

# Department of Computer Science and Engineering (CSE) BRAC University

Fall 2023

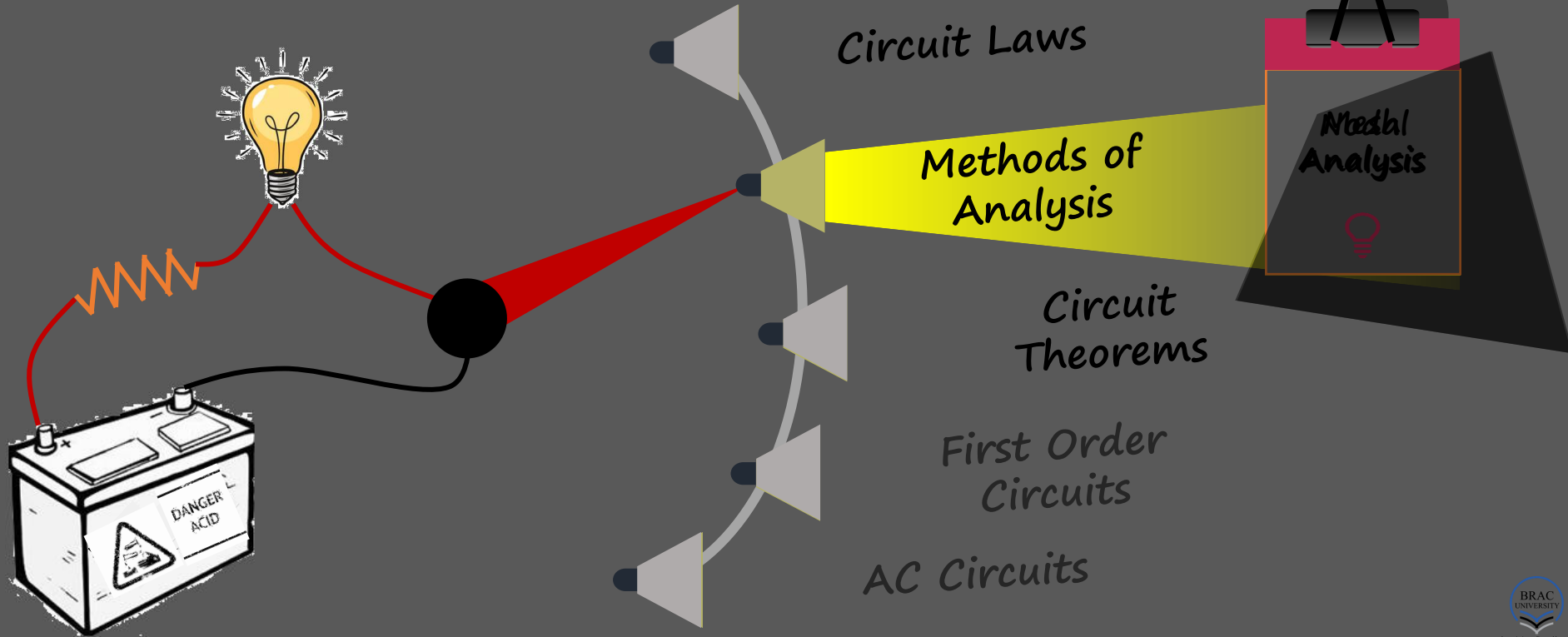
CSE250 - Circuits and Electronics

## MESH ANALYSIS



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# Course Outline: broad themes



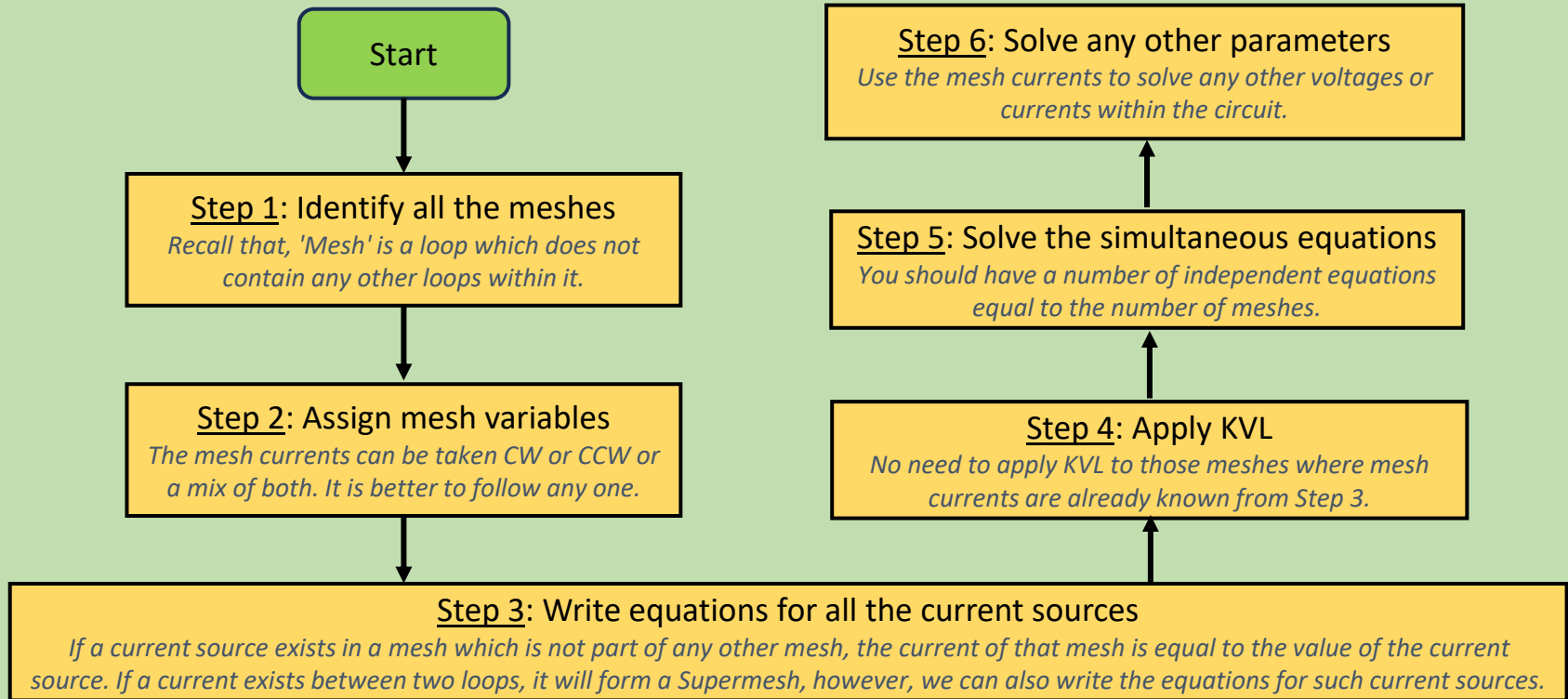
# Mesh Analysis

- *Mesh analysis* provides another general procedure for analysing circuits, using mesh currents as the circuit variables. Mesh analysis applies KVL to find unknown currents in a given circuit.
- A *mesh* is a loop that does not contain any other loops within it.
- *Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. Nonplanar circuits cannot be handled with mesh analysis.*
- *A nonplanar circuit is one that has branches that cross each other and cannot be redrawn without doing so.*

## Steps to Determine Mesh Currents:

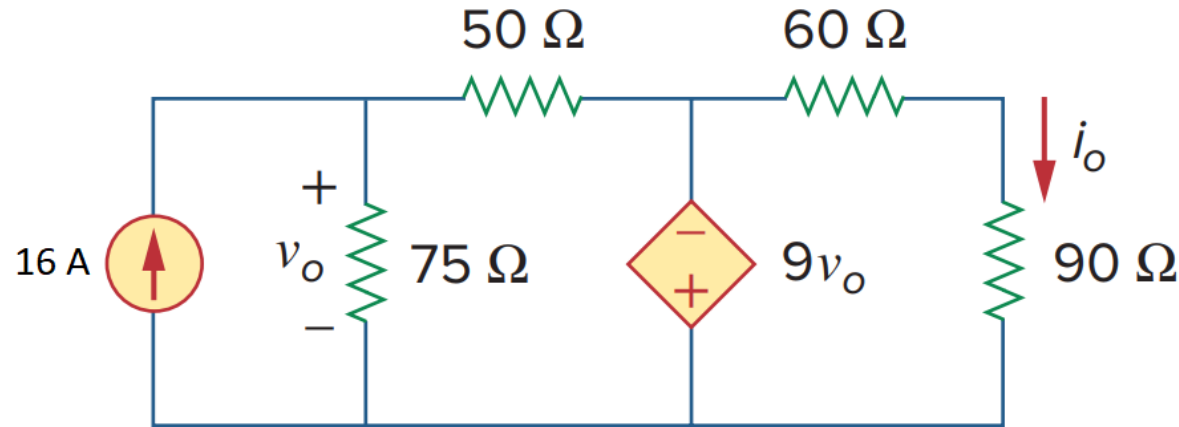
1. Assign mesh currents  $i_1, i_2, \dots, i_n$  to the  $n$  meshes.
2. Apply KVL to each of the  $n$  meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting  $n$  simultaneous equations to get the mesh currents.

