BJT: Mathematical Problems

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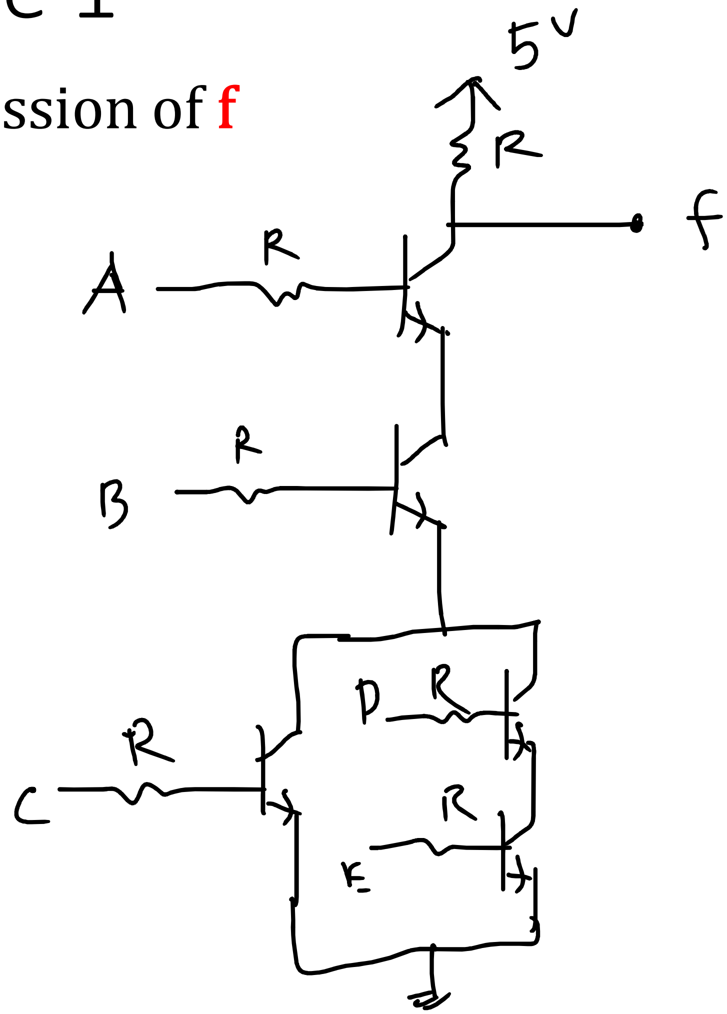
Course No: CSE 251

Course Title: Electronic Devices and Circuits

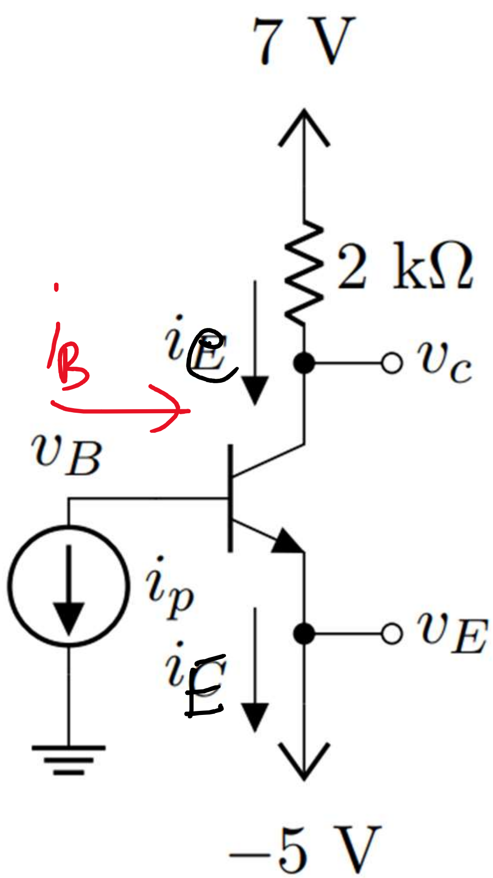
Example 1

Find expression of **f**



$$f = \overline{AB(C+DE)}$$

Example 2



Analyze the circuit above to find i_B, i_C, i_E and v_{CE}. Assume, the BJT is in **Saturation.** Here, use the Method of Assumed State. You must validate your assumptions. Assume, i_p= -1mA

