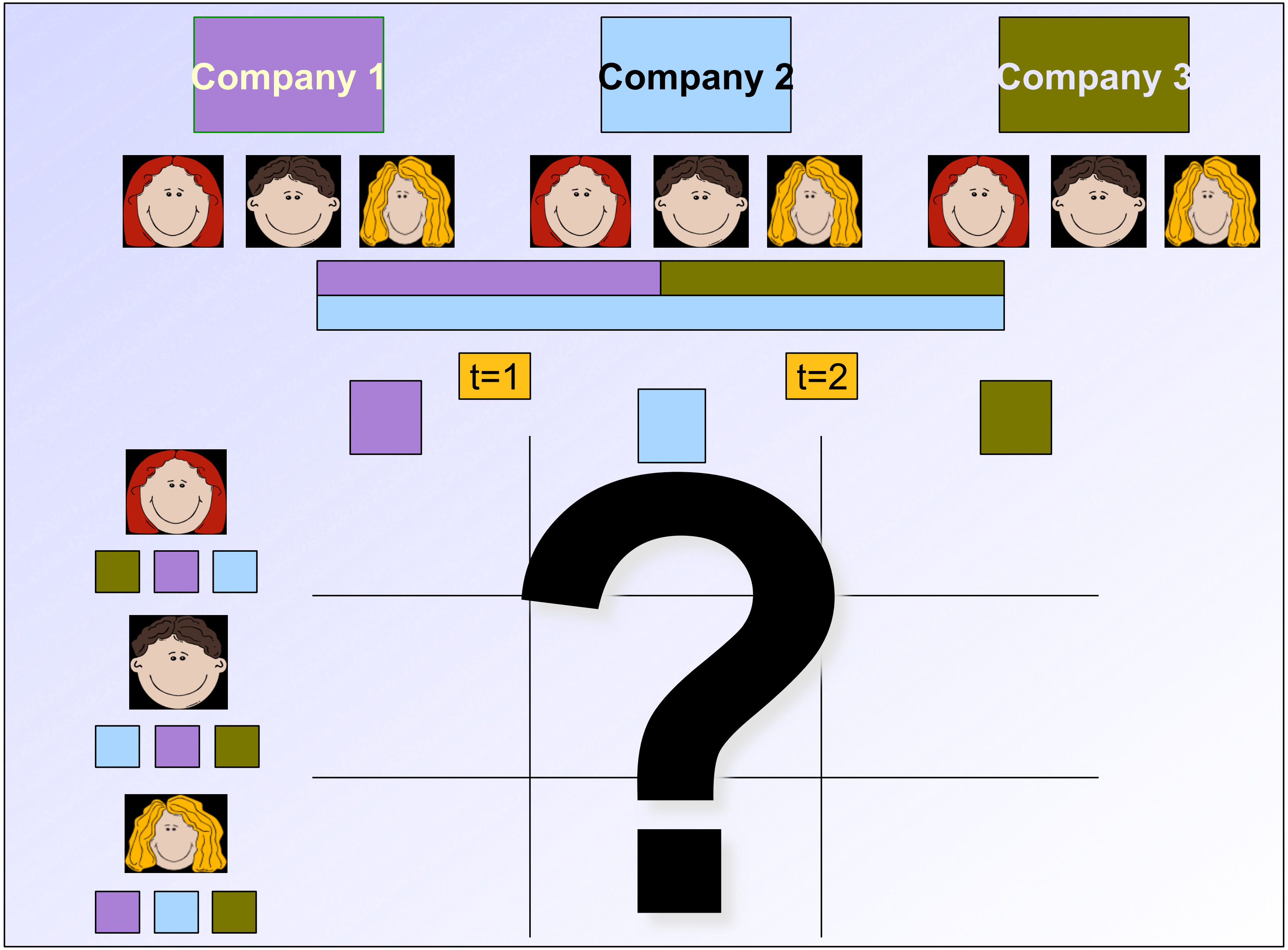


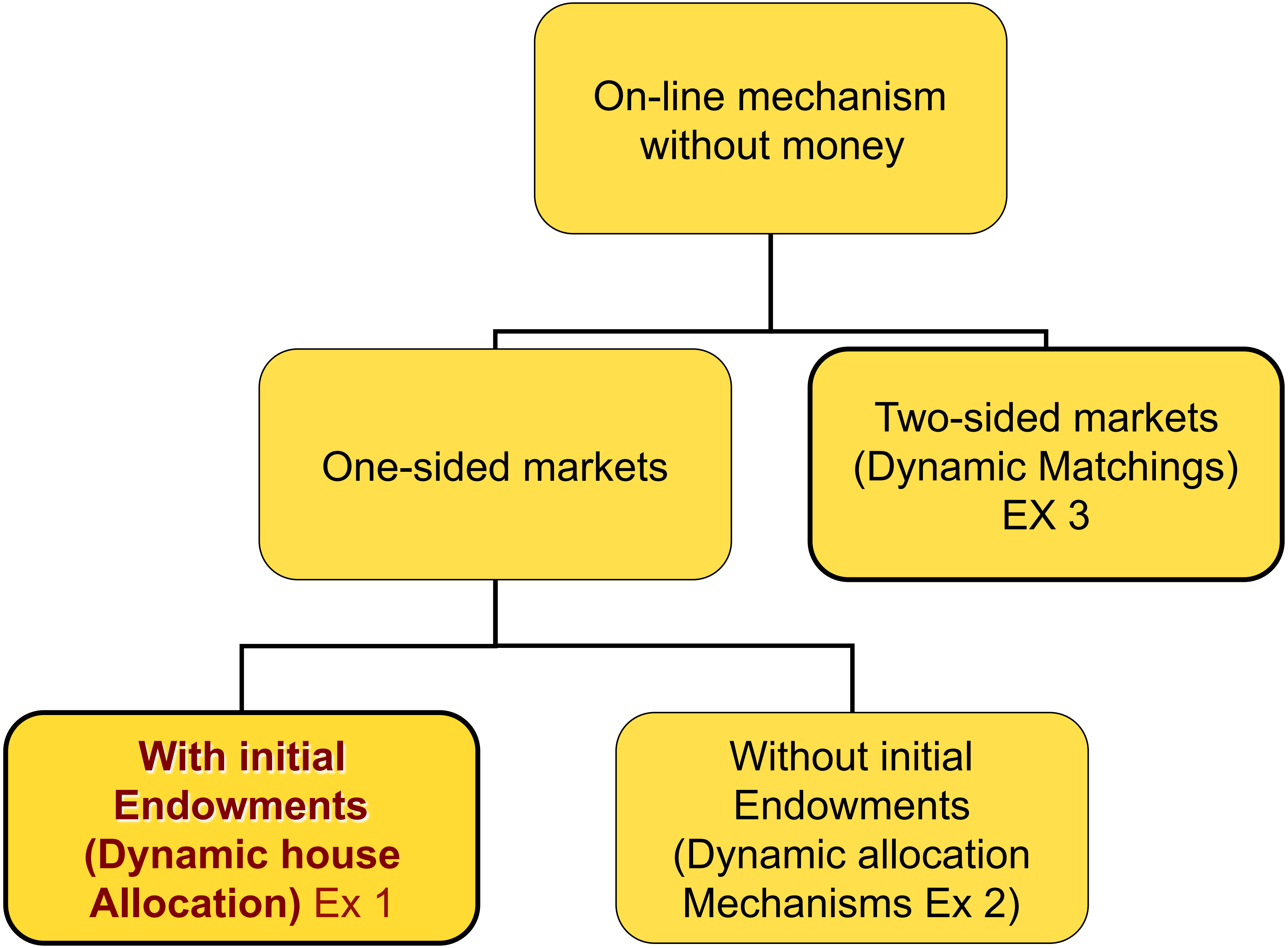
Online Mechanisms without Money



Online Mechanisms without Money

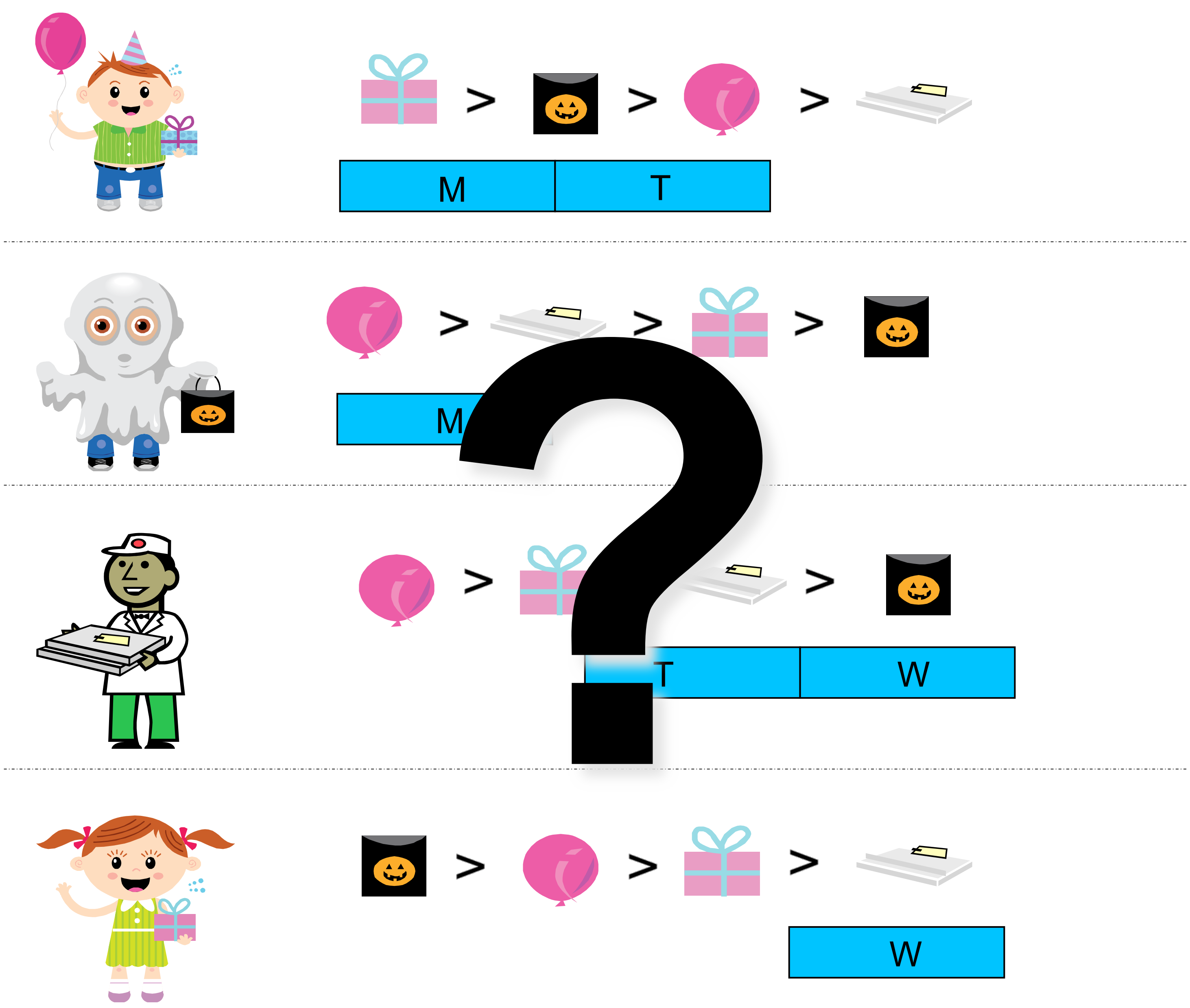
Intellectual Challenges

- Preferences of the agents are private
  - (Incomplete information)
- Manipulative agents
- On-line nature of agents
  - any decision pertaining to the agents should be taken before he/ she leaves the system
- No Money
- Repeated usage of static solutions fail
- All the above characteristics naturally fit into mechanism design problem solving framework


On-line mechanism without money
One-sided markets
Two-sided markets (Dynamic Matchings) EX 3
With initial Endowments (Dynamic house Allocation) Ex 1
Without initial Endowments (Dynamic allocation Mechanisms Ex 2)

Dynamic House Allocation[1]

[1] Sujit Gujar, James Zou, David C Parkes, “Dynamic House Allocation”, In the proceedings of 5th Multidisciplinary Workshop on Advances in Preference Handling. (MPREF-2010)



Online Mechanisms without
Money

Dynamic House Allocation

- The above problem fits in dynamic house allocation framework
- Object owned by an agent – house
- Agents have private preferences
- Agents arrive-depart dynamically
- Feasibility: No agent can receive a house that does not arrive before his departure

Desirable Properties

- ✓ Strategyproof

  Mechanism is strategyproof if at all type profiles and at all arrival-departure schedules reporting preferences truthfully is dominant strategy equilibrium

- ✓ Individual Rationality (IR)

  Agent either receives better house than current one or keeps its own house

- ✓ Pareto Efficiency

  No other way of reallocation such that every agent is equally happy and at least one is strictly better off.

- ✓ Rank efficiency = average true rank for allocated houses.

