Multi-Armed Bandit Mechanisms

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Agenda

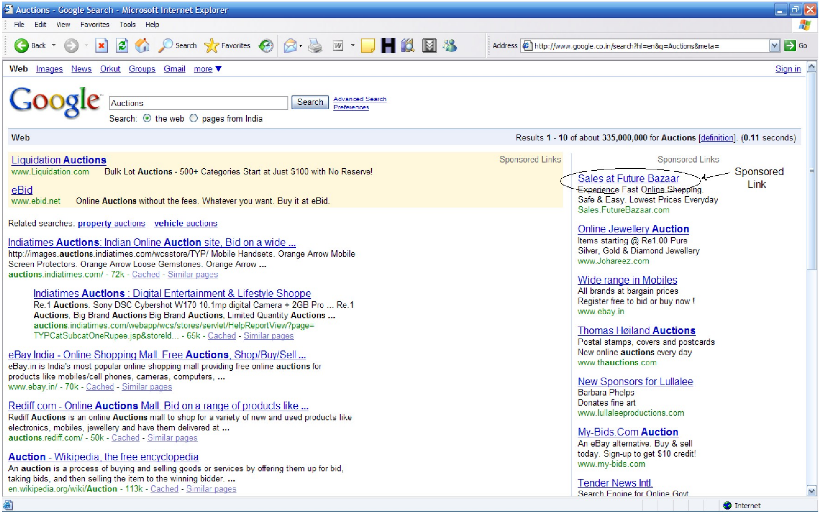
- Introduction
  - Motivating Examples
  - Research Challenges
  - Multi-Armed Bandit (MAB) Mechanisms
- (i) MAB Mechanisms for Crowd-Sourcing
  - Assured Armed Bandit Framework
  - Constraint Confidence Bound Algorithm
- (II)MAB Mechanisms for Sponsored Search Auctions
  - Single Slot Auctions vs Multi-Slot Auctions
  - Summary of our results
- Summary

Motivating Examples 1: Crowdsourcing

A company provides financial advice to clients by hiring some consultants from a pool of workers available.

- The goal of the company is to provide an advice meeting certain accuracy threshold.
- Getting opinion from many consultants increases accuracy but entails cost.

Motivating Example 2: Sponsored Search Auctions



Key Observations

- How to choose the workers for the task to ensure quality?
- The cost incurred by a worker is private information
- The qualities of the workers are unknown
- Need for an selection mechanism that elicits true valuations from the workers (agents) and learns their qualities simultaneously

- How do we assign the slots to advertisers to maximize efficiency?
- A valuation of a click on ad is private information of the advertisers
- The click probabilities are unknown
- Need for an assignment mechanism that elicits true valuations from the advertiser (agents) and learns the click probabilities simultaneously

Focus

certain parameters are to be learnt while eliciting true information

Quick Introduction to Mechanism Design

- Game Theory: Analysis of strategic interaction among players
- Mechanism Design: Reverse engineering of game theory

Mechanism Design

Mechanism Design is the art of designing rules of a game to achieve a specific outcome in presence of *multiple self-interested agents*, each with *private information* about their preferences

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Multi-Armed Bandit (MAB) Mechanism

- Regret: Measure of a performance of a learning algorithm

$$\boxed{\text{Regret} = \text{Expected Loss in Rewards}}$$

  - Extra costs incurred in Example 1, Social Welfare in Example 2
- One would like to learn qualities/CTRs with minimal regret
- Agents may manipulate the learning algorithm for the underlying MAB problem
- This calls for combining techniques from Mechanism Design Theory and Machine Learning
- Goal is to design an auction mechanism that is truthful as well as learns certain parameters
- Such mechanisms are called as MAB Mechanisms

Assured Armed Bandits Mechanisms for Crowd Sourcing

The State of the Art

- Ittai Abraham, Omar Alonso, Vasilis Kandylas, and Aleksandrs Slivkins. Adaptive crowdsourcing algorithms for the bandit survey problem. Computing Research Repository, abs/1302.3268, 2013.