Sol^n:

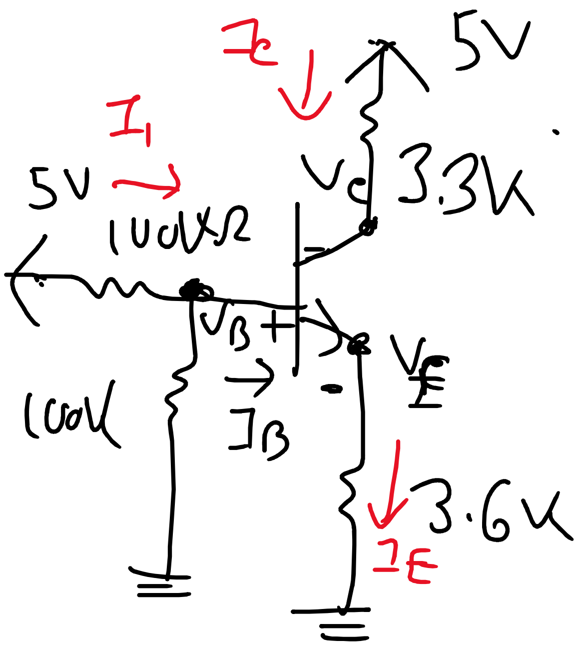
$i_B = -i_P = -(-1) = 1$ mA
$V_E = -5$V
In saturation mode
So, $V_BE = 0.8$ V &
$V_{CE} = 0.2$ V
$\Rightarrow V_C - V_E = 0.2V \Rightarrow V_C = 0.2 + V_E = 0.2 + (-5)$
$\therefore V_C = -4.8$ V
$i_c = (7 - V_C)/2 = (7+4.8)/2 = 5.9$ mA
$i_E = i_c + i_B = 6.9$ mA

<u>Verify:</u>

$\beta = 100$
$i_c / i_B = 5.9/1 = 5.9 < \beta$

$\therefore$ Assumption Correct

Example 3



Analyze the circuit above to find i_B, i_C, i_E and v_{CE}. Here, use the Method of Assumed State. You must validate your assumptions. **β= 100, α= (β)/(β+1)= 0.99**

Sol^n:

Assume, Active mode

So, V_{BE}= 0.7 V & I_C= βI_B= αI_E

Nodal analysis (at base terminal):

$$\frac{V_B - 5}{100\ k\Omega} + \frac{V_B}{100\ k\Omega} + I_B = 0 ----(1)$$

KVL:

$$5 - 0 = 100I_1 + V_{BE} + 3.6I_E$$

$$5 = 100\frac{5 - V_B}{100\ k\Omega} + 0.7 + 3.6\frac{\beta}{\alpha}I_B ---(2)$$