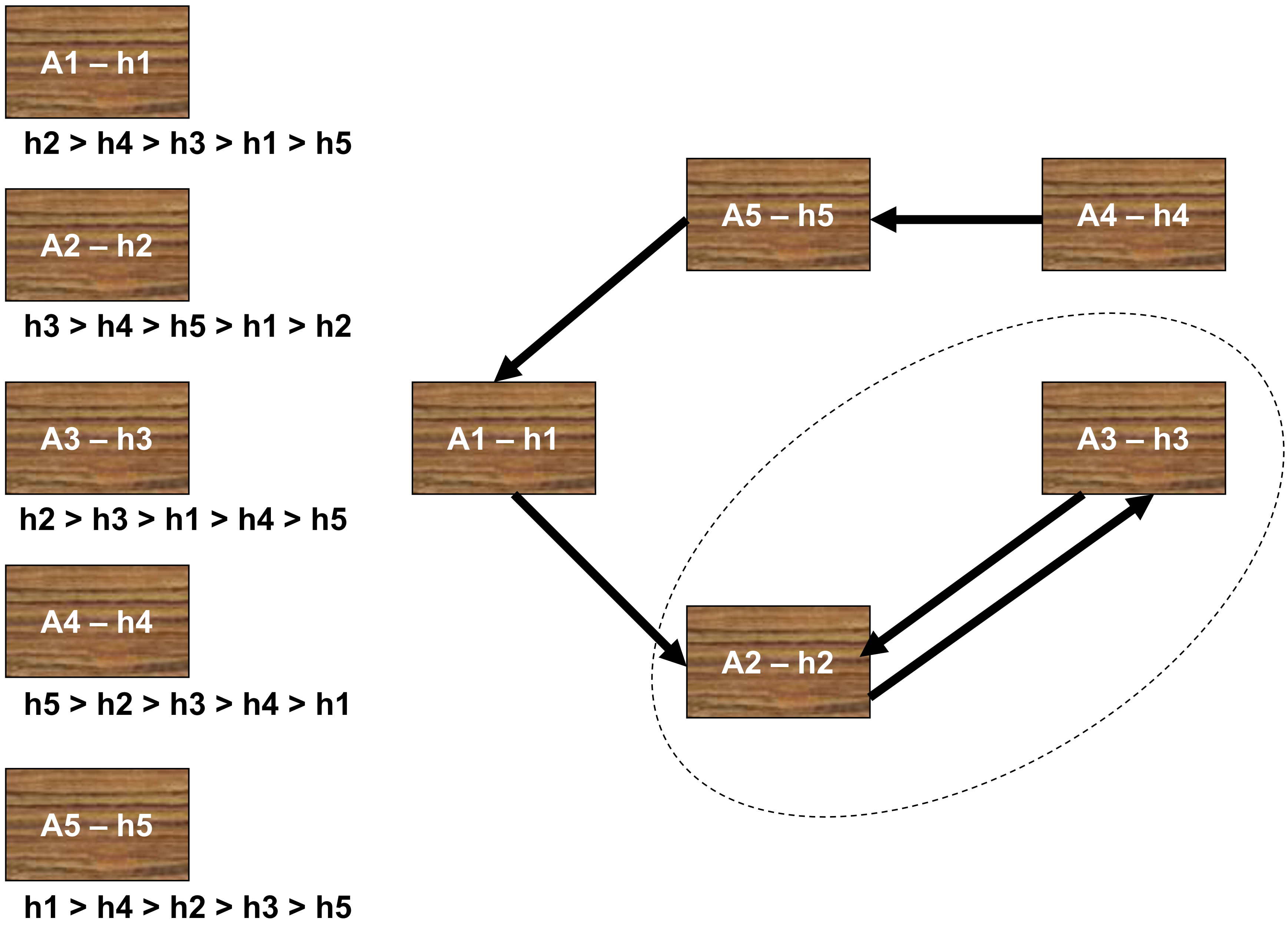
  Minimize expected rank of allocated houses

State of the Art
(Static Problem)

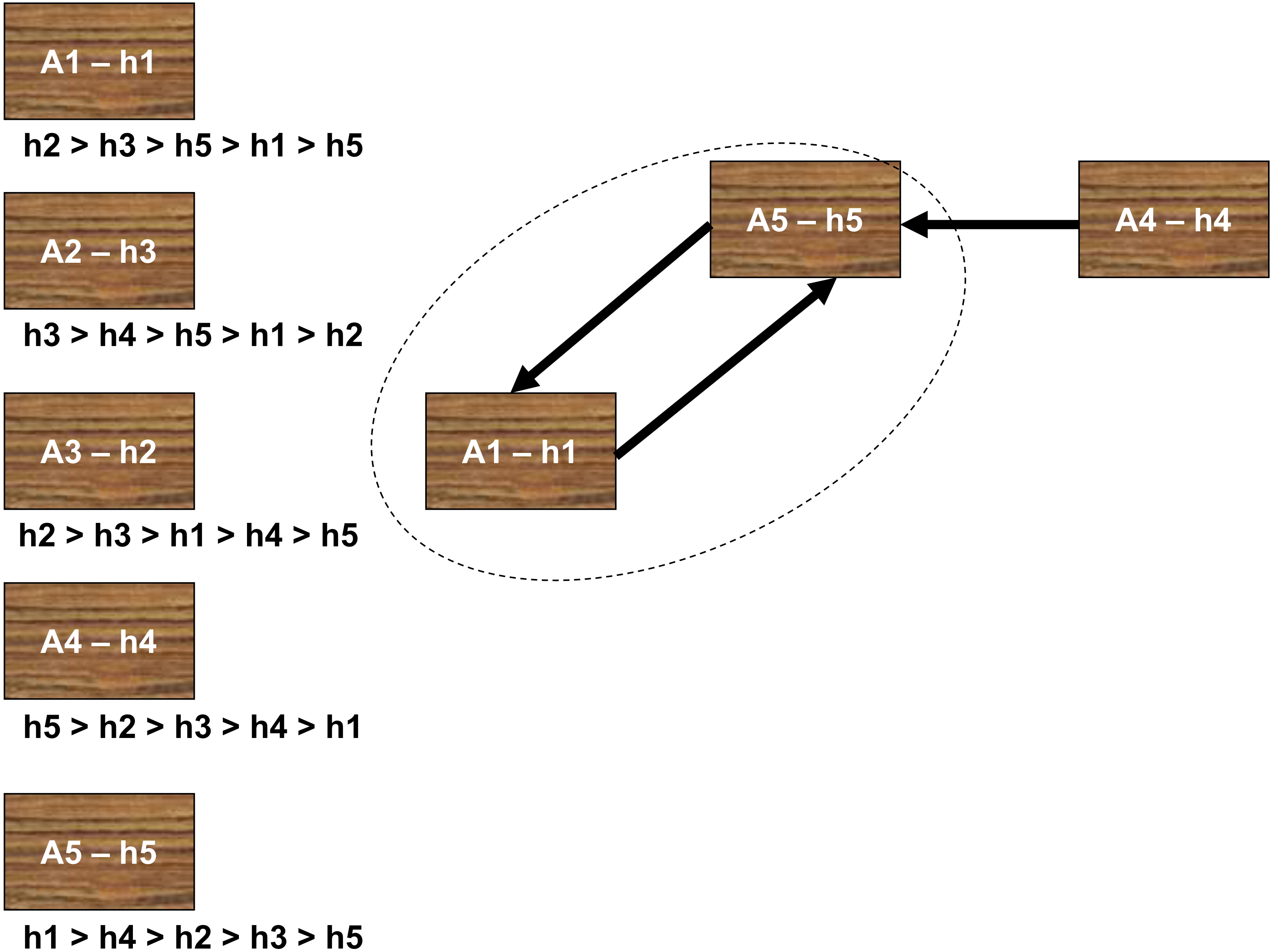
Top Trading Cycle Algorithm (TTCA)[1]

- Each agent points to his most preferred house among the available houses
- Cycles – Trades
- Remove all agents in the cycles
- Continue till nobody is left

[1] L. Shapley and H. Scarf, 'On cores and indivisibility', *J. of Math. Econ.*, 1(1), 23–37, (1974).



Online Mechanisms without Money



A1 – h1
h2 > h3 > h5 > h1 > h5
A2 – h3
h3 > h4 > h5 > h1 > h2
A3 – h2
h2 > h3 > h1 > h4 > h5
A4 – h4
h5 > h2 > h3 > h4 > h1
A5 – h5
h1 > h4 > h2 > h3 > h5
A5 – h5
A4 – h4
A1 – h1

A1 – h5

h2 > h3 > h5 > h1 > h5

A2 – h3

h3 > h4 > h5 > h1 > h2

A3 – h2

h2 > h3 > h1 > h4 > h5

A4 – h4

h5 > h2 > h3 > h4 > h1

A5 – h1

h1 > h4 > h2 > h3 > h5

A4 – h4

Final house allocation:

A1- h5, A2-h3, A3-h2, A4-h4, A5-h1

Static Case Results

TTCA

- Strategyproof[1]
- Core (No subset of agents can improve allocation)
- $\Rightarrow$ Pareto optimality, IR

Ma[2]

– TTCA is unique mechanism that is strategyproof, individually rational and in the core

[1]A. E. Roth, ‘Incentive compatibility in a market with indivisible goods’, *Economics Letters*, 9(2), 127–132, (1982).