  Selecting an optimal crowd consisting of homogeneous crowd workers

- Long Tran-Thanh, Archie C. Chapman, Alex Rogers, and Nicholas R. Jennings. Knapsack based optimal policies for budget-limited multi-armed bandits In Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence, July 22-26, 2012, Toronto, Ontario, Canada. AAAI Press, 2012.

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Research Gaps

- No work considered learning qualities and eliciting true cost in crowdsourcing environment
- While leaning qualities, we need to ensure the target accuracy
- This calls for developing new framework for MAB in our setting

The Model

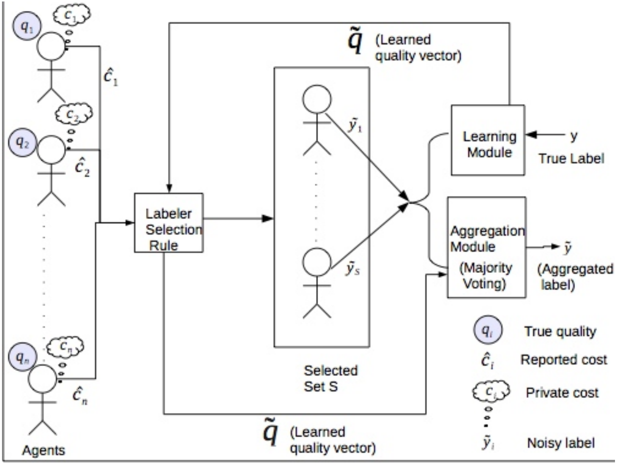


Figure : Crowdsourcing Model

Assured Armed Bandit Framework

Bound the probability of occurrence of most probable noisy label vector of the set S that leads to an error if y is revealed later

$$\tilde{y}_w(S) = arg \max_{\tilde{y}(S)} \mathbb{P}(\tilde{y}(S), \hat{y} \neq y | y)$$

The following optimization problem is solved:

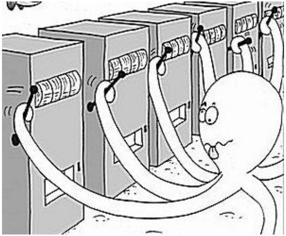
$$\boxed{\begin{array}{c} \min_{S \subseteq \mathcal{N}} \sum_{i \in S} c_i \\ \text{s.t.} \\ \mathbb{P}(\tilde{y}_w(S), \hat{y} \neq y | y) < e \end{array}} \tag{1}$$

Definition

AAB Framework We refer the instance where the qualities are learnt over period (rounds) while solving optimization problem 1 as an Assured Armed Bandit (AAB) framework.

Solution Approach

- Map the problem to multiple pull multi-armed bandit mechanism.
- Maintain lower confidence and upper confidence bounds on the qualities similar to UCB algorithm.
- Solve the optimization problem with respect to upper confidence bound.



- Check the accuracy constraint with respect to lower confidence bound.
- If the constraint is not satisfied select all the players (exploration round).
- If the constraint is satisfied then the selected set is the optimal set.
- For all the remaining rounds select the optimal set (exploitation rounds).

Properties of CCB Algorithm

- CCB is an adaptive exploration separated learning Algorithm

Theorem

Allocation rule by the proposed algorithm is ex-post monotone and thus can be converted to a randomized mechanism which is ex-post incentive compatible and universally ex-post individually rational.

Theorem

Expected number of exploration rounds are bounded by $(1-\mu)\frac{2}{\Delta^2}n^2 ln(\frac{1}{\mu'}) + \mu T$ where $(1-\mu) = (1-\mu')^{n/2}$ and Δ is the minimum difference between tolerance e and error probability of any set.