Solving, eq^n 1 & 2, V_B= 2.28 V , I_B= 4.35 uA

I_C= βI_B= 0.435 mA, I_E= I_c+ I_B=0.43935 mA

V_C= 5-3.3I_C= 3.5645 V

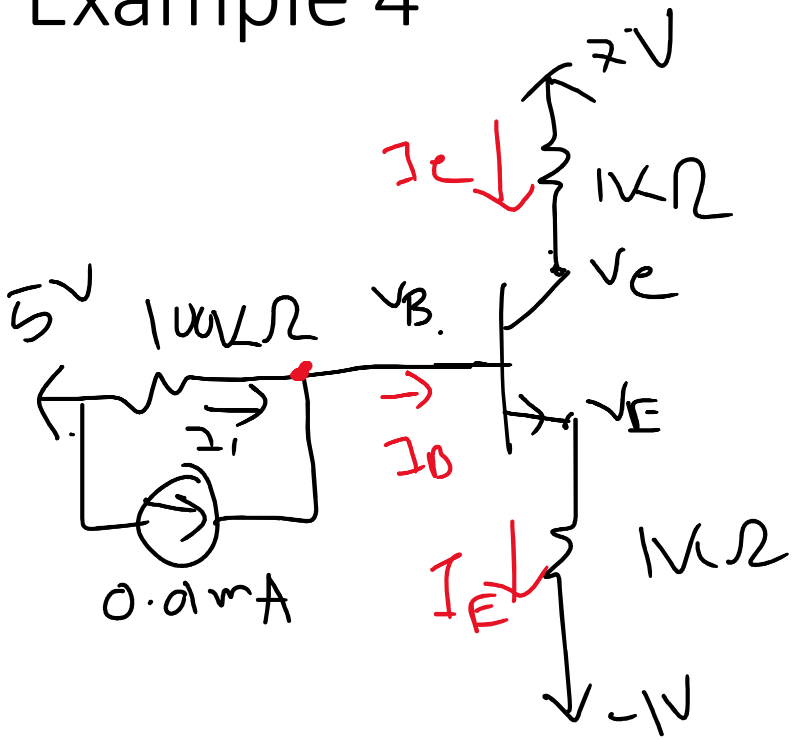
V_E= 3.6I_E= 1.582 V

V_{CE}= V_C-V_E= 1.983 V

So, V_{CE}> 0.2 V

$\therefore$ Assumption Correct!

Example 4



Analyze the circuit above to find v_{CE}. Here, use the Method of Assumed State. You must validate your assumptions. β= 100, α= (β)/(β+1)= 0.99

Sol^{n}:

Assume, Active mode

So, V_{BE}= 0.7 V & I_C= βI_B= αI_E

KCL (at base terminal):

$$I_1 + 0.01 = I_B$$
$$\therefore I_1 = I_B - 0.01$$

KVL:

$$5 - (-1) = 100I_1 + V_{BE} + 1I_E$$
$$6 = 100(I_B - 0.01) + 0.7 + \frac{\beta}{\alpha} I_B \;---(2)$$

$\therefore$ I_B= 0.0313 mA

I_C= βI_B= 3.13 mA, I_E= I_c+ I_B=3.1613 mA

V_C= 7-1I_C= 3.87 V

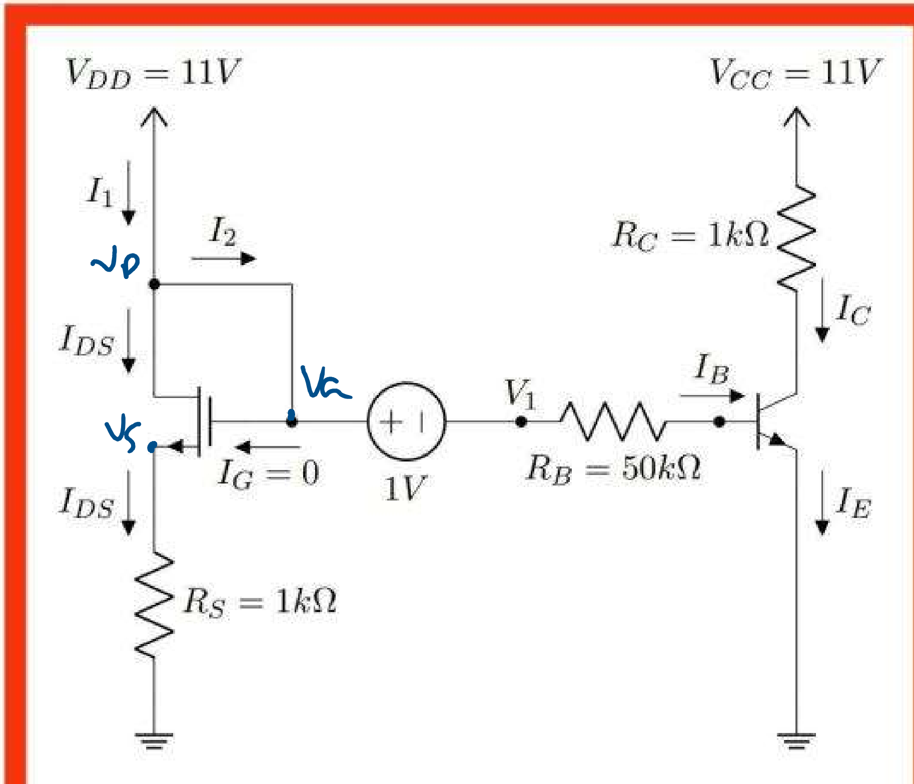
V_E= 1I_E-1= 2.16 V

V_{CE}= V_C-V_E= 1.709 V

So, V_{CE}> 0.2 V

$\therefore$ Assumption Correct!