[2]J. Ma, ‘Strategy-proofness and the strict core in a market with indivisibilities’, *Int. J. of Game Theory*, 23(1), 75–83, (1994).

Coming to On-line Settings

- In on-line settings our main result is:

A simple strategyproof mechanism cannot use agent's reported preference to decide at which time to let it participate in TTCA

Recall: in Static Settings, TTCA achieves efficiency + Individual Rationality

We show:

In on-line setting no mechanism can achieve efficiency as well as individual rationality

Compromise IR?♪

Look for maximal efficiency in mechanisms that are IR?

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We show:

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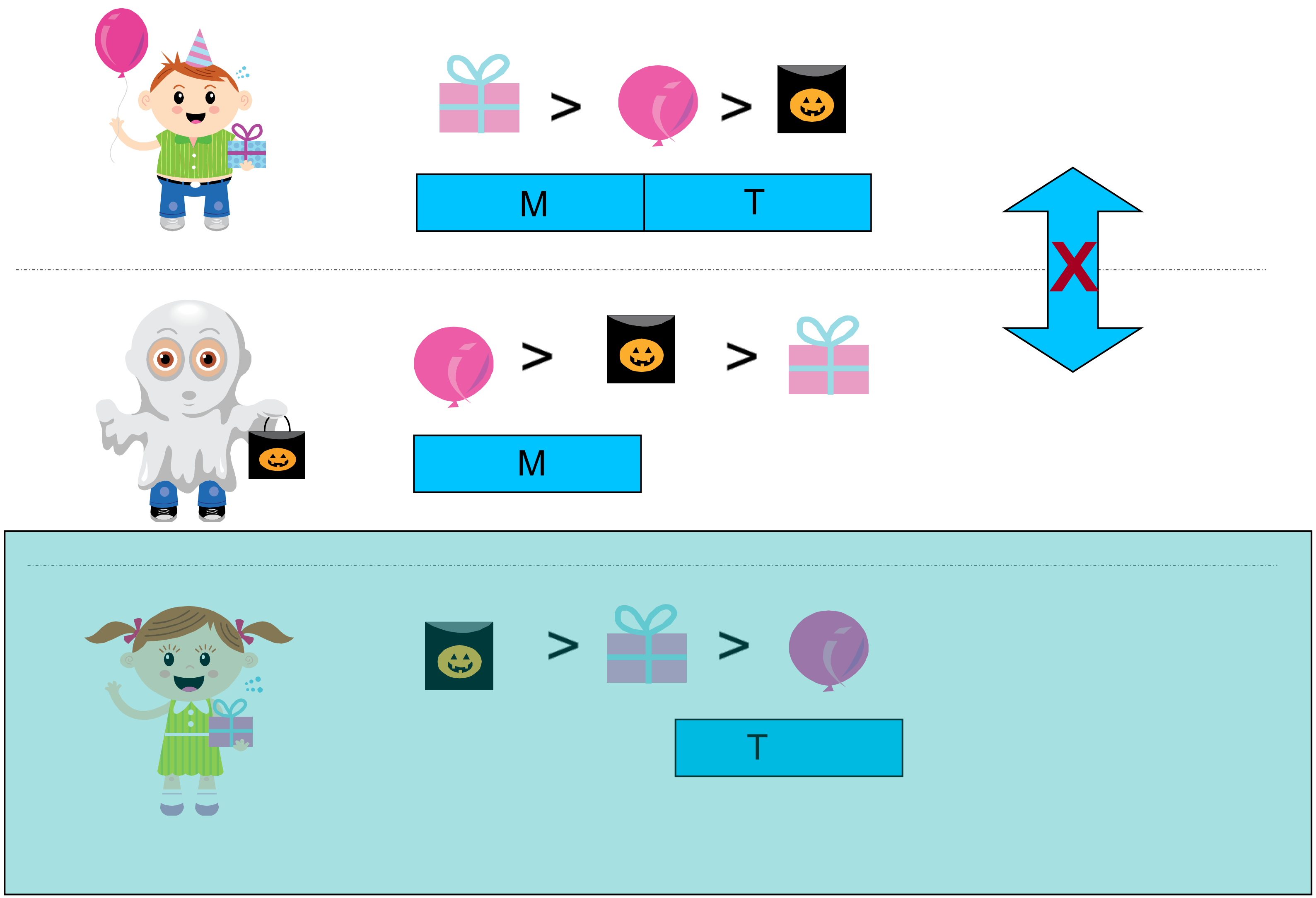
Compromise IR?♪

Look for maximal efficiency in mechanisms that are IR?