Summary So Far

Our Contributions

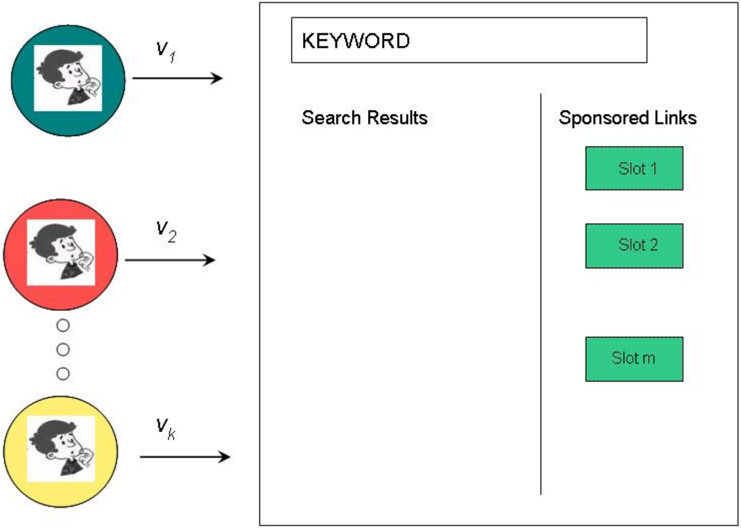
- Proposed a novel framework Assured Accuracy Bandit (AAB) where the constrained optimization problem is formulated for selecting an optimal subset of crowd workers to assure accuracy.
- Proposed an adaptive exploration separated algorithm, Constrained Confidence Bound (CCB).
- CCB Algorithm is ex-post monotone in costs and hence leads to an ex-post truthful and ex-post individual rational mechanism
- Provided an upper bound on the number of exploration steps.

Directions for Future Work

- Investigating existence of non exploration-separated algorithms satisfying desirable mechanism properties with lower regret.
- Investigating other constraint which is closer to actual probability of error as the current accuracy constraint is stiff.
- Providing an approximate mechanism so as to solve the optimization problem efficiently.
- Investigate different aggregation rules like weighted majority voting.
- Provide lower bounds on the regret in this setting.

Multi-Armed Bandit (MAB) Mechanisms for Multi-Slot Sponsored Search Auctions

Sponsored Search Auction Scenario



Challenges in Allocating the slots to the Agents

- Billions of dollars
- Consider an example
  - 2 agents competing for a single slot
  - Agent 1 bids \$1 and Agent 2 bids \$0.7
  - The probability of receiving click on ad for agent 1 is 0.5 and agent 2 receives a click if her ad is displayed
  - Displaying ad of the agent 2 is beneficial for the search engine
- The probability of ad receiving a click is referred to as Click-Through-Rate (CTR)
- While allocating one should consider the CTRs

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Need for Mechanism Design

- Agents are strategic
- It is critical to know the value that an agent obtains for each click she receives, but this is private information
- × Popular Mechanisms GFP and GSP are not truthful
- Mechanism Design is a natural tool
- Design of truthful sponsored search auctions: Aggarwal *et. al.*[1]
- Design of optimal sponsored search auctions: Garg and Narahari [2]

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[1] G. Aggarwal, A. Goel, and R. Motwani. Truthful auctions for pricing search keywords. ACM EC'06.

[2] D. Garg and Y. Narahari. An Optimal Mechanism for Sponsored Search Auctions and Comparison with other Mechanisms. In, IEEE Transactions on Automation Science and Engineering, 2009.

State of The Art

- If agents are not strategic: $R(T) = O(T^{1/2})$
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Research Gaps

- MAB Mechanisms for sponsored search: Previous work assumes a single slot
- The techniques do not immediately generalize to multi-slot case
- The problem that we address

Goal

Characterize truthful Multi-Armed Bandit (MAB) mechanisms for the allocation of advertisers to multiple slots in sponsored search auctions under various assumptions on Click-Through Rates (CTRs).

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Our Contributions

For truthfulness allocation rule needs to satisfy certain properties. We define following properties for deterministic allocation rules.