Example 5



Circuit-1

a. Find out the gate voltage of the MOSFET.
b. Calculate V_1.
c. Find out the expression for V_{GS}, V_{DS} and V_{OV}.
d. Find the operating mode of the MOSFET using the expressions from ©. [Hint: You don't need any assumption]
e. Calculate I_{DS} and V_{DS} using the given parameters.
f. Assume that the BJT is in the saturation mode. Now, calculate I_B, I_C, I_E. You must validate the given assumption.

The MOSFET and BJT in Circuit-1 have the following parameters,
$k = 5\ mA/V^2$, $V_T = 0.7$ V, $\beta = 85$, $V_{BE(Active)} = 0.7$ V, $V_{BE(Saturation)} = 0.8$ V, $V_{CE(Saturation)} = 0.2$ V

Sol^n:

a) $V_G= V_D= 11$ V [Drain & Gate short]

b) Applying voltage difference formula, $V_G-V_1= 1 \Rightarrow V_1=V_G-1= 11-1 \therefore V_1= 10$ V

c) Assume, $I_{DS}= x$

Now, $V_S= 1\ k\Omega \times I_{DS}= I_{DS} \therefore V_S= x$

$V_{GS}= V_G- V_S= 11-x$, $V_{OV}= V_{GS}-V_T= 11-x-0.7= 10.3-x$

$V_{DS}= V_D- V_S= V_G- V_S= V_{GS} = 11-x$ [as $V_D= V_G$]

d) As Drain & Gate are short ($V_D= V_G$ or $V_{DS}= V_{GS}$), for this MOSFET, V_{DS} will be always greater than V_{OV} ($=V_{GS}-V_T$)

So, the MOSFET is in saturation mode

$$I_{DS} = \frac{k}{2} {V_{OV}}^2$$

$$x = \frac{5}{2}(10.3 - x)^2$$

$\therefore$ x= 12.54 or 8.46

If x= 12.54, then $V_{OV}= 10.3-12.54= -2.24$ V

But V_{OV} cannot be negative as $V_{OV}<0$ means $V_{GS}<V_T$ which will result the MOSFET to be in cutoff mode.

$\therefore$ x= 8.46

Sol^n:

e) I_{DS}= x= 8.46 mA

V_{DS}= 11-x= 2.54 V

f) Given, BJT is in saturation mode

So, V_{BE}=0.8 V & V_{CE}= 0.2 V

V_E= 0V (ground connected)→ V_B= 0.8 V, V_C= 0.2 V

$$I_B = \frac{V_1 - V_B}{R_B} = \frac{10 - 0.8}{50\ k\Omega} = 0.184\ mA$$

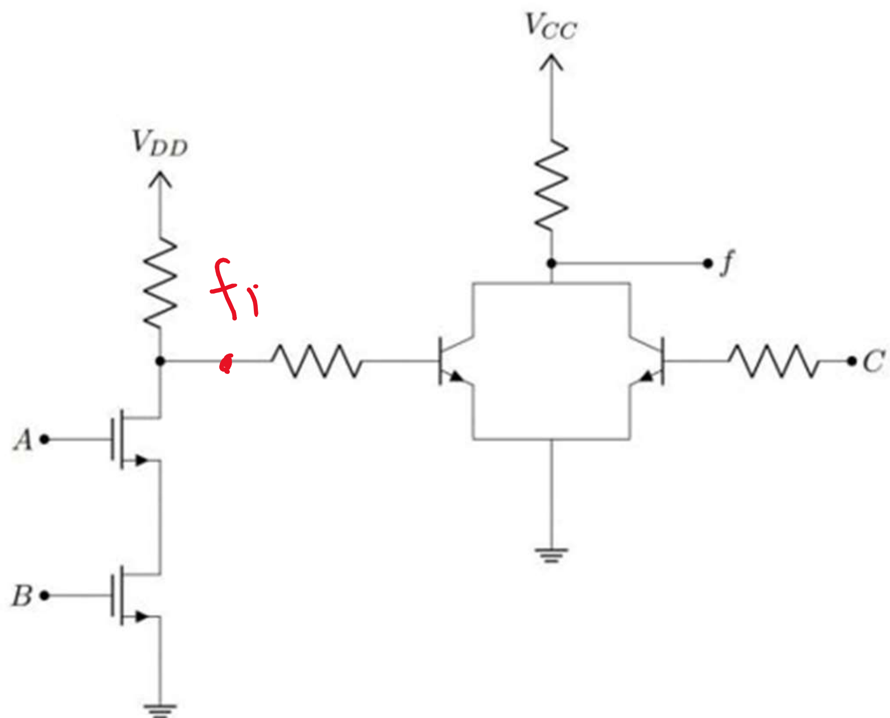
$$I_C = \frac{V_{CC} - V_C}{R_C} = \frac{11 - 0.2}{1\ k\Omega} = 10.8\ mA$$