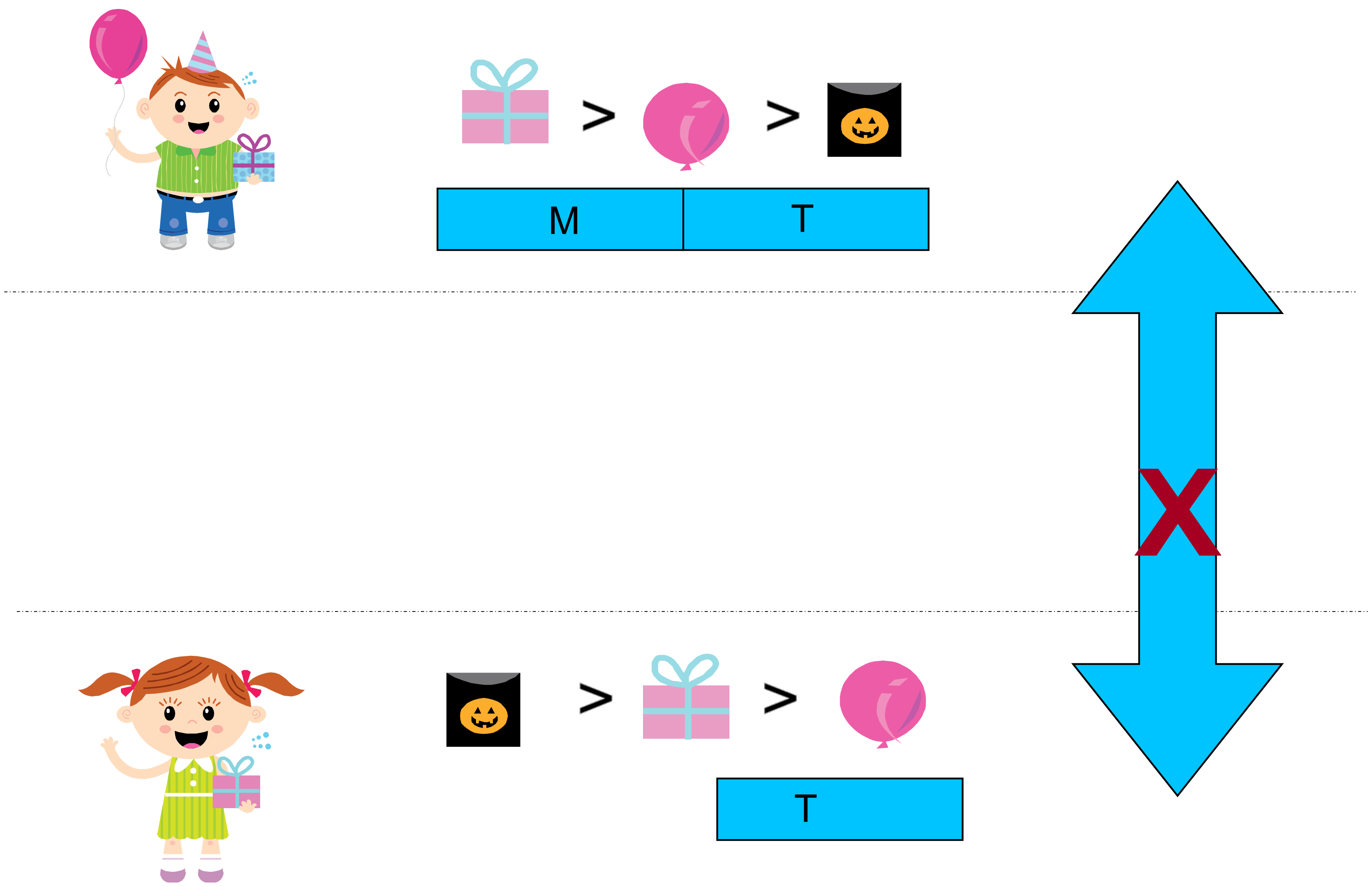
Naïve Mechanism: On-line TTCA

- In each period: The agents those are present, trade using TTCA
- Fails to be strategyproof

  (For more than three agents and two or more period)



Online Mechanisms without Money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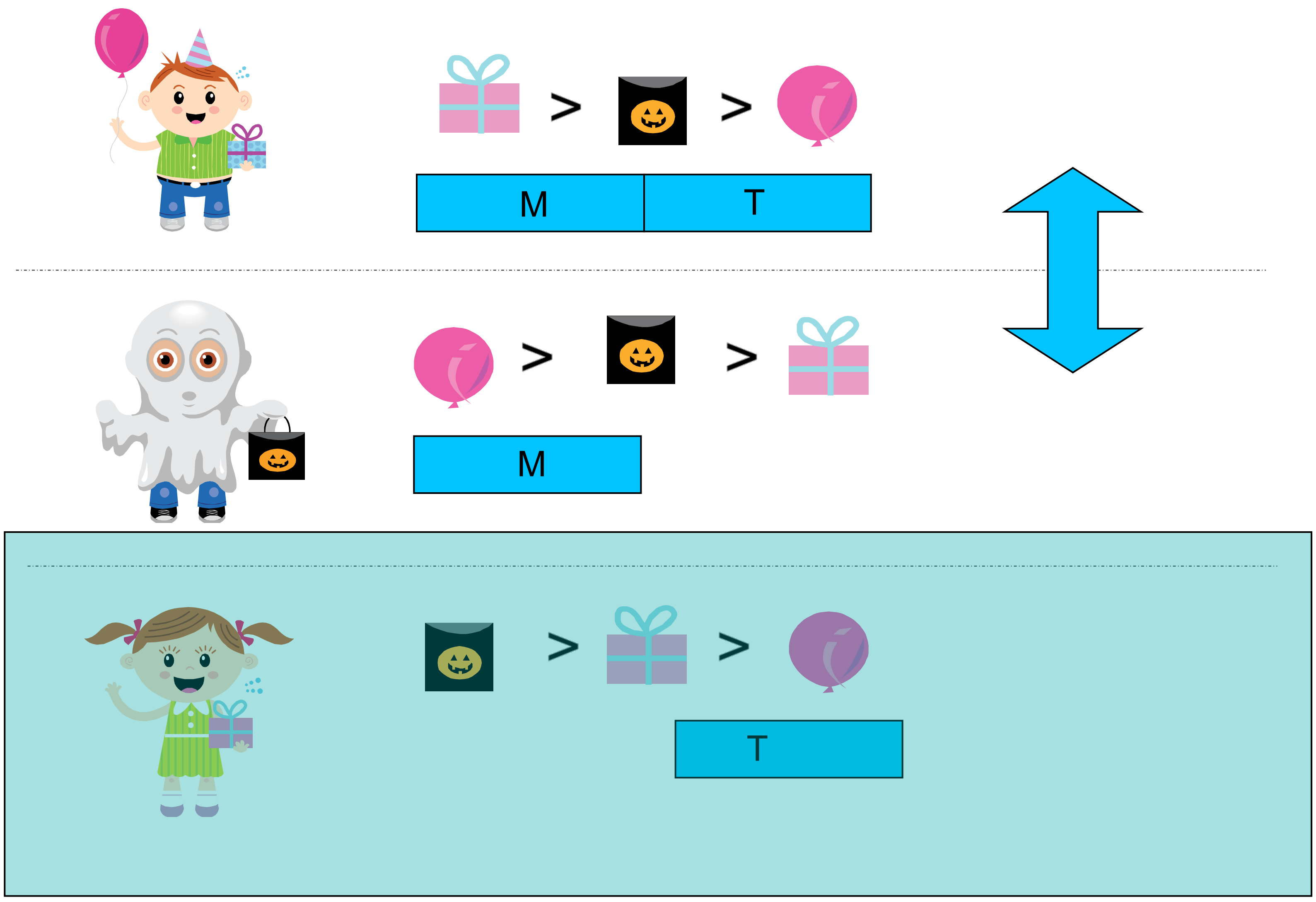
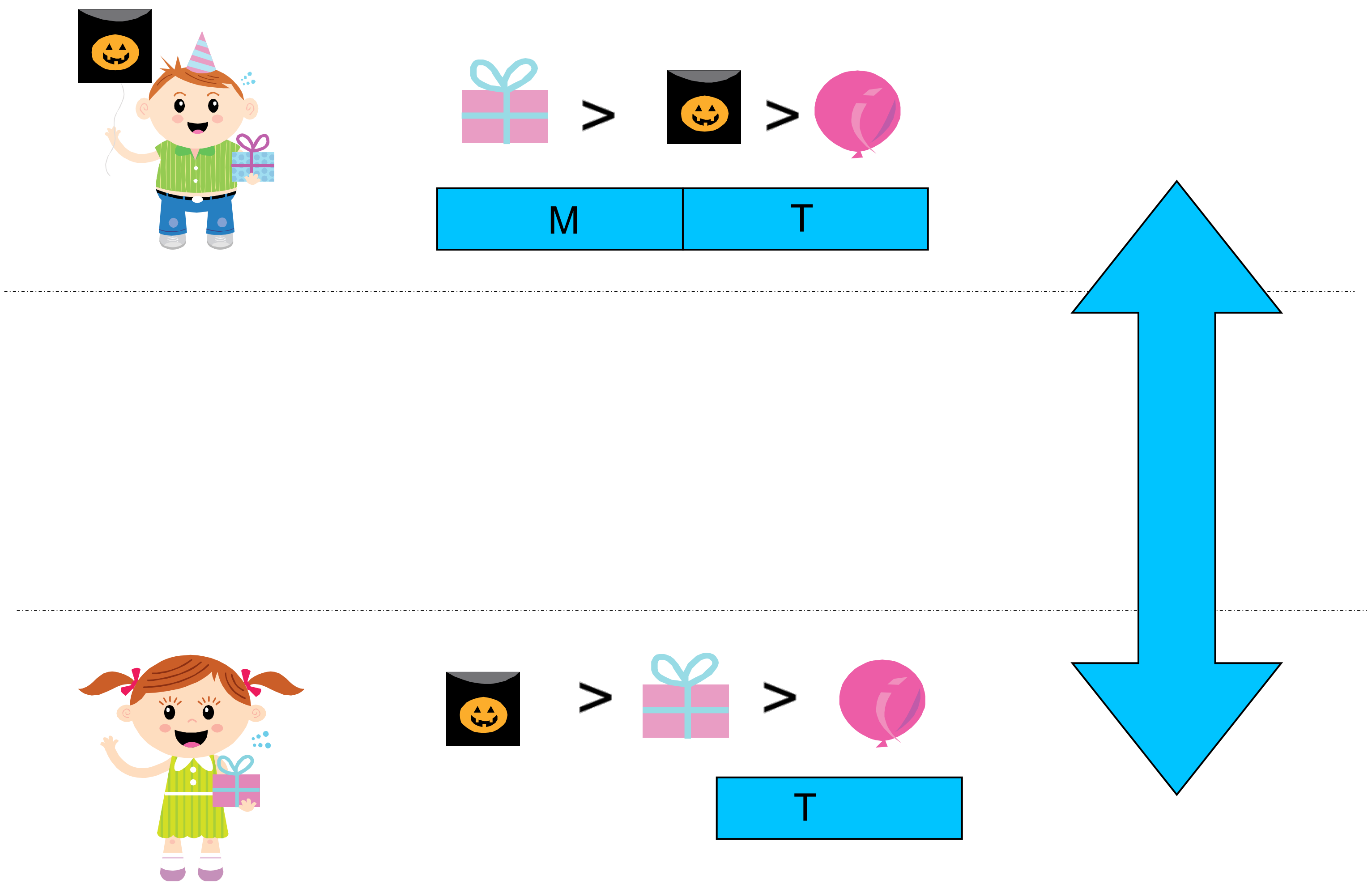


