

**BRAC UNIVERSITY**  
**Department of Computer Science and Engineering**

Examination: Mid Semester Exam  
 Duration: 1 Hour 30 Minutes

Semester: Spring 2023  
 Full Marks: 40

**CSE 221: Algorithms**

Answer the following questions.  
 Figures in the right margin indicate marks.

Name:	ID:	Section:
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- 1 a. In the primary scholarship exam in Bangladesh, four lakh ( $n=4,00,000$ ) students take part but only the top 50 students are given an award. 02  
**CO3**

**Write** the asymptotic time complexity to give the awards. Assume that each award is given in a constant time.

- b. Write the asymptotic time complexity of the following function. 04  
**CO1**

```

1. def contains_duplicates(elements):
2.     for outer in range(len(elements)):
3.         for inner in range(len(elements)):
4.             if outer == inner:
5.                 continue
6.
7.             if elements[outer] == elements[inner]:
8.                 return True
9.
10.    return False
  
```

- c. Express the following running time  $T(n)$  with an asymptotic bound. 04  
**CO3**

$$T(n) = 625T\left(\frac{n}{5}\right) + n^3$$

Any method is acceptable as long as you show calculations.

- 2 a. You are given an array containing  $N$  distinct integers in a wave-like sequence. Meaning, the numbers in the beginning are in ascending order, and after a specific position, they are in descending order. For example: [1, 3, 4, 5, 9, 6, 2, -1] 04  
**CO4**

You have to find the maximum number of this sequence. Can you devise an efficient algorithm such that the time complexity will be less than  $O(N)$ ?

- a) **Present** your solution idea as a pseudocode/ python code/ flowchart/ step-by-step instructions/ logical explanation in one-two paragraphs. 04  
 b) **Write** the time complexity of your algorithm. 01

b. In case of sorting an array in ascending order, *Bubble Sort* extends a sorted subarray at the rightmost side and, *Insertion Sort* extends a sorted subarray at the leftmost side. Your friend, Jimmy was told to find the first five largest and smallest numbers from a list of  $N$  distinct integers ( $N > 10$ ). To solve the task, he modified the Bubble sort and Insertion sort algorithm for only 5 iterations and used the rightmost and leftmost 5 numbers as the 5 largest and smallest numbers respectively. Do you support his strategy? **Explain** with logical reasons. **02**

c. While sorting a list of 10 integers in ascending order using Quick Sort, after the first partition (using the first element as a pivot), the list looks like this : [13, 11, 19, 7, 23, 37, 29, 53, 59, 41]. Which element was the pivot before partitioning? **02**  
**Explain** your answer in brief.

d. Consider the scenario in 2(c). **Show** how the list will look like after partitioning using 13 as the pivot. **01**

3 CO4 You are the coach of the renowned football team “Real Madrid”. Your team is behind one goal and there is still some time left to back in the game by scoring a goal. Benzema is your main striker and if you can pass the ball to him he will give the goal for sure. Currently the ball is in your goalkeeper’s (Courtois) hand. So, your main target should be Passing the ball in minimum steps from Courtois to Benzema as the time left is very low.

Courtois can pass the ball to Rudiger or Alaba  
Rudiger can pass the ball to Modric  
Alaba can pass the ball to Modric, Hazard or Nacho  
Modric can pass the ball to Benzema  
Hazard can pass the ball to Kroos  
Nacho can pass the ball to Vini  
Kroos can pass the ball to Benzema  
Vini can pass the ball to Benzema

Now answer the following questions.

- a) Using a suitable algorithm, **find** the minimum number of passes your team will give to reach the ball from Courtois to Benzema. **Mention** the name of the algorithm, **Show** each step of the simulation properly to find out the minimum number of passes. **07**
- b) **Find** out the players who are required for this minimum number of passes scenario. **03**

**Answer only one of 4, 4(or)**

4 Suppose the company, "Sparkling Ideas," has been new in the market for the past few years and you work as a financial analyst hired for this company.

You have been provided a list of the company’s monthly incomes over the last year. However, the incomes are mixed with positive and negative figures, which indicates the company has faced some difficulties during certain months. The company's CFO has a special skill to identify the trend in the company's income details, which helps her make informed decisions. She believes that there is a particular set of consecutive months where the company earns the most, and she wants you to find out which one(s) it is.

Your task is to find the maximum earnings from the given monthly incomes and determine in which part of the year the company earned the most.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
7	-15	9	-4	12	-8	3	-11	16	-2	6	-10

CO4

The figures are in millions of USD.

07

CO4

- Try to think of an efficient algorithm to solve this problem and provide the result in terms of the starting and ending months, along with the corresponding total income. **Show** a simulation of your algorithm.
- Calculate** the time complexity of your algorithm. Show proper mathematical logic.