- Generalize notion of pointwise monotonicity: Strong Monotonicity and Weak Monotonicity
- Introduce notion of **Type-I Separatedness** and **Type-II Separatedness**
- Characterization of truthful MAB mechanisms for unknown and unconstrained CTRs
- Necessary conditions for a MAB mechanism to be truthful with various assumptions on CTRs
- Sufficient conditions for a MAB mechanism to be truthful with various assumptions on CTRs

Our Results

Number of Slots (m)	Learning Parameter (CTR)	Solution Concept	Allocation rule Characterization	Worst Case Regret
$m = 1$	Unrestricted	DSIC	Pointwise monotone & Exploration separated	$\Theta(T^{2/3})$
$m \geq 1$	Unrestricted	DSIC	Strongly pointwise monotone and Type-I separated	$\Theta(T)$
$m \geq 1$	Higher Slot Click Precedence	DSIC	Weakly pointwise monotone & Type-I separated (Necessary Condition)	regret analysis open
$m \geq 1$	CTR Pre-estimates available	Truthful in expectation	Weakly Pointwise monotone & Type-I separated (Necessary) Type-II Separated (Sufficient)	regret analysis open
$m \geq 1$	Separable CTR	Truthful in expectation	Weakly Pointwise monotone & Type-I separated (Necessary) Type-II Separated (Sufficient)	$\Omega(T^{2/3})$ (Experimental Evidence)

Table : Results

Story So Far...

We have seen,

- Characterization of truthful mechanisms with Unknown and Unconstrained CTRs
- Need for strong monotonicity puts severe restrictions on truthful allocation rules
- How do we exploit the relation across the CTRs

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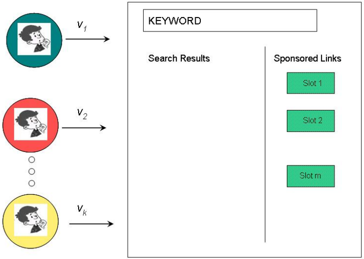
Directions for Future Work



- Look for regret minimizing MAB mechanism
- Weaker solution concepts. BIC is natural in learning settings.
- Budget constrained bidders
- Bidders wish to revise their bids in each round

Typically, these techniques are useful for designing truthful MAB mechanisms. For example, learning from the data for which the source itself may be selfish (document labelers).

Summary of the Talk

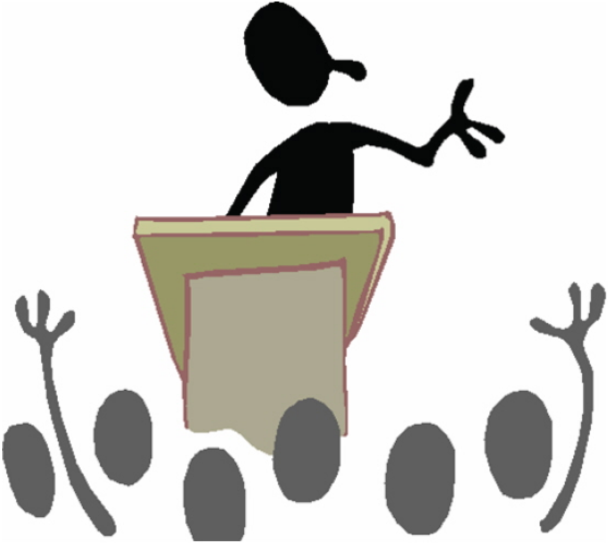
PART - I

- A new Framework for learning qualities of workers while achieving target accuracy, namely AAB
- An algorithm to solve AAB, namely CCB

PART - II

- Characterization of truthful MAB mechanism for multi-slot sponsored search auctions under general settings
- High regret, leading to restrict the settings and then characterization of truthful MAB mechanisms

Questions?



Thank You!!!

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Notation

m	Number of slots
M	$= \{1, 2, \ldots, m\}$ Set of slots
i	Index of an agent, $i = 1, 2, \ldots, k$
j	Index of a slot, $j = 1, 2, \ldots, m$
T	Total number of rounds
t	A particular round. $t \in \{1, 2, \ldots, T\}$
$A_{ij}(t)$ $A(t)$	$= 1$ if an agent i is allocated slot j in round t $= 0$ otherwise $(A_{ij}(t))_{i \in K, j \in M}$
A	$= (A(1), A(2), \ldots, A(T))$, Allocation rule
$\rho_{ij}(t)$ $\rho(t)$	$= 1$ if agent i gets a click in slot j in round t $= 0$ otherwise $(\rho_{ij}(t))_{i \in K, j \in M}$
ρ	$= (\rho(1), \rho(2), \ldots, \rho(T))$
v_i	Agent i's valuation of a click to her ad
b_i	Bid by agent i
b	Bid vector, indicating bids of all the agents $= (b_i, b_{-i}) = (b_1, b_2, \ldots, b_k)$