Online Mechanisms without Money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Preliminaries:

- Instead of houses: Classes of houses

(To avoid corner cases in the proofs)

- Simple mechanism:

Suppose for a given mechanism, for any agent, **A** there exists, a perfect match set, occurrence of which guarantees the agent his most preferred house and the agents not similar to **A** do not trade with the perfect set agents

Characterization Result

If an online house allocation mechanism is SP and simple and agent **A** participates in TTCA in period $t(>_a)$ for some report $>_a$, then fixing scenario w(0,t) in regard to all agents except **A**, agent **A** continues to participate in period $t(>_a)$ for all reports $>_a'$.

(Informal)
In any strategyproof, simple on-line house allocation mechanism, if agent A participates in TTCA trade in period t by reporting some preference, then he continues to participate in TTCA trade in the period t for any other report

A simple strategyproof mechanism cannot use agent's reported preference to decide at which time to let it participate in TTCA

Partition Mechanisms

Partition agents into sets such that

all the agents in a set are simultaneously present in some period

The partition is independent of agent's preferences. It may depend upon a_i-d_i

Execute TTCA among the agents in each partition

Easy: Partition Mechanisms are strategyproof and IR

DO-TTCA, T-TTCA

DO-TTCA (**D**eparting agents **O**n-line **TTCA**)

Partition agents based on departure time (All agents with $d_i = t$ trade in period t)

X a large number of agents that are present but may depart at distinct times

T-TTCA (**T**hreshold **TTCA**)

If more than THRSHLD number of agents that have not participated in TTCA,are present, execute TTCA with these agents. Otherwise, if there are any departing agents, execute TTCA with these agents

SO-TTCA

Stochastic **O**ptimization **TTCA**

Adopts a sample-based stochastic optimization[1] method for partitioning the agents

- In each period, generate samples of possible future arrival-departure
- For each sample, find an offline partition using greedy heuristic (the bigger the each set in the partition, the better)
- For each agent, find out how often he is getting scheduled in the current period. Include the agent in current period if he is getting scheduled in current period more often than the fraction of agents yet to arrive

[1]P. Van Hentenryck and R. Bent, *Online Stochastic Combinatorial Optimization*, MIT Press, 2006.

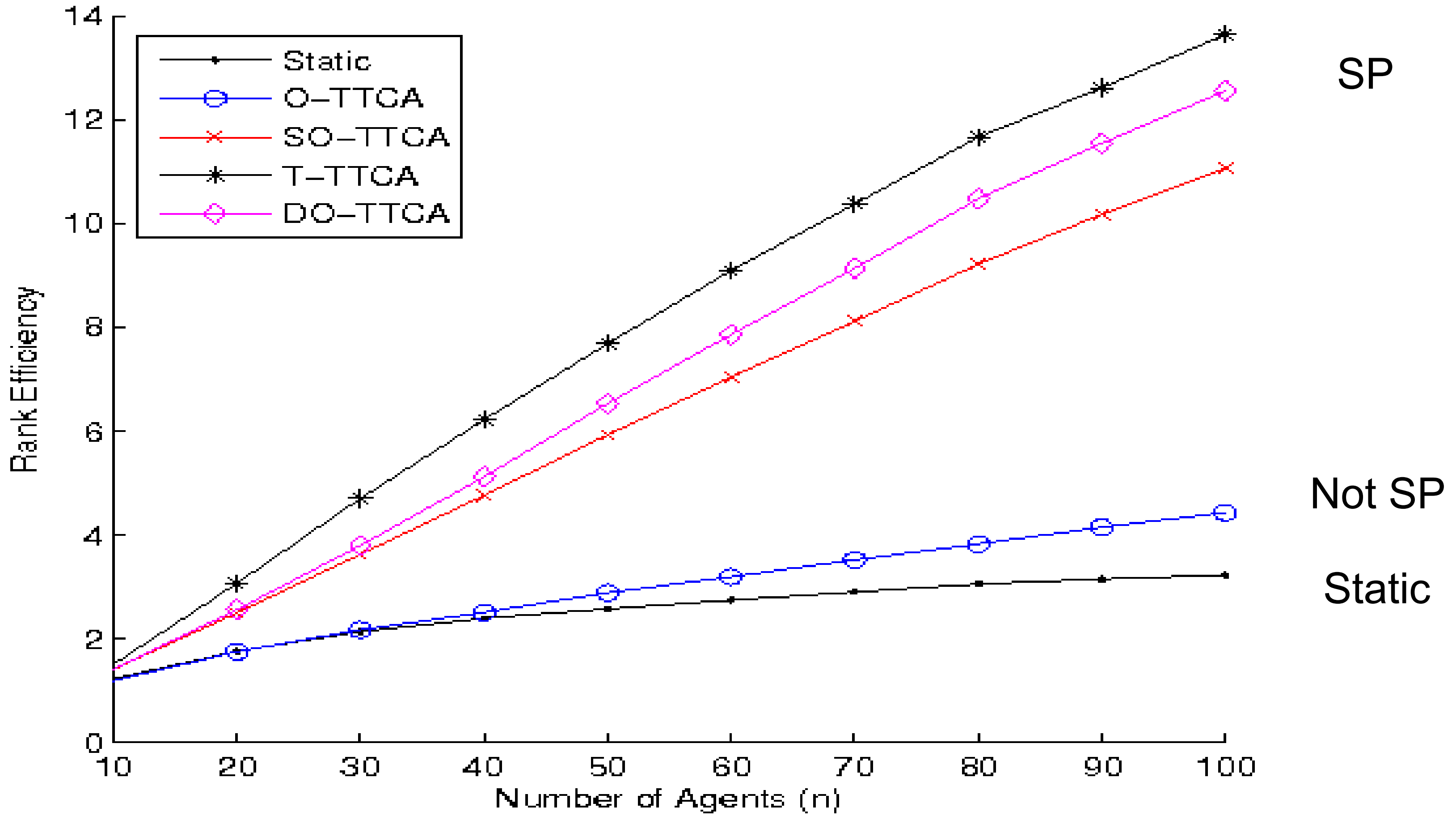
Partition Mechanism Simple

- DO-TTCA, T-TTCA and SO-TTCA induce partition of the agents which is independent of the agents preference reports and hence partition mechanisms