I_E= I_c+ I_B= 10.984 mA

$$\frac{I_C}{I_B} = 58.7 < \beta$$

So, Assumption Correct!

Example 5



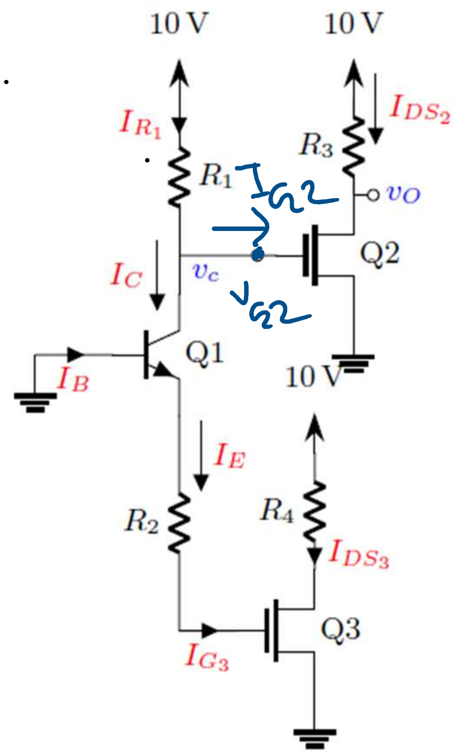
$$f_1 = \overline{AB}$$

$$f = \overline{f_1 + C}$$

$$= \overline{\overline{AB} + C}$$

Example 6

For the following circuit, $R_1 = 1\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 3\text{k}\Omega$, $R_4 = 4\text{k}\Omega$



For BJT

$\beta = 100$

$\alpha = 0.99$

$v_{BE(\text{Active})} = 0.7\,\text{V}$

$v_{BE(\text{Saturation})} = 0.8\,\text{V}$

$v_{CE(\text{Saturation})} = 0.2\,\text{V}$

for MOSFETs

$V_T = 1\,\text{V}$

$k = 5\,\text{mA/V}^2$

(a) [**5 marks**] **Analyze** the circuit to **determine** the values of I_C, I_B, I_E, v_C.

(b) [**5 marks**] **Analyze** the circuit to determine the value of I_{DS_2}, v_O.

Sol^n:

a) As gate current of MOSFET is zero

b) So, I_{G3}= 0 mA= I_E

$\therefore I_B$= 0 mA, I_C= 0 mA

So, the BJT is in cutoff mode

V_C= 10-1*I_{R1}= 10- 1*(I_C+I_{G2})= 10- 1*(0+0)= 10 V

d) Assume, saturation mode

V_{G2}=V_C= 10 V [Collector of BJT and Gate of MOSFET (top one) are short]

V_{GS}= V_{G2}-V_S= 10-0= 10 V, V_{OV}= 10-1= 9 V

$$I_{DS2} = \frac{k}{2} {V_{OV}}^2$$
$$= \frac{5}{2}(9)^2 = 202.5\ mA$$

V_{DS}= 10-3*I_{DS2}= -597.5 V

V_{DS}< V_{OV}

$\therefore$ Assumption wrong!