# Mesh Analysis: steps



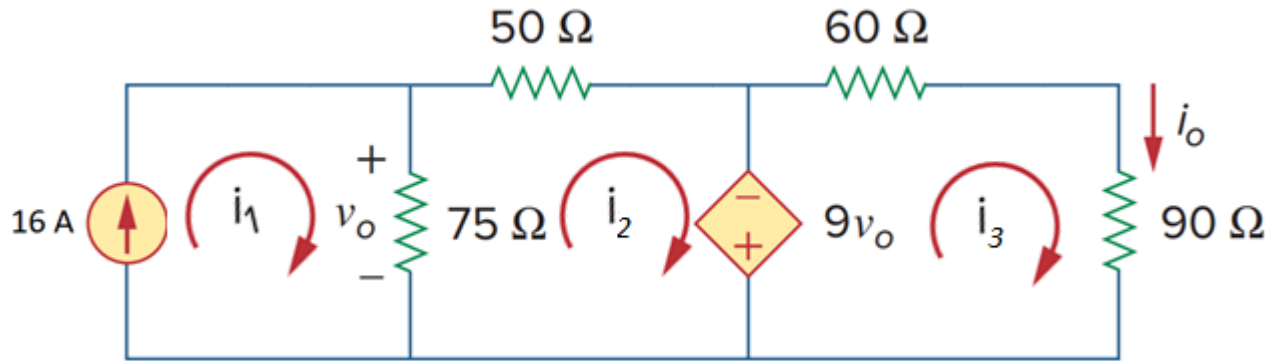
# Example 1

Use mesh analysis, determine  $v_o$ . What is the current supplied by the dependent voltage source? What is the power of it? Is it absorbing or supplying?



Before solving the circuit using mesh analysis, recall that, "*For passive elements, current enters through the positive terminal of the voltage drop across it.*" This is according to the *passive sign convention*, current must always flow from a higher potential to a lower potential through a passive element that is absorbing power.

# Example 1: step 1

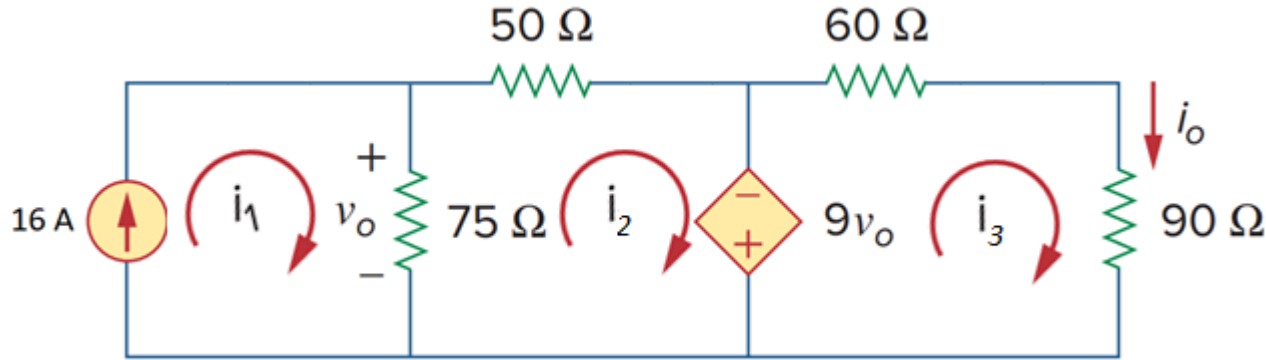


☞ First identify all the meshes (independent loops) in this circuit.

There are 3 meshes as identified in the circuit.

☞ Assign mesh currents ( $i_1$ ,  $i_2$ , and  $i_3$ ) to all the meshes. The assigned currents can be clockwise, anti-clockwise, or a combination of the two.

# Example 1: step 2



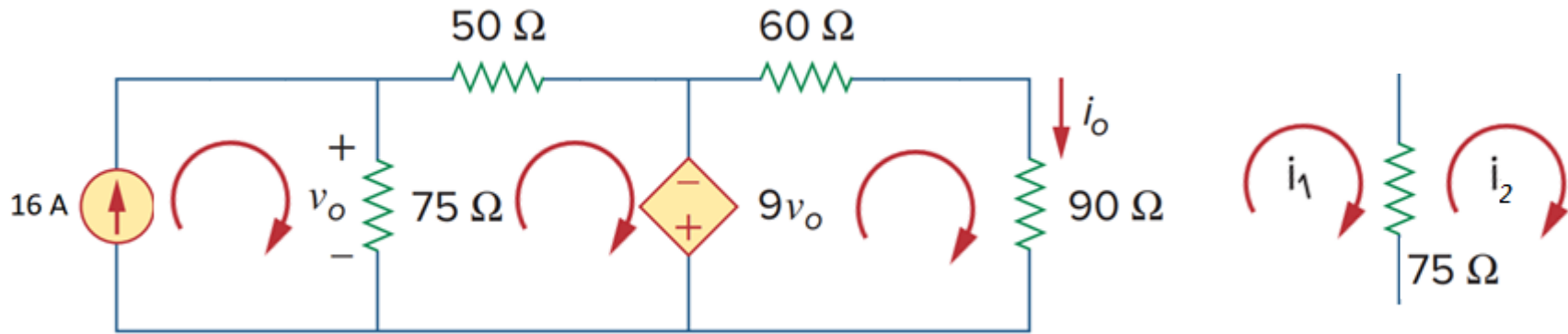
👉 The 2<sup>nd</sup> step is to apply KVL to each mesh.

Note that, we already know the mesh 1 current.  $i_1$  and the 16 A current flow through the same wire in the same direction. We can write directly,

$$i_1 = 16 \text{ A} \text{ ----- } (i)$$

For meshes whose mesh currents are already known, we don't need to apply KVL.

# Example 1: step 2 (contd ... 3)



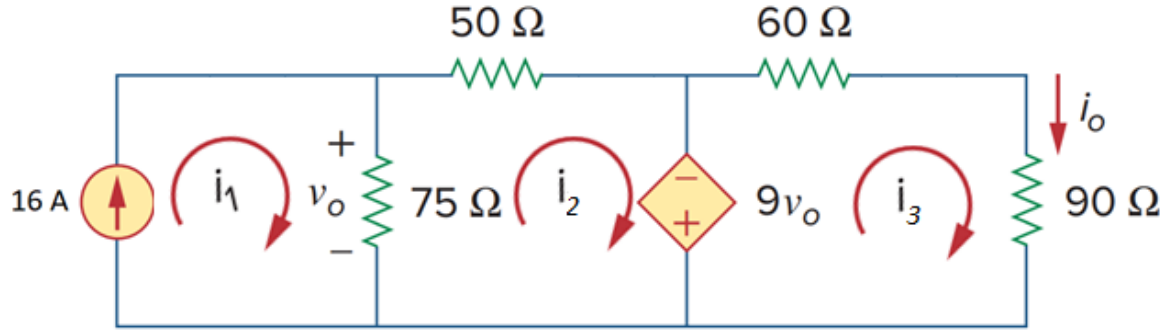
👉 Next, apply KVL to mesh 2.

$$75(i_2 - i_1) + 50i_2 - 9v_o = 0$$

Notice that, the two mesh currents ( $i_1$  and  $i_2$ ) overlap through the  $75\ \Omega$ . As there can be no more than a current in a wire, the resulting current through the  $75\ \Omega$  will be either  $i_1 - i_2$  or  $i_2 - i_1$ . But we won't know exactly before solving. As we are moving in the direction of  $i_2$ , we take  $i_2 - i_1$  as the resulting current and the KVL equation is written accordingly.



# Example 1: step 2 (contd ... 4)



$$75 (i_2 - i_1) + 50i_2 - 9v_o = 0 \text{ [from the previous slide]}$$

Now we have to replace  $v_o$  in terms of the mesh currents as the mesh equations should not contain unknowns other than the mesh currents.