- DO-TTCA T-TTCA and SO-TTCA are simple mechanisms

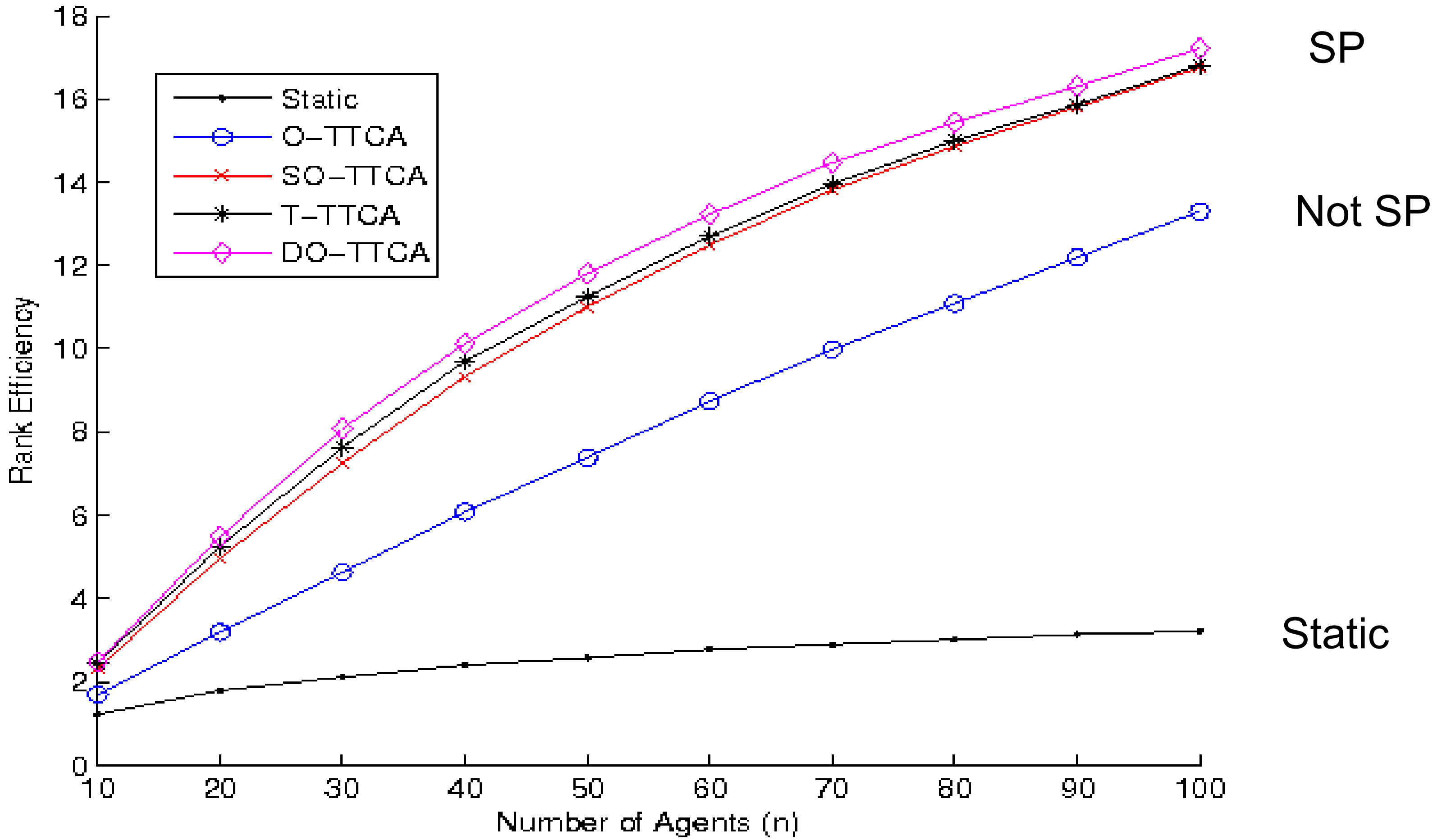
Simulation Results

E1:
Poisson Arrival rate L = n/8
waiting period exponential distribution u = 0.01 L