Sol^n:

Now, assume triode mode

If I_{DS2}= x, V_{DS}= 10-3*I_{DS2}= 10-3x

$$I_{DS2} = \frac{k}{2}(2(V_{GS} - V_T)V_{DS} - {V_{DS}}^2)$$

$$x = \frac{5}{2}(2(10-1)(10-3x) - (10-3x)^2)$$

x= 3.31 or -2.69

As current cannot be negative, so x= 3.31

I_{DS2}= x= 3.31 mA

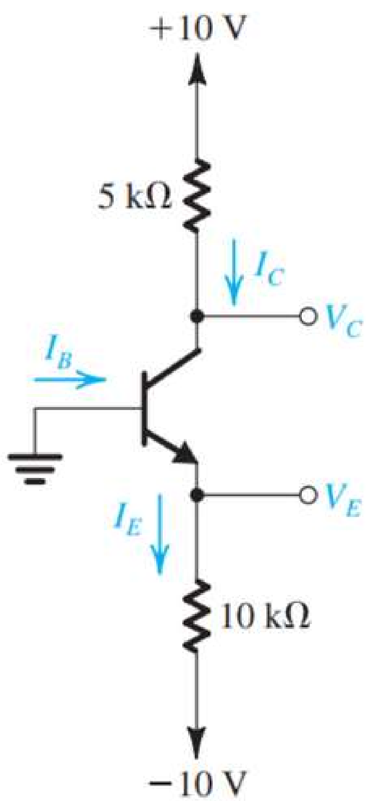
V_{DS}= 10-3x= 0.07 V

$V_{DS} < V_{OV}$

$\therefore$ Assumption correct!

Example 7

In the circuit shown in Fig. the voltage at the emitter was measured and found to be –0.7 V. If $\beta = 50$, find I_E, I_B, I_C, and V_C.



Sol[n:]

$$I_E = \frac{V_E - (-10)}{10\ k\Omega} = \frac{-0.7 + 10}{10} = 0.93\ mA$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{50}{51}$$

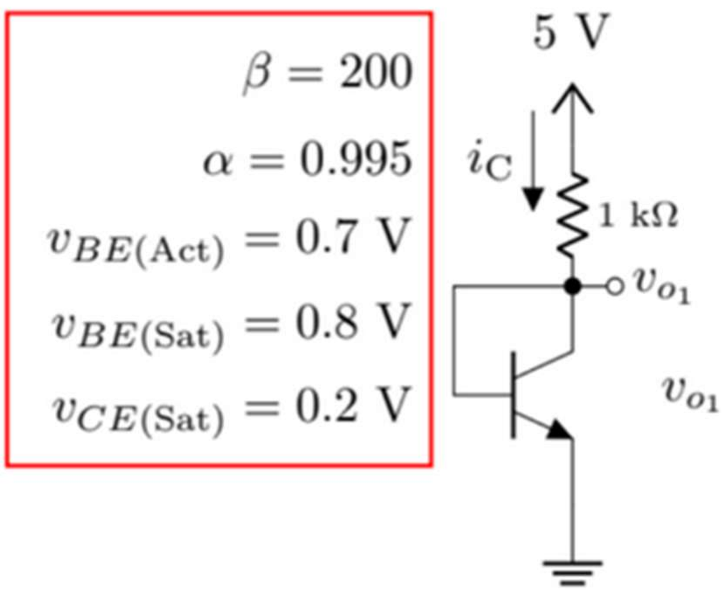
Assuming forward active mode,

$I_C = \alpha I_E = 0.9118$ mA
$V_C = 10\text{-}5I_C = 5.44$ V

So, $V_{CE} = V_C\text{-}V_E = 6.141$ V > 0.2 V so, forward active mode
Assumption correct!

$I_B = I_E\text{-}I_C = 18.2$ uA

Example 8



Analyze the circuit above to find i_C, i_E, i_B, and v_{01}. Here, use the Method of Assumed State.