$v_o$  is the voltage drop across the  $75 \Omega$  resistor. With the polarity of  $v_o$  given,

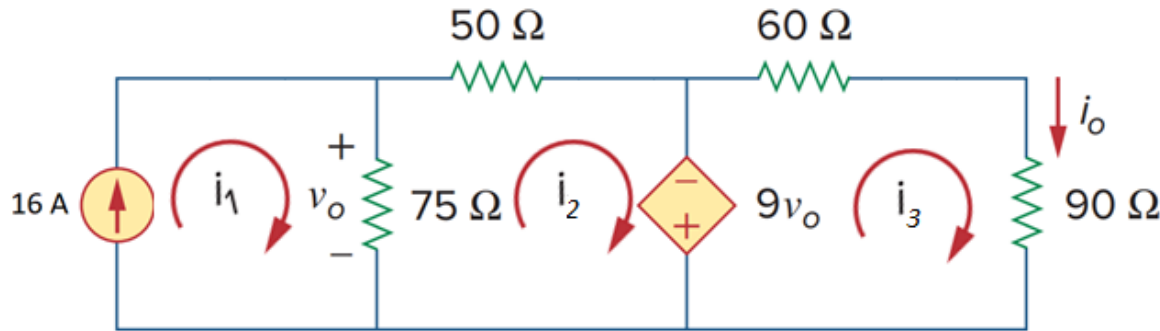
$$v_o = 75 (i_1 - i_2)$$

Substituting,

$$75 (i_2 - i_1) + 50i_2 - 9 \times 75 (i_1 - i_2) = 0$$

$$750 i_1 - 800 i_2 = 0 \text{ ----- (ii)}$$

# Example 1: step 2 (contd ... 5)



👉 Next, apply KVL to mesh 3.

$$9v_o + 60i_3 + 90i_3 = 0$$

Substituting  $v_o = 75 (i_1 - i_2)$  for  $v_o$ ,

$$9 \times 75 (i_1 - i_2) + 60i_3 + 90i_3 = 0$$

After simplifying,

$$675 i_1 - 675 i_2 + 150 i_3 = 0 \text{ ----- (iii)}$$

# Example 1: step 2 (contd ... 6)

We have derived the three mesh equations,

$$i_1 = 16 A$$

$$750 i_1 - 800 i_2 = 0$$

$$675 i_1 - 675 i_2 - 150 i_3 = 0$$

Solving ... ..,

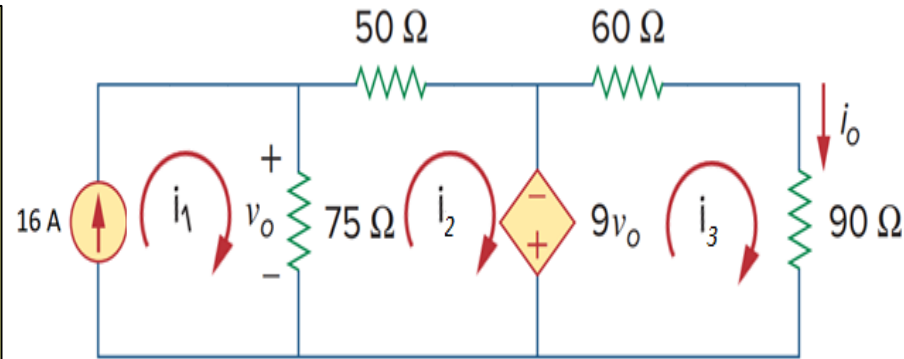
$$i_1 = 16 A; \quad i_2 = 15 A; \quad i_3 = -4.5 A;$$

So,

$$v_o = 75(i_1 - i_2) = 75(16 - 15) = 75 V$$

Current supplied (entering into the -ve terminal) by the dependent source is,

$$i_2 - i_3 = 15 - (-4.5) = 19.5 A$$

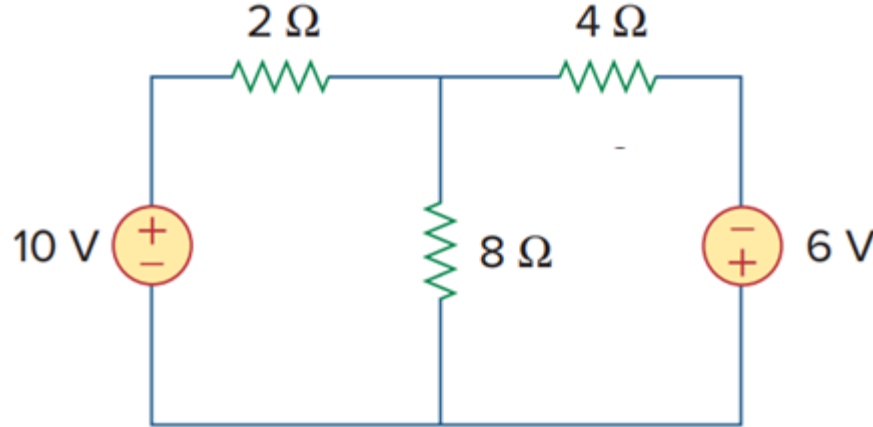


Power supplied by the dependent source is thus,

$$\begin{aligned} p &= -vi = 9v_o \times 19.5 \\ &= 9 \times 75 \times 19.5 \\ &= 13162.5 W \end{aligned}$$

# Problem 1

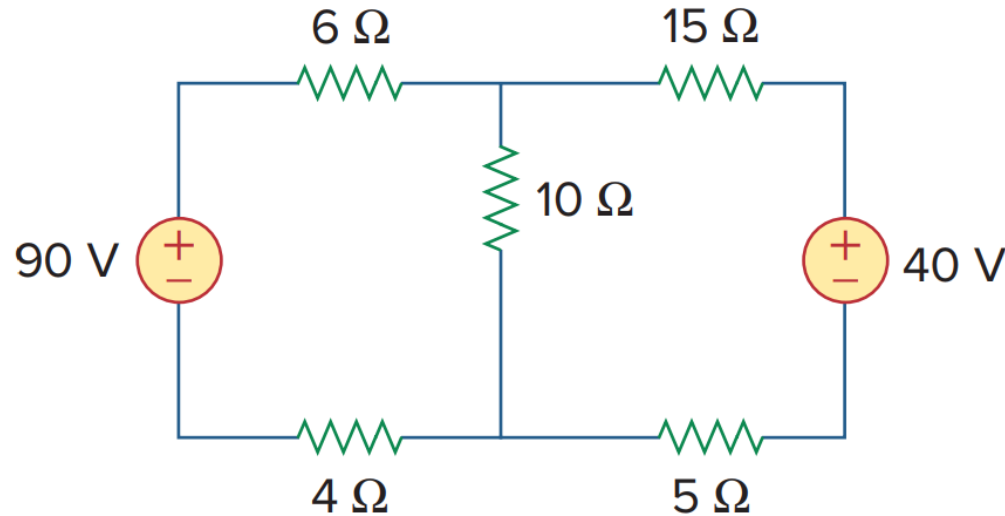
- i. Perform *branch current analysis* to determine the current absorbed by the 6 V source in the following circuit.
- ii. Perform *mesh analysis* to determine the current absorbed by the 6 V source in the following circuit.



Ans:  $-2.5 \text{ A}$

# Problem 2

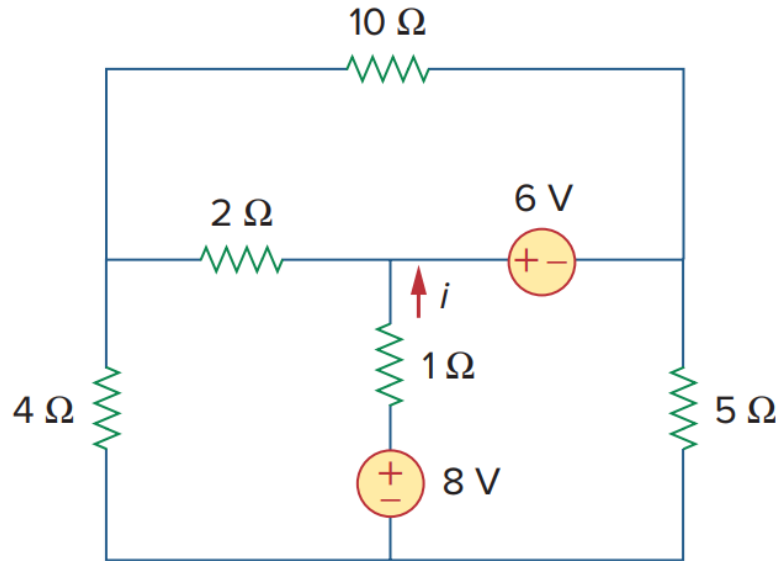
- Calculate the current through the  $10\ \Omega$  resistor using mesh analysis.



Ans:  $I_{10\Omega} = 4.4\text{ A}$

# Problem 3

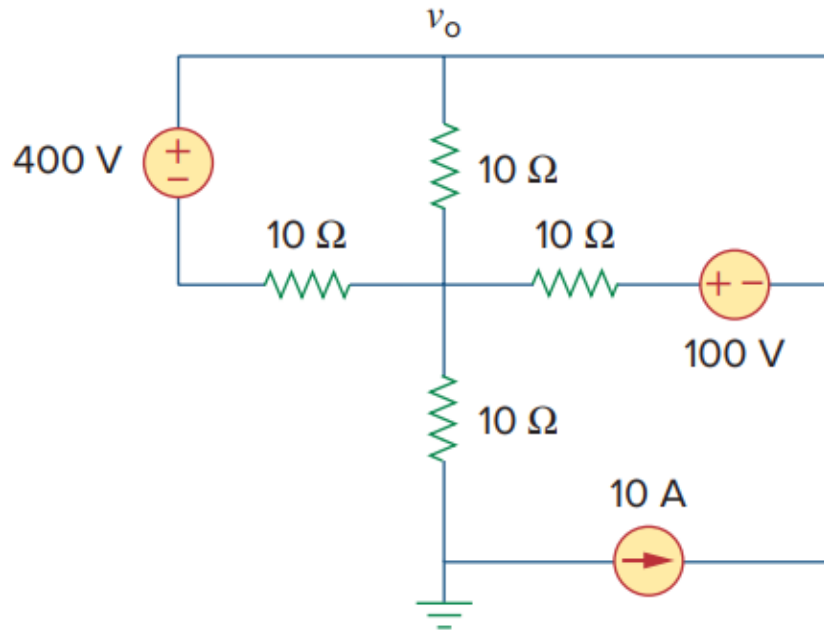
- Calculate the current  $i$  using mesh analysis.