$C_i(b, \rho)$	Total number of clicks obtained by an agent i in T rounds
$P_i(b, \rho)$	Payment made by agent i
$P(b, \rho)$	$= (P_1(.), P_2(.), \ldots, P_k(.))$, Payment rule
$U_i(v_i, b, \rho)$	Utility of an agent i in T rounds $= v_i C_i(b, \rho) - P_i(b, \rho)$
b_i^+	A real number $> b_i$
α_i	Click probability associated with agent i
β_j	Click probability associated with slot j
μ_{ij}	The probability that an ad of an agent i receives click when the agent is allotted slot j.
$N(b, \rho, i, t)$	Set of slot agent pairs in round t that influence agent i in some future rounds

Table : Notation

- Myerson[3] designed an optimal auction for the seller.
  - He characterized the truthful mechanisms
- Technique developed by him applies to most single parameter domains
- **Myerson Characterization**: Let (A, P) be a normalized mechanism for the MAB mechanism design problem. It is truthful with unrestricted payment computation *if and only if* for any given realization ρ the corresponding click-allocation C is non-decreasing and the payment rule is given by,

$$P_i(b_i, b_{-i}; \rho) = b_i \cdot C_i(x, b_{-i}; \rho) - \int_0^{b_i} C_i(x, b_{-i}; \rho) dx$$

[3] R. B. Myerson. Optimal auction design. *Math. Operations Res.*, 6(1):58-73, Feb. 1981.

Characterization of Truthful MAB mechanism for single slot case

Babaioff[4] *et. al.* showed,

Theorem

Consider the MAB mechanism design problem. Let A be a non-degenerate, deterministic allocation rule. Then mechanism (A, P) is normalized and truthful for some payment rule P if and only if A is pointwise monotone and weakly separated.

[4] M. Babaioff, Y. Sharma, and A. Slivkins. Characterizing truthful multi-armed bandit mechanisms. ACM EC'09

Allocation Rule Properties

- Normalized: Payment is non negative and no agent pays more than what she has bid for. That is,

$$\forall\, i, \forall\, b, \rho, \quad P_i(\cdot) \geq 0 \text{ and } P_i(\cdot) \leq b_i \cdot C_i(\cdot)$$

- Non-degenerate: For any agent at each bid, small perturbation in bid does not affect her allocation. That is, $\forall\, i$, $\forall\, b_i\ \forall b_{-i}, \rho,\ \exists\, I \ni b_i$ such that,

$$\boxed{A_i(x, b_{-i}; \rho) = A_i(b_i, b_{-i}; \rho)\ \ \forall x \in I}$$

Monotonicity

Definition (Strong Pointwise Monotonicity)

An allocation rule is said to be strongly pointwise monotone if it satisfies: For any fixed (b_{-i}, ρ), if an agent i with bid b_i is allocated a slot j in round t, then $\forall\ b_i^+ > b_i$, she is allocated the same slot j in round t.

Higher bid: agent receives the same slot in round t,
Lower bid:, she may receive the same slot or may lose the impression.

Definition (Weak Pointwise Monotonicity)

We call an allocation rule weak pointwise monotone if, for any given (b_{-i}, ρ), and bid $b_i^+ > b_i$, $A_{ij}((b_i, b_{-i}), \rho, t) = 1 \Rightarrow A_{ij'}((b_i^+, b_{-i}), \rho, t) = 1$ for some slot $j' \leq j$, $\forall t$.

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Example: Monotonicity

Example

- Consider, $k = 4$ agents, $m = 2$ slots, $T = 1000$ rounds
- CTRs, μ_{ij} decreasing for each agent i
- Let an allocation A be:
    - For the first 100 rounds, advertisements of four agents displayed in round robin fashion
    - For the remaining 900 rounds, the advertisements are displayed that maximize the expected sum of valuations of the clicks
- A described here is weakly pointwise monotone

More Definitions

Definition (Influential Set)

Given a bid vector, b, a realization ρ and round t, an influential set $I(b, \rho, t)$ is the set of all agent-slot allocation pairs (i, j), such that (i) $A_{ij}(b, \rho, t) = 1$ and (ii) a change in $\rho_{ij}(t)$ will result in a change in the allocation in a future round. t is referred to as an *influential round*. Agent i is referred to as an *influential agent* and j as *influential slot* w.r.t round t.