Sol^{n}:

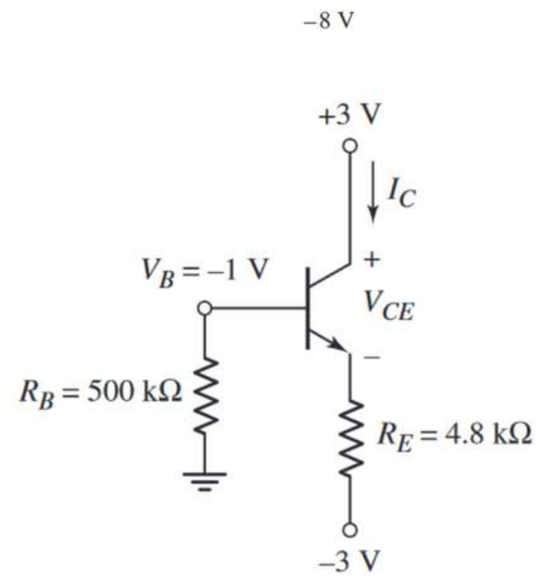
Assuming forward active mode,
V_{BE}=0.7 V → V_B-V_E= 0.7 V ∴ V_B= 0.7 V= V_C
So, V_{CE}= V_C-V_E= 0.7 V > 0.2 V so, forward active mode
Assumption correct!

$$i_C = \frac{5 - V_C}{1\ k\Omega} = \frac{5 - 0.7}{1} = 4.3\ mA$$

$I_C = \alpha I_E$
∴ I_E= 4.3/0.995= 4.3216 mA
I_B=I_E-I_C= 21.6 uA

Example 9

In the circuit shown in adjacent figure, the values of measured parameters are shown. Determine $\boldsymbol{\beta}, \boldsymbol{\alpha}$, and the other labeled currents and voltages.



Sol^n:

Assuming Active mode,

V_{BE}=0.7 V → V_B-V_E= 0.7 V ∴ V_E= (-1)-0.7 V= -1.7 V

$$I_B = \frac{0 - V_B}{R_B} = \frac{0 - (-1)}{500\ k\Omega} = 2\ uA$$

$$I_E = \frac{V_E - (-3)}{R_E} = \frac{-1.7 + 3}{4.8\ k\Omega} = 0.271\ mA$$

I_C=I_E-I_B=0.269 mA

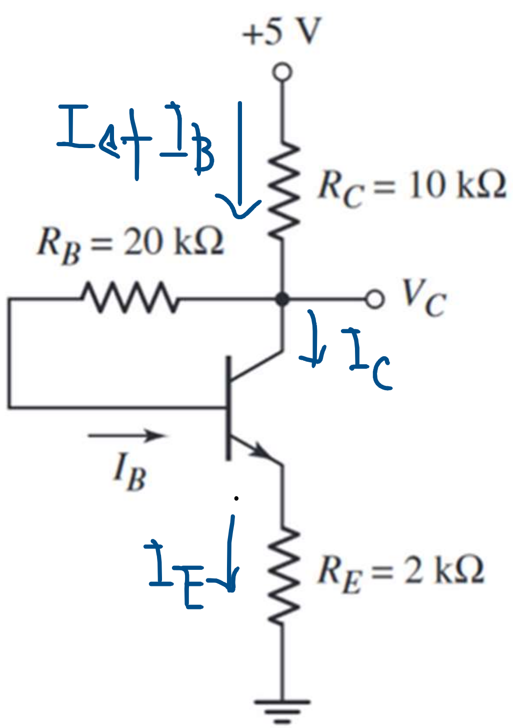
$$\beta = \frac{I_C}{I_B} = 134.5$$

$$\alpha = \frac{I_C}{I_E} = 0.9926$$

V_{CE}= V_C-V_E= 3-(-1.7)= 4.7 V > 0.2 V so, forward active mode

Assumption correct!

Example 10



$\beta = 75$. Find the labelled voltages and currents.

Sol^n:

Assuming Active mode,

V_{BE}=0.7 V

$$5 - 0 = R_C(I_C + I_B) + R_B I_B + V_{BE} + R_E I_E$$

$$5 = 10(\beta I_B + I_B) + 20 I_B + 0.7 + 2\frac{\beta}{\alpha} I_B$$

$\therefore$ I_B= 4.604 uA

$I_C = \beta I_B$= 0.345 mA

V_C= 5-10(I_C+I_B)= 1.5 V

I_E= I_c+ I_B=0.3496 mA

V_E= 2I_E= 0.7 V

V_{CE}= V_C-V_E= 1.5-0.7= 0.8 V> 0.2 V so, forward active mode

Assumption correct!