Ans:  $i = 1.188\text{ A}$

# Problem 4

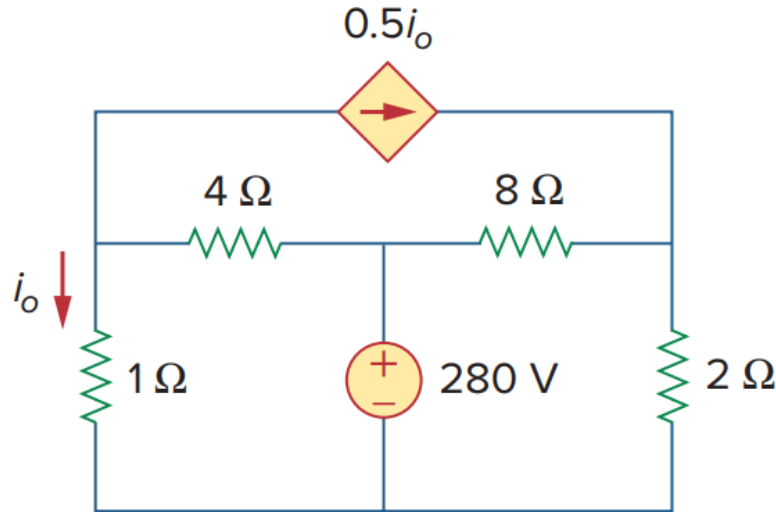
- Apply mesh analysis to find  $v_o$  in the following circuit.



Ans:  $v_o = 233.3 \text{ V}$

# Problem 5

- Find  $i_0$  using mesh analysis. What is the voltage across the  $0.5i_0$  source?

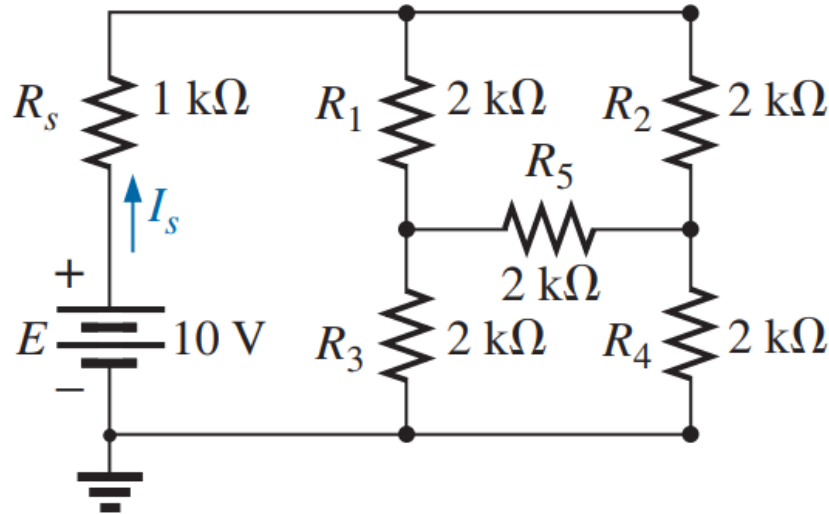


Ans:  $i_0 = 40 \text{ A}; \pm 48 \text{ V}$



# Problem 6

- Determine the current through the source resistor  $R_s$  using mesh analysis.



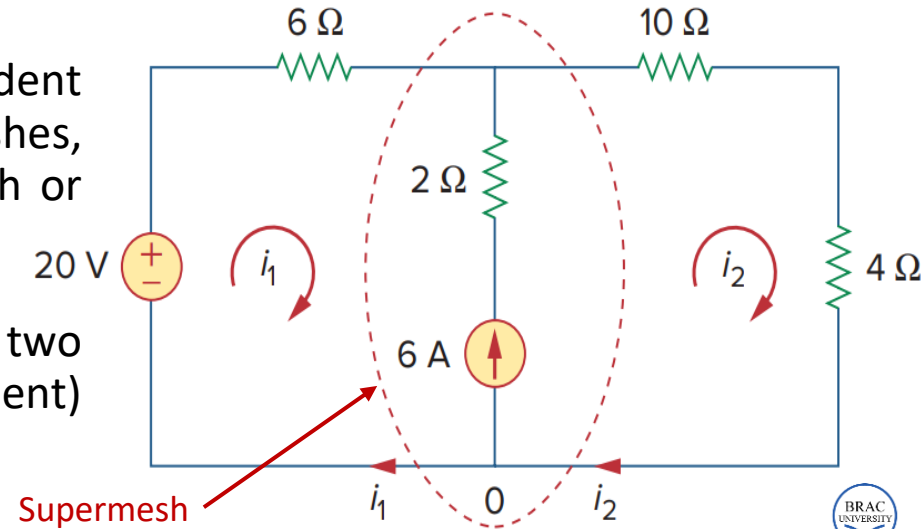
Ans:  $i_s = 3.33 \text{ mA}$

# Mesh Analysis with current source bet<sup>n</sup> loops

■ **CASE 1** When a current source (dependent or independent) exists only in one mesh, we simply set the current at that mesh equal to the current of the current source. (We have already seen this in [example 1](#)).

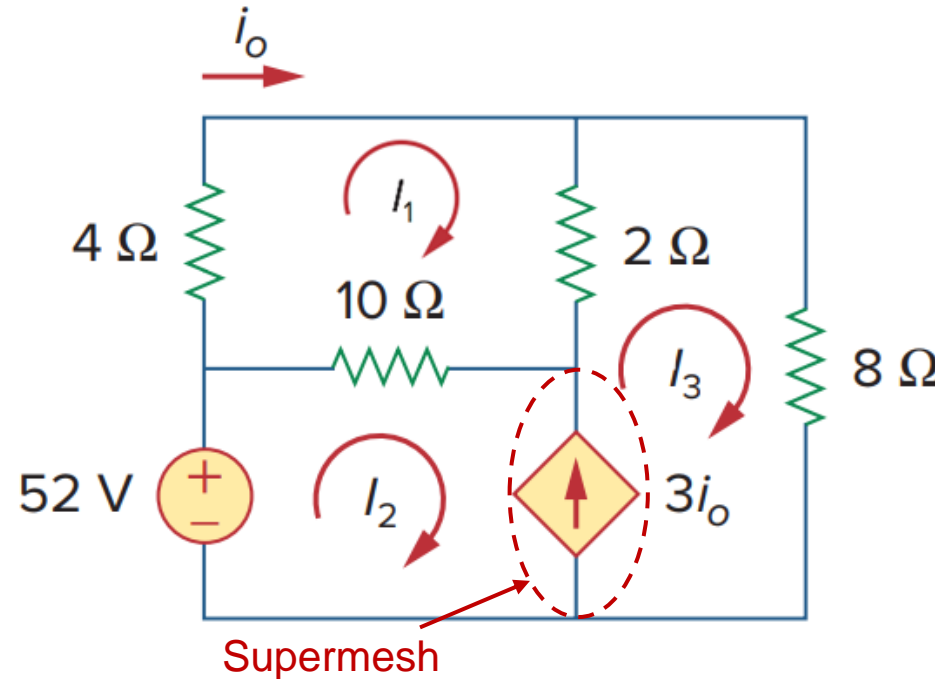
■ **CASE 2** When a current source (dependent or independent) exists between two meshes, the two meshes form a generalized mesh or supermesh.

In other words, a *supermesh* results when two meshes have a (dependent or independent) current source in common.



# Example 2: step 1

- Find  $i_o$  using mesh analysis. Also, calculate the voltage across the  $3i_o$  source.



Step 1: Identify all the meshes and assign mesh variables to each of the meshes.

