

$$a) \text{ L2 norm of } x = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \quad \text{L2 norm of } y = \sqrt{9^2 + 11^2 + 13^2} = \sqrt{371}$$

$$\text{L2 norm of } y = \sqrt{9^2 + 11^2 + 13^2} = \sqrt{371}$$

$$b) \vec{u}_1 = \frac{1}{\sqrt{5}} (2, -1, 0)^T$$

$$\vec{u}_2 = \frac{1}{\sqrt{30}} (1, 2, -5)^T$$

$$\vec{u}_3 = \frac{1}{\sqrt{24}} (2, 4, 2)^T$$

$$\begin{aligned} \vec{u}_1 \cdot \vec{u}_2 &= 0 \\ \vec{u}_2 \cdot \vec{u}_3 &= 0 \\ \vec{u}_3 \cdot \vec{u}_1 &= 0 \end{aligned}$$

orthogonal

$$|\vec{u}_1| = 1$$

$$|\vec{u}_2| = 1$$

$$|\vec{u}_3| = 1$$

Orthonormal

$$(C) \quad x_0 = 4 \quad x_1 = 9 \quad x_2 = -6$$

$$f(x_0) = -0.757 \quad f(x_1) = 0.412 \quad f(x_2) = 0.279$$

$$P_1(x) = a_0 + a_1 x$$

$$-0.757 = a_0 + 4a_1$$

$$0.412 = a_0 + 9a_1$$

$$0.279 = a_0 - 6a_1$$

$$\begin{bmatrix} 1 & 4 \\ 1 & 9 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -0.757 \\ 0.412 \\ 0.279 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 9 & -6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 9 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 9 & -6 \end{bmatrix} \begin{bmatrix} -0.757 \\ 0.412 \\ 0.279 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 8 \\ 8 & 142 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} -0.0667 \\ -1.751 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0.0128 \\ -0.013 \end{bmatrix}$$

$$P_1(x) = 0.013 - 0.013x$$