It is collection of agent-slot pairs which clicked or not clicked changes future allocations.

Separatedness

Definition (Type-I Separated)

We call an allocation rule Type-I separated if for a given (b_{-i}, ρ), if $N((b_i, b_{-i}), \rho, i, t)$ is an i-influential set, then $\forall\,(i', j') \in N((b_i, b_{-i}), \rho, i, t)$, $A_{i'j'} = 1$ when the agent i increases her bid to b_i^+.

When an agent i increases her bid, while the other parameters are kept fixed, the allocation in the originally influential slots does not change, though the influentiality of that agent-slot pair may be lost.

Separatedness

Definition (Type-II Separated)

We call an allocation rule Type-II separated if for a given (b_{-i}, ρ) and two bids of agent i, b_i and $b_i^+ > b_i$, $N((b_i, b_{-i}), \rho, i, t) \subseteq N((b_i^+, b_{-i}), \rho, i, t)$.

This means that when an agent i increases her bid, while the other parameters are kept fixed, the allocation in the originally influential slots does not change and they remain influential.

Example: Type-I Separated Allocation Rule

Example

- Consider, $k = 4$ agents, $m = 2$ slots, $T = 2$ rounds
- Let an allocation A be:
  - First round, the ad of the agent i is displayed in the slot i, $i = 1, 2$.
  - If any ad receives click, same ad is retained in same slot
  - If the ad in slot i is not clicked
    - i If $b_i < b_{i+2}$, in round 2, the ad of the agent $i + 2$ is displayed in slot i
    - ii Else, the original ad is retained
- A described here is Type-I separated
- A described here is not Type-II separated (If $b_1 > b_3$, agent 1 is not influential for herself)

Unconstrained and Un-related CTRs

Theorem

Let (A, P) be a deterministic, non-degenerate mechanism for the MAB, multi-slot sponsored search auction, with unconstrained and unknown μ_{ij}. Then, mechanism (A, P) is DSIC iff A is strongly pointwise monotone and Type-I separated. Further, the payment scheme is given by,

$$P_i((b_i, b_{-i}), \rho) = b_i C_i((b_i, b_{-i}), \rho) - \int_0^{b_i} C_i((x, b_{-i}), \rho)dx.$$

Sketch of the Proof

- Monotonic Allocation - # clicks should increase with her bid
- For the unknown and unconstrained CTRs this leads to a need of strong pointwise monotonicity
- Payment should be in the form as given by Theorem 1
  This can be derived from Myerson Characterization
- Payment should be computable from the observed clicks: Type -I separatedness is necessary
- Type-I separatedness along with the strong pointwise monotonicity is sufficient for truthful implementation

Implications of Strong Monotonicity

- In a particular round, for a fixed game instance, if an agent is allocated a particular slot, she has to be allocated same slot at all higher bids
- This is a very strong necessity
- It leads to instances on which regret is $O(T)$

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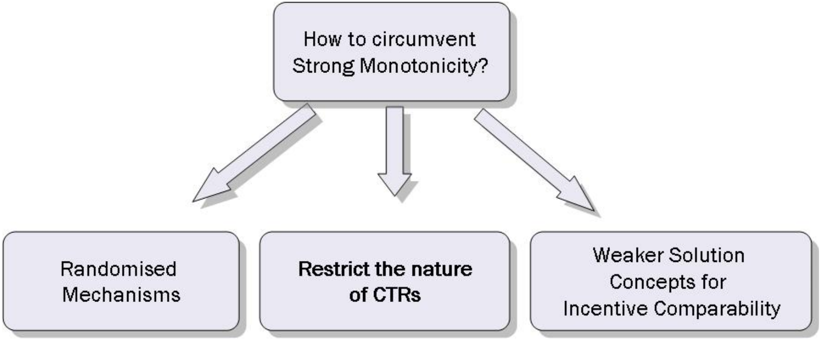
What Next?