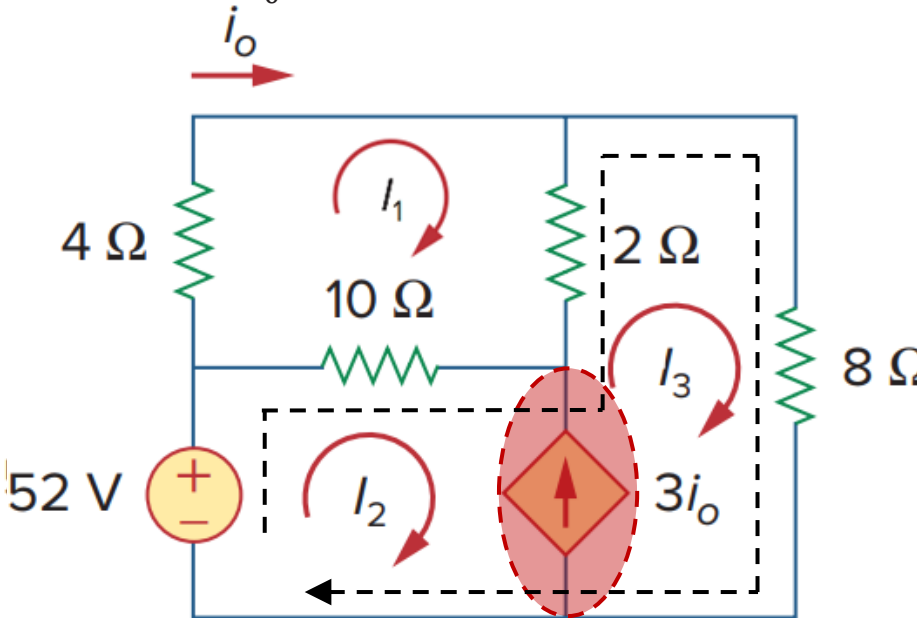
Check for supermeshes. Check if a current source (dependent or independent) is connected between two meshes. There can be multiple supermeshes in a circuit.

In this circuit, the  $3i_o$  current source forms a supermesh between meshes 2 and 3.

We need to handle such conditions differently because there is no way to know the voltage across a current source in advance.

# Example 2: step 2

- Find  $i_0$  using mesh analysis. Also, calculate the voltage across the  $3i_0$  source.



Step 2: Apply KVL to each of the meshes.

KVL to the mesh 1,

$$4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$$

$$\Rightarrow 16i_1 - 10i_2 - 2i_3 = 0$$

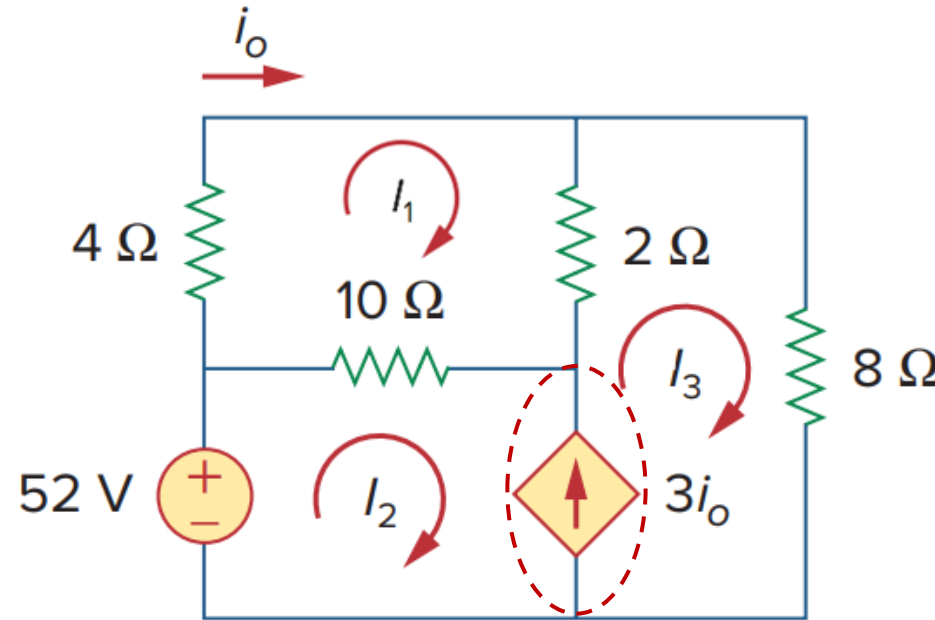
Next, ignore the current source that forms the supermesh and apply KVL to the corresponding meshes together. Careful with the current notations. Applying KVL to the supermesh along the black dotted line shown in the figure,

$$-52 + 10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 0$$

$$\Rightarrow 12i_1 - 10i_2 - 10i_3 = -52$$

# Example 2: step 2 (contd ... 2)

- Find  $i_0$  using mesh analysis. Also, calculate the voltage across the  $3i_0$  source.



We have 2 equations, 3 variables, and no remaining mesh for KVL.

The 3<sup>rd</sup> equation required, can be found by applying KCL to the supermesh.

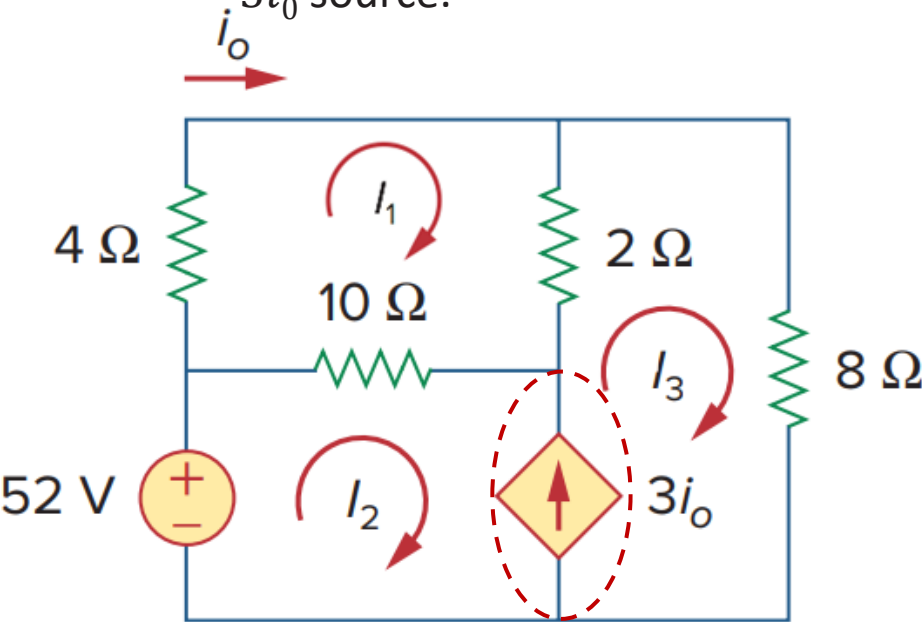
$$i_3 - i_2 = 3i_0$$

Now replace  $i_0$  in terms of the mesh currents. It can be seen from the figure that,  $i_0 = i_1$ . Substituting,

$$\begin{aligned} i_3 - i_2 &= 3i_1 \\ \Rightarrow 3i_1 + i_2 - i_3 &= 0 \quad \text{----- (iii)} \end{aligned}$$

# Example 2: step 3

- Find  $i_0$  using mesh analysis. Also, calculate the voltage across the  $3i_0$  source.



We have derived the three equations,

$$16i_1 - 10i_2 - 2i_3 = 0$$

$$12i_1 - 10i_2 - 10i_3 = -52$$

$$3i_1 + i_2 - i_3 = 0$$

Solving ... ..,

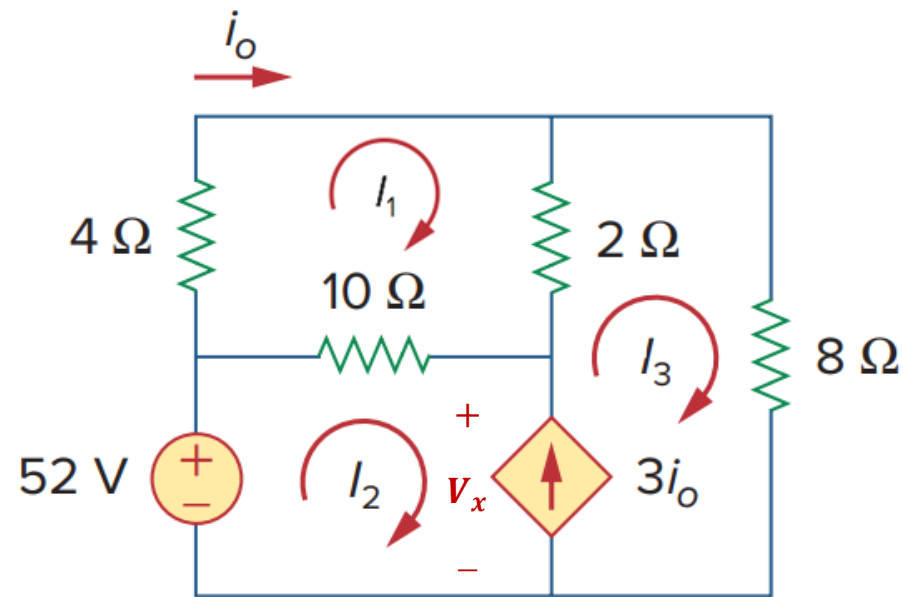
$$i_1 = 1.5 \text{ A}; \quad i_2 = 1.25 \text{ A}; \quad i_3 = 5.75 \text{ A}$$

So,  $i_0 = i_1 = 1.5 \text{ A}$

To calculate the voltage across the  $3i_0$  dependent source, we have to apply KVL to either loop 2 or loop 3.

# Example 2: voltage across $3i_0$

- Find  $i_0$  using mesh analysis. Also, calculate the voltage across the  $3i_0$  source.



Let the voltage across the  $3i_0$  source is  $V_x$  as indicated in the figure.

