

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
- **Solve only the first question(1).**
- Start answering a question (not the part of the question) from the top of a new page.
- Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
- Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.

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1. A student has decided to sell the jerseys of football club as a relaxation after the stressful final exam. There are  $x_1$  number of jerseys of PSG club and  $x_2$  number of jerseys of Barcelona club in his shop. The total number of jersey is 30. On the first day, he sold each jersey of two clubs by 400 tk. and totally he earned 12000 tk by selling all the jerseys. But when Messi left Barcelona club and went to PSG club, each jersey of Barcelona costs 300 tk. and each jersey of PSG costs 500 tk. Then, he earned total 13000 tk. In the following, this overdetermined system will be solved by using the QR Decomposition Method by answering the following step by step:
    - (a) **[1.5 marks]** Write down the linear equations that relate the variable  $x_1$  and  $x_2$ .
    - (b) **[1.5 marks]** Identify the matrices A, x and b so that the equations in the previous question can be expressed in the standard matrix equation form  $Ax = b$ .
    - (c) **[5 marks]** From matrix A in the previous question, compute the matrices Q and R such that  $A = QR$ , where the symbols have their usual meanings.
    - (d) **[2 marks]** Evaluate  $Q^T b$ , and finally solve the system by evaluating x (that is, evaluate  $x_1$  and  $x_2$ ).
  2. Consider the following function:  $f(x) = e^x - x$ , which is continuous on the interval  $[1, 3]$ . Use this function to answer the following:
    - (a) **[2 marks]** Find the actual integral value for this function.
    - (b) **[5 marks]** Use Composite Newton-cotes formula to find the numerical integration for 4 segments.
    - (c) **[3 marks]** Compute the error in percentage between the results obtained in the previous two parts. How can we decrease the error more?
    - (d) **[3 marks]** Use the Simpson rule to find the numerical integration.
  3. Consider the following function:  $f(x) = 6x^2 - 4x - 9$ .
    - (a) **[3 marks]** Use the Trapezium rule to numerically integrate over the interval  $[-2, 2]$ .
    - (b) **[2 marks]** Compute the exact integrated value of the given function.
    - (c) **[2 marks]** Calculate the relative error in percentage.