Three escape routes



Higher Order Click Precedence

Assume, if there is a click on ad in slot j, then all previous slots receive click.

Proposition

Consider the setting in which realization ρ follows Higher Slot Click Precedence. Let (A, P) be a deterministic non-degenerate DSIC mechanism for this setting. Then the allocation rule A must be weak pointwise monotone and Type-I separated. Further, the payment scheme is given by,

$$P_i(b_i, b_{-i}; \rho) = b_i C_i(b_i, b_{-i}; \rho) - \int_0^{b_i} C_i(x, b_{-i}; \rho) dx$$

Higher Order Click Precedence

Assume, if there is a click on ad in slot j, then all previous slots receive click.

Proposition

Consider the setting in which realization ρ follows Higher Slot Click Precedence. Let (A, P) be a deterministic non-degenerate DSIC mechanism for this setting. Then the allocation rule A must be weak pointwise monotone and Type-I separated. Further, the payment scheme is given by,

$$P_i(b_i, b_{-i}; \rho) = b_i C_i(b_i, b_{-i}; \rho) - \int_0^{b_i} C_i(x, b_{-i}; \rho) dx$$

Truthful In Expectation

Definition (Truthful in Expectation)

A mechanism is said to be truthful in expectation over μ, the CTR pre-estimate, if each of the agents believes that the number of clicks she obtains is indeed $\sum_t \sum_j (\mu_{ij} A_{ij}(\cdot))$, which is the number of clicks she will obtain if the CTR pre-estimate is perfectly accurate.

CTR Pre-estimates are Available

Theorem

Let (A, P) be a mechanism for this stochastic multi-round auction setting where A is a non-degenerate, deterministic and fair allocation rule. Then, (A, P) is truthful in expectation over μ if A is weakly pointwise monotone and Type-II separated and the payment scheme is given by,

$$P_i(b, \rho) = \sum_{t=1}^{T} \sum_{j=1}^{m} \mu_{ij} \{ b_i A_{ij}(b, \rho, t) - \int_0^{b_i} A_{ij}(x, b_{-i}, \rho, t) dx \}$$

Also, if a mechanism (A, P) is truthful, then it is weakly pointwise monotone, Type-I separated, the payment is given as above and is computable.

Separable CTRs

Theorem

Let (A, P) be a mechanism for the stochastic multi-round auction setting where A is a non-degenerate, deterministic and fair allocation rule. Then, (A, P) is truthful in expectation over μ' if A is weakly pointwise monotone and Type-II separated and the payment scheme is given by,

$$P_i(b, \rho) = \sum_{t=1}^{T} \sum_{j=1}^{m} \mu'_{ij} \{ b_i A_{ij}(b, \rho, t) - \int_0^{b_i} A_{ij}(x, b_{-i}, \rho, t) dx \}$$

Also, if a mechanism (A, P) is truthful, then it is weakly pointwise monotone, Type-I separated, the payment is given as above, and is computable.

Our Results

Number of Slots (m)	Learning Parameter (CTR)	Solution Concept	Allocation rule Characterization	Worst Case Regret
$m = 1$	Unrestricted	DSIC	Pointwise monotone & Exploration separated	$\Theta(T^{2/3})$
$m \geq 1$	Unrestricted	DSIC	Strongly pointwise monotone and Type-I separated	$\Theta(T)$
$m \geq 1$	Higher Slot Click Precedence	DSIC	Weakly pointwise monotone & Type-I separated (Necessary Condition)	regret analysis open
$m \geq 1$	CTR Pre-estimates available	Truthful in expectation	Weakly Pointwise monotone & Type-I separated (Necessary) Type-II Separated (Sufficient)	regret analysis open
$m \geq 1$	Separable CTR	Truthful in expectation	Weakly Pointwise monotone & Type-I separated (Necessary) Type-II Separated (Sufficient)	$\Omega(T^{2/3})$ (Experimental Evidence)

Table : Results