Applying KVL to the loop 2,

$$-52 + 10(i_2 - i_1) + V_x = 0$$

$$\Rightarrow V_x = 52 - 10(1.25 - 1.5) = 54.5 \text{ V}$$

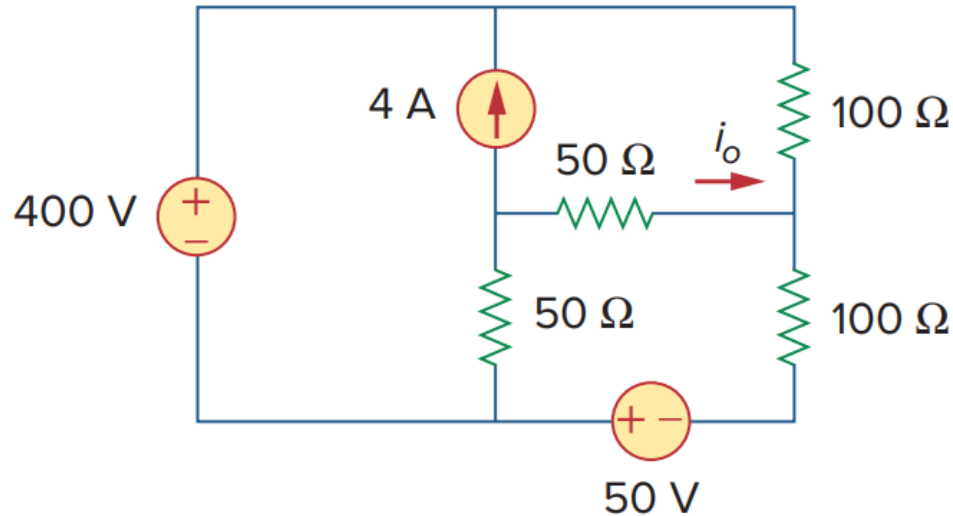
As observed by the polarities of voltage and current, the dependent source is supplying power.

$$p = +vi = 54.5 \times 3i_0 = 54.5 \times 3i_1$$

$$\Rightarrow p = 54.5 \times 3 \times 1.5 = 245.25 \text{ W}$$

# Problem 7

- Find  $i_0$  using mesh analysis.

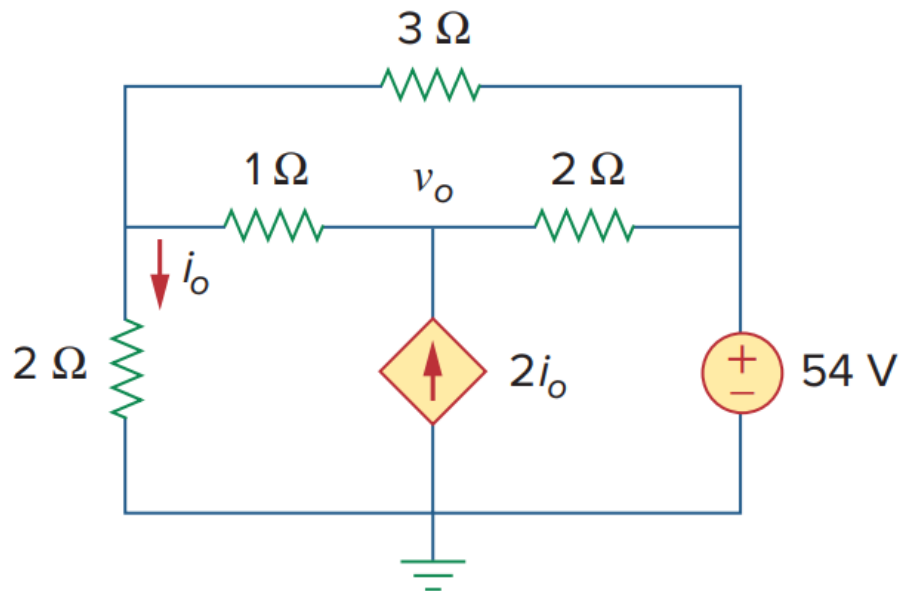


Ans:  $i_0 = -2.5 \text{ A}$



# Problem 8

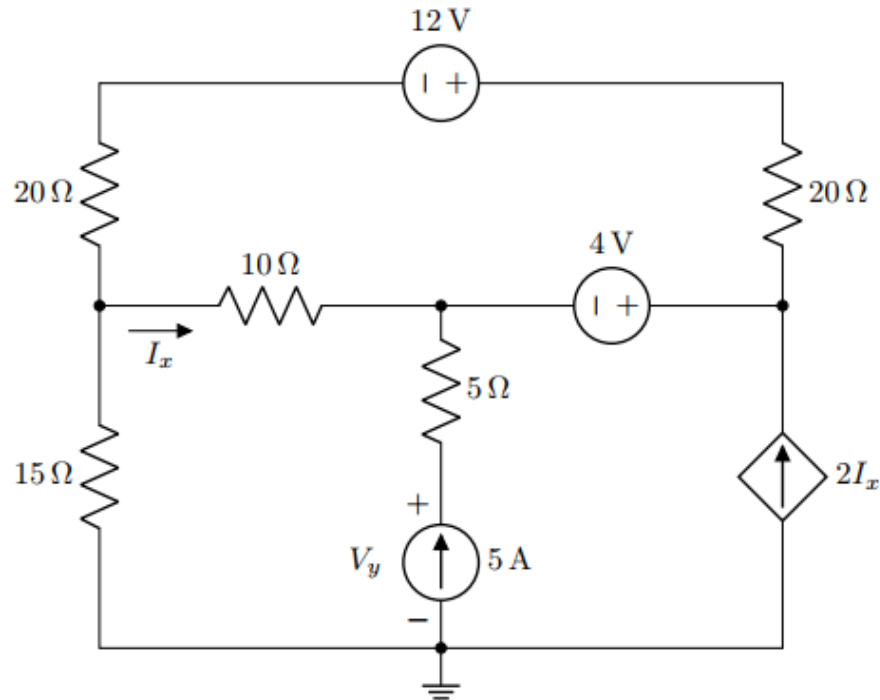
- Find  $i_0$  using mesh analysis. Determine the node voltage  $v_0$ .



Ans:  $i_0 = 36 \text{ A}$ ;  $v_0 = 114 \text{ V}$

# Problem 9

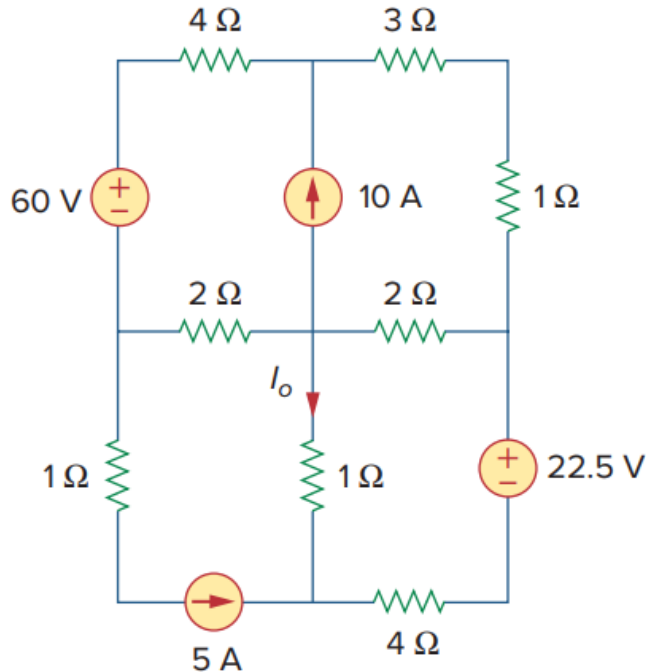
- Use Mesh Analysis to analyze the circuit. Find  $V_y$ .



Ans:  $V_y = 68 \text{ V}$

# Problem 10

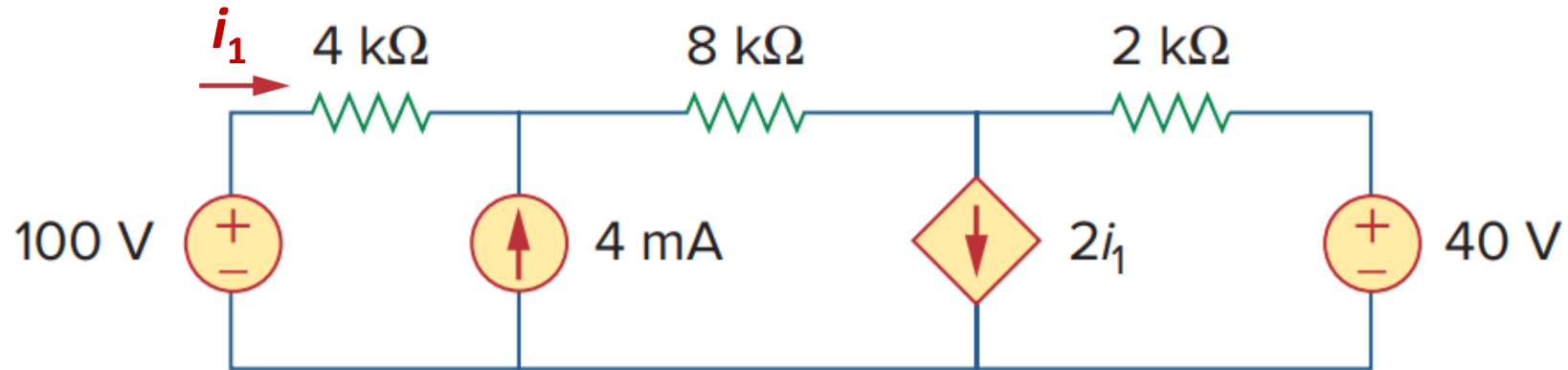
- Derive the mesh equations for the following circuit. Determine  $i_0$ .