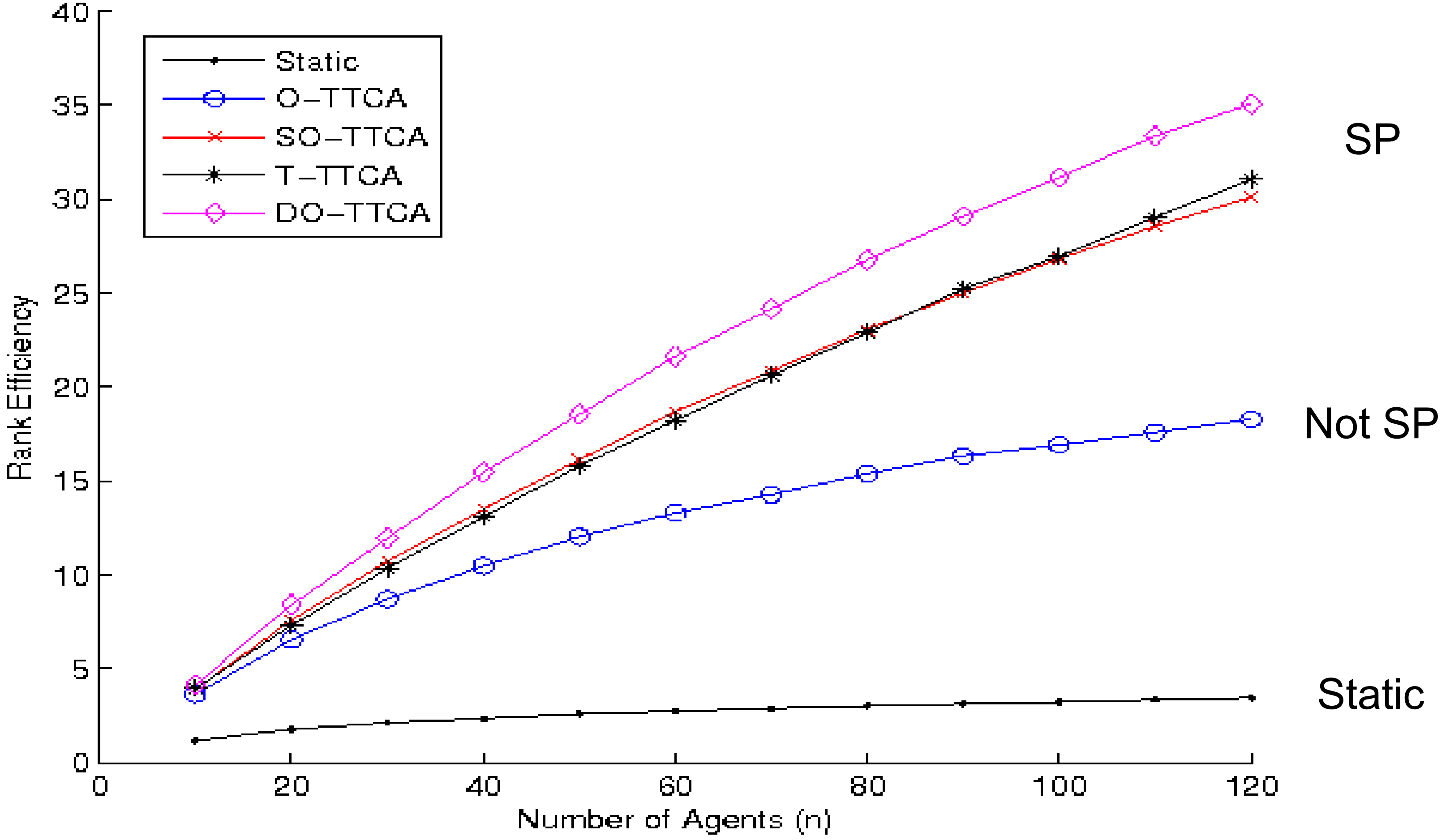
Simulation Results

E2:
Poisson Arrival rate L = n/8
waiting period exponential distribution u = 0.1 L



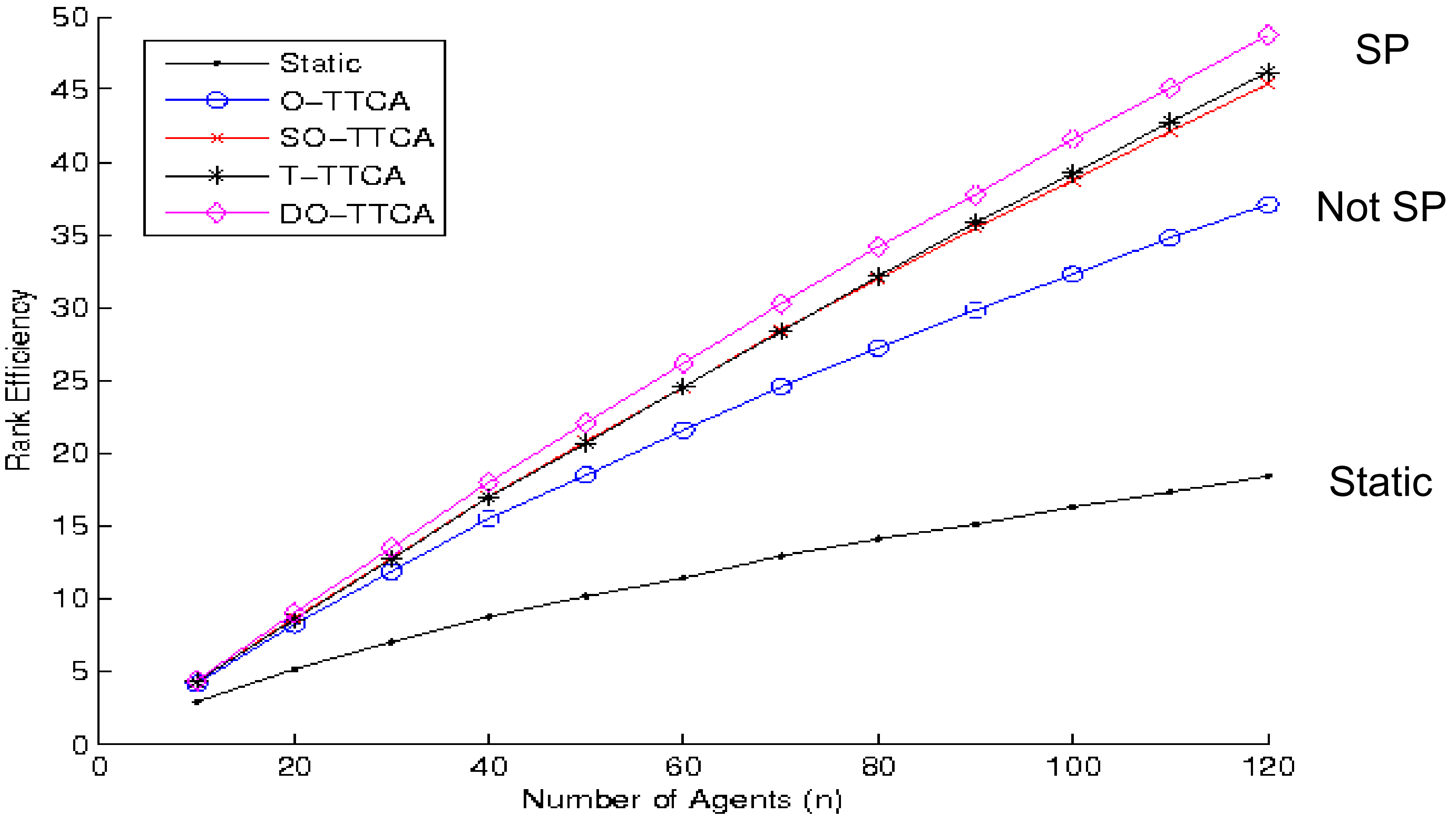
Simulation Results

E3:
Uniform Arrival in period {1,2,…,T} (T=30)
Departure period uniform between {ai, ai+T/8} truncate to T if di > T



Simulation Results

E4:
Preferences are skewed (Some houses are more demanded than other)
Arrival Departure as in **E1**



Story so far…

We have seen,

- Trade-off between efficiency and individual rationality in the on-line version
- For strategyproofness: agents cannot trade with different subsets of agents by changing their preference report
- Partition Mechanisms

  SO-TTCA, T-TTCA out perform DO-TTCA

- Immediate question: What if agents can mis-report arrival-departure schedules?

Thank You!

Preliminaries:

- Instead of houses: Classes of houses

  (To avoid corner cases in the proofs)

- Sample Path: $w(>,\rho)$ An instance of an dynamic house allocation problem
- $w(t)$: Restriction of $w(<,\rho)$ to the agents arrived before period t
- $w(t,t')$: instance of reports of agents arrived in period $[t,t']$
- Λ: Perfect Match Set $\{A_i, h_i, >_i, t > a_i, d_i\}$

Simple Mechanism

A simple mechanism:
Given a scenario w(0, t) and agent **A** available at t, there exists a perfect match set agents $\Lambda = \{A_i, h_i, >_A, t > a, d\}$ such that

(a) If a continuation w(t+) contains Λ, then **A** receives his most preferred house under the scenario w(0, t)+w(t+), and

(b) if **B** is present in w(0,t) and not similar to **A** then **B** does not trade with any agent in Λ

Partition Mechanism Simple

- DO-TTCA: For agent **A** (**A**,h_0, $>_{\mathbf{A}}$,a, d), the perfect match set:

  A set of n' identical agents (A^, h^, $>_{A\wedge}$,d,d)

  h^ : **A** most preferred house and A^'s second most

  h_0 : A^'s most preferred house and

  n' : the number of agents similar to A^

- T-TTCA and SO-TTCA are simple mechanisms