Ans:  $i_0 = -3.62 \text{ A}$

# Problem 11

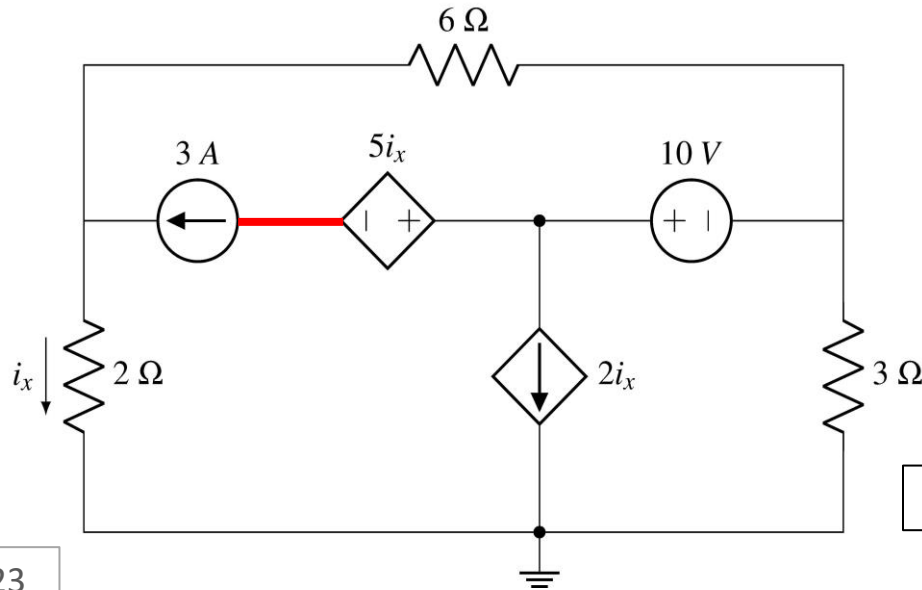
- Find the mesh currents.



Ans:  $\pm 2 \text{ mA}$ ;  $\pm 6 \text{ mA}$ ;  $\pm 2 \text{ mA}$

# Problem 12

- (i) Use nodal analysis to find  $i_x$ . Determine the voltage across the 3 A source.
- (ii) Use mesh analysis to find  $i_x$ . Determine the voltage of the **red** colored node.



Ans:  $i_x = 1.059 \text{ A}$ ;  $v_{3A} = \pm 298 \text{ V}$ ;  $v_{\text{red}} = 172 \text{ V}$

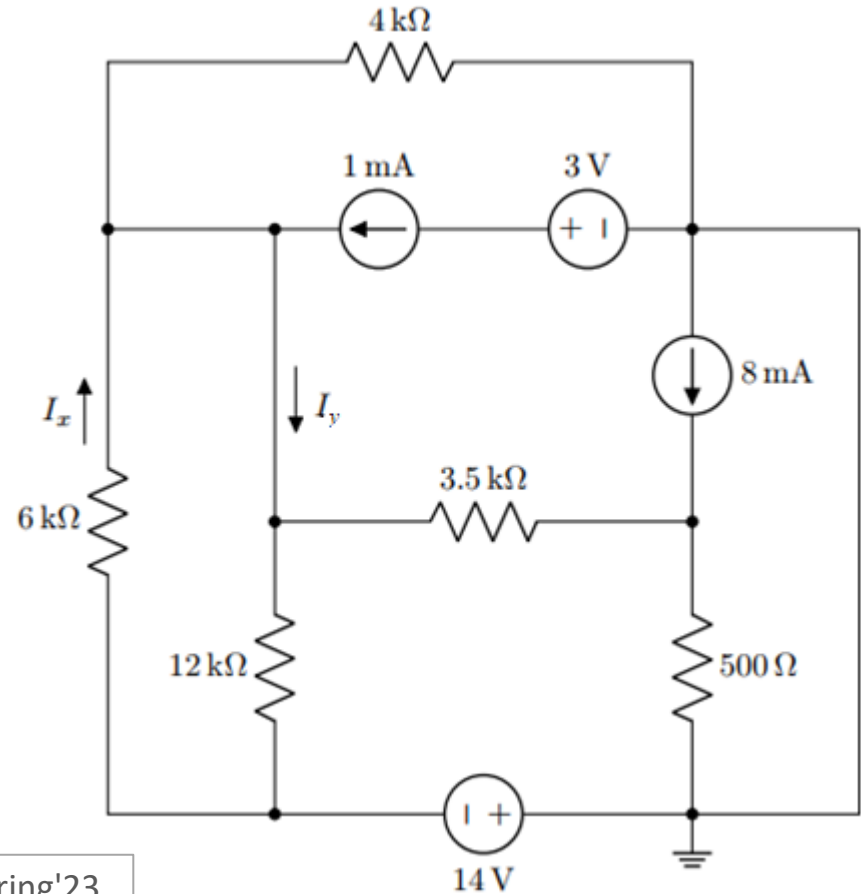
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# Problem 13

- Use mesh analysis to analyze the circuit. Find  $I_x$ .
- Determine the current  $I_y$ .
- Now repeat using Nodal analysis.
- Compare the two methods in solving this circuit.

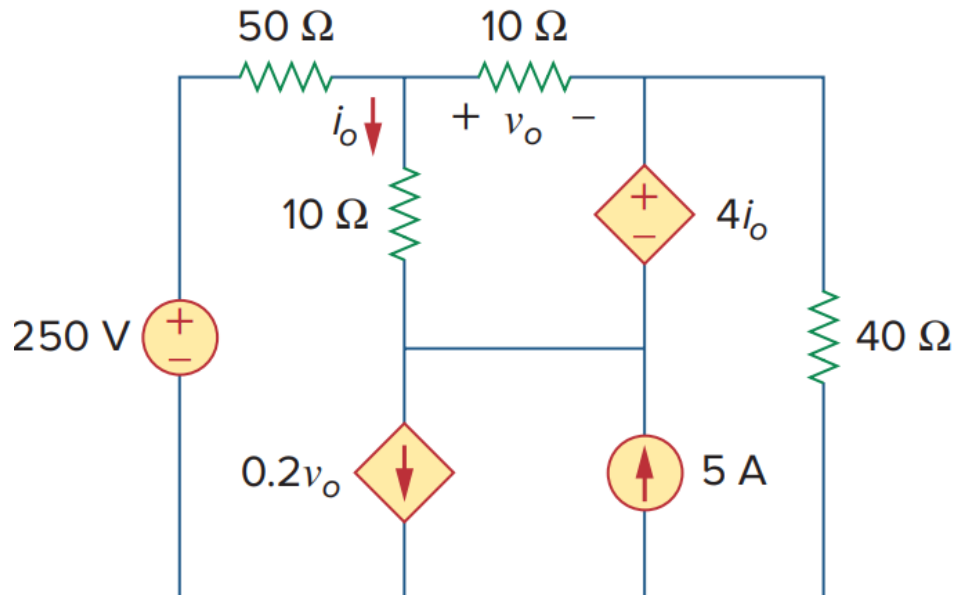
Ans:  $I_x = -2 \text{ mA}$ ;  $I_y = -0.5 \text{ mA}$

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# Problem 14

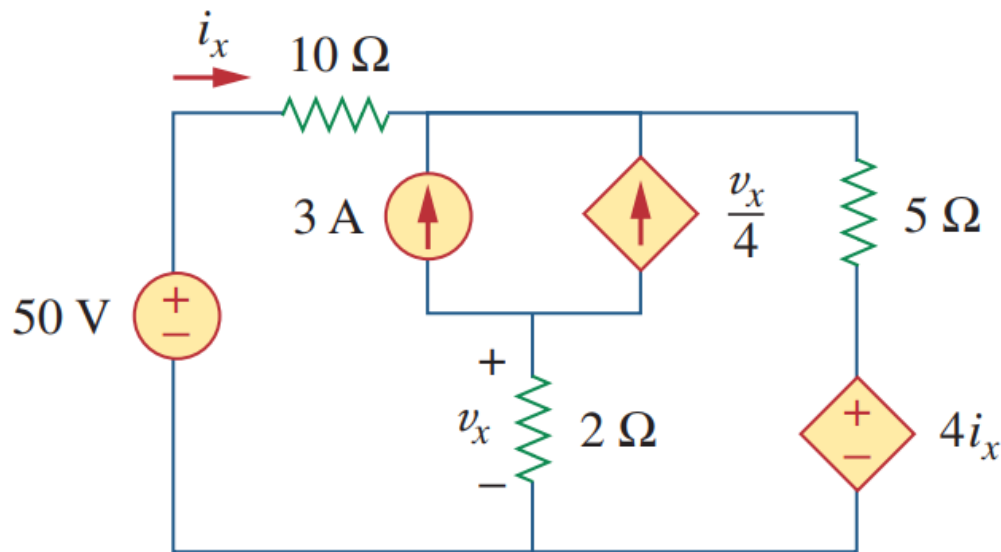
- Use mesh analysis to determine  $v_o$  and  $i_o$ . What is the voltage across the 5 A source?



Ans:  $v_o = 2.941 \text{ V}$ ;  $i_o = 0.49 \text{ A}$

# Problem 15

- Use mesh analysis to determine  $v_x$  and  $i_x$ . What is the voltage across the  $3\text{ A}$  source?



Ans:  $v_x = -4\text{ V}$ ;  $i_x = 2.105\text{ A}$



# Nodal vs Mesh Analysis

- Given a network to be analysed, how do we know which method is better or more efficient?  
*The choice of the better method is dictated by two factors:*

## ■ Nature of the network

**Mesh analysis is easier** for networks that contain many series-connected elements, voltage sources, or supermeshes

**Nodal analysis is easier** for networks with parallel connected elements, current sources, or supernodes.

A circuit with fewer nodes than meshes is better analysed using nodal analysis, and vice versa. The key is to select the method that results in the smaller number of equations.

## ■ Information required

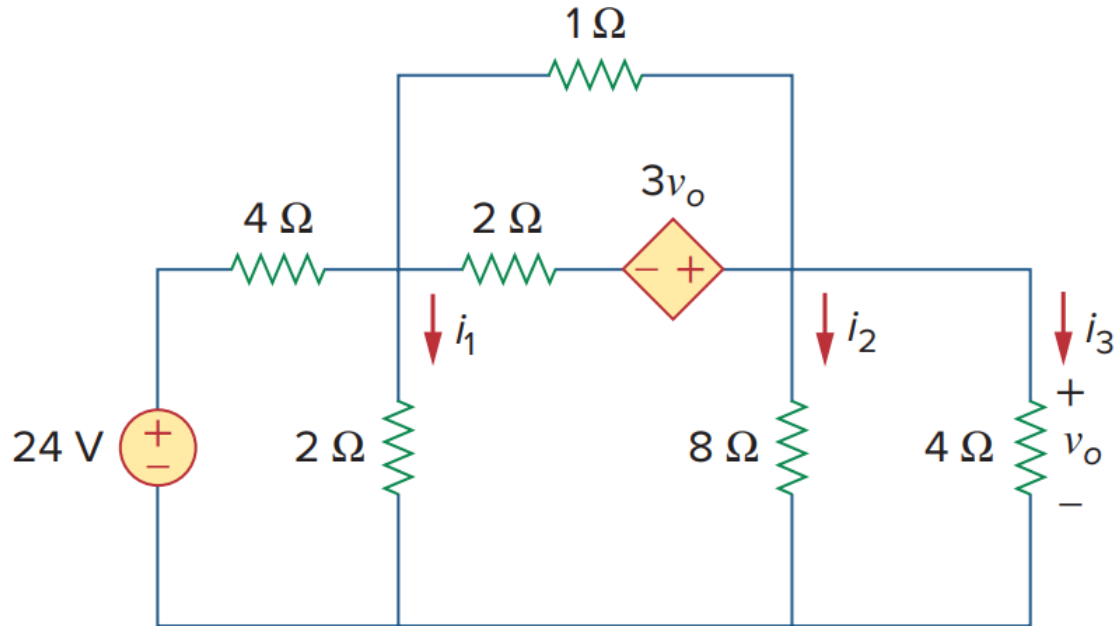
**Mesh analysis is easier** if branch or mesh currents are required

**Nodal analysis is easier** if node voltages are required

As we shall see in CSE251, mesh analysis is the only method to use in analysing transistor circuits. But mesh analysis cannot easily be used to solve an op amp circuit, because there is no direct way to obtain the voltage across the op amp itself. For nonplanar networks, nodal analysis is the only option.

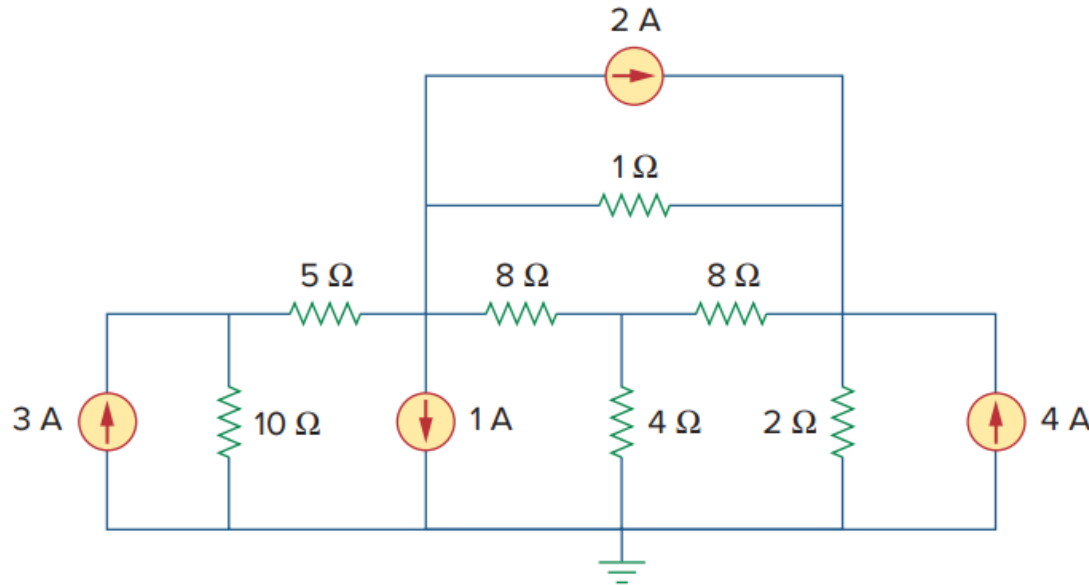
# Problem 16

- Which method, nodal or mesh, is more convenient for solving the circuit? Derive the equations that correspond to the convenient one.



# Problem 17

- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.

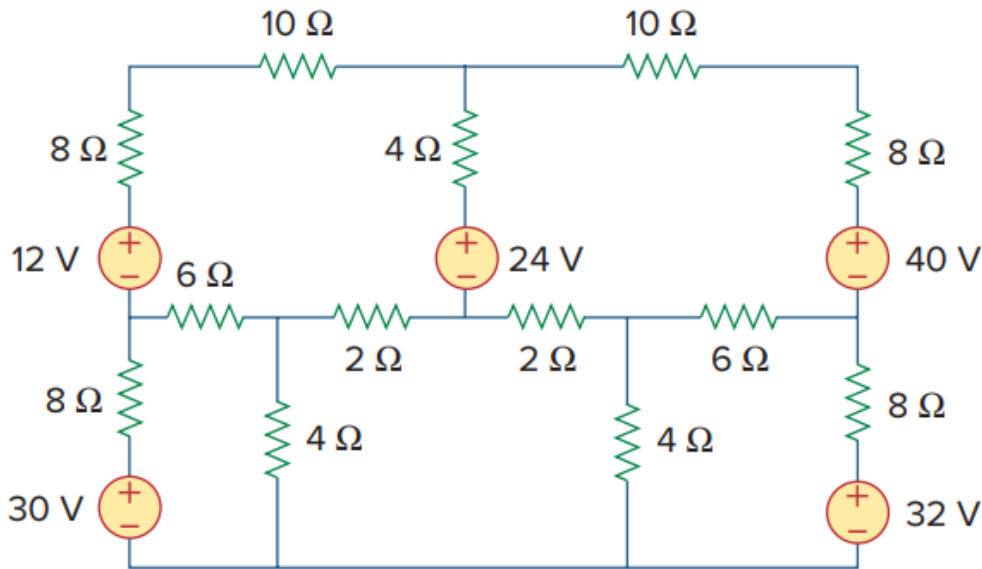


Ans:

- # of nodes = 4;
- # of meshes = 7;
- minimum # of variables for nodal analysis = 4;
- minimum # of variables for mesh analysis = 7.

# Problem 18

- Count how many nodes and meshes there are in this circuit. What is the bare minimum of variables that need to be considered for both nodal and mesh analysis? Which of these methods is the most convenient for solving the circuit? Determine the equations that correspond to the convenient one.



Ans:

- # of nodes = 14;
- # of meshes = 5;
- minimum # of variables for nodal analysis = 6;
- minimum # of variables for mesh analysis = 5.

# Practice Problems

- Additional recommended practice problems: [here](#)
- Other suggested problems from the textbook: [here](#)

Thank you for your attention

# Course Outline: broad themes