03

4 or

Inspired by Karatsuba's algorithm for multiplying integers, your friend has come up with the following divide-and-conquer algorithm for squaring  $n$ -digit numbers. Pseudocode for the algorithm is given below. Your job is to fill in the details. You may assume that  $n$  is a power of two.

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**Algorithm 1: KARAT-SQUARE( $x$ )**

---

```

Input : An  $n$ -digit number  $x$ 
Output:  $x^2$ 
1 if  $n = 1$  then
2   | return  $x^2$ 
3 end
4  $a \leftarrow$  the leftmost  $n/2$  digits of  $x$ ;
5  $b \leftarrow$  the rightmost  $n/2$  digits of  $x$ ;
6  $S_1 \leftarrow$  KARAT-SQUARE( $a$ ) ;                               /*  $S_1$  equals  $a^2$  */
7  $S_2 \leftarrow$  KARAT-SQUARE( $b$ ) ;                               /*  $S_2$  equals  $b^2$  */
8  $S_3 \leftarrow$  KARAT-SQUARE( _____ ) ;                   /* fill in the blank */
9  $P \leftarrow$  _____ ;                                     /*  $P$  should equal  $2ab$  */
10 return _____  $\times 10^n + P \times 10^{n/2} +$  _____;

```

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Now carefully read the algorithm above, and answer the following questions.

CO4

- Write  $x$  in terms of  $a$  and  $b$ . Looking at lines 4 and 5 of the algorithm should help.
- Square your answer from (i). The formula  $(m + n)^2 = m^2 + 2mn + n^2$  should help.
- Fill in the blank in line 8. Your answer should involve  $a$  and  $b$  in some way. The idea is to compute a square that, along with  $a^2$  and  $b^2$ , helps you compute  $2ab$ . You may want to think about questions (iii) and (iv) at the same time.
- Fill in the blank in line 9. Your answer should involve  $S_1$ ,  $S_2$ , and  $S_3$  in such a way that  $P$  equals  $2ab$ .
- Fill in the blank in line 10.
- Write the running time of your algorithm above.
- Say you have another algorithm KARAT-SQUARE-2( $x$ ) that takes an  $n$ -digit number and squares it by recursively squaring six  $n/3$ -digit numbers, and combining them in  $O(n)$  time. Which algorithm is asymptotically faster: KARAT-SQUARE( $x$ ) or KARAT-SQUARE-2( $x$ )?

0.5

0.5

02

03

01

01

02

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- 1 a. In the primary scholarship exam in Bangladesh, two lakh ( $n=2,00,000$ ) students take part but only the top **25** students are given an award. **02**

**CO3**

**Write** the asymptotic time complexity to give the awards. Assume that each award is given in a constant time.

- b. Write the asymptotic time complexity of the following function. **04**

**CO1**

```
1. def cumulative_sum(elem):
2.     for outer in range(len(elem)):
3.         for inner in range(outer+1, len(elem)):
4.             elem[outer] = elem[outer] + elem[inner]
5.
6.     return elem
```

- c. **Express** the following running time  $T(n)$  with an asymptotic bound. **04**

**CO3**

$$T(n) = 25T\left(\frac{n}{5}\right) + n^3$$

Any method is acceptable as long as you show calculations.

- 2 a. You are given an array containing  $N$  distinct integers in a wave-like sequence. Meaning, the numbers in the beginning are in descending order, and after a specific position, they are in ascending order. For example: [9, 7, 6, 4, 2, 1, 3, 5, 8]

**CO4**

You have to find the minimum number of this sequence. Can you devise an efficient algorithm such that the time complexity will be less than  $O(N)$ ?

- c) **Present** your solution idea as a pseudocode/ python code/ flowchart/ step-by-step instructions/ logical explanation in one-two paragraphs. **04**
- d) **Write** the time complexity of your algorithm. **01**

**b.** In case of sorting an array in ascending order, *Bubble Sort* extends a sorted subarray at the rightmost side and, *Insertion Sort* extends a sorted subarray at the leftmost side. Your friend, Jimmy was told to find the first five largest and smallest numbers from a list of  $N$  distinct integers ( $N > 10$ ). To solve the task, he modified the Bubble sort and Insertion sort algorithm for only 5 iterations and used the rightmost and leftmost 5 numbers as the 5 largest and smallest numbers respectively. Do you support his strategy?  
**CO2** **Explain** with logical reasons. **02**

**c.** While sorting a list of 10 integers in ascending order using Quick Sort, after the first partition (using the first element as a pivot), the list looks like this : [31, 19, 3, 17, 23, 37, 59, 61, 71, 43]. Which element was the pivot before partitioning?  
**CO2** **Explain** your answer in brief. **02**

**d.** Consider the scenario in 2(c). **Show** how the list will look like after partitioning using **31** as the pivot.  
**CO2** **01**

**3 CO4** You are the coach of the renowned football team “FC Barcelona”. Your team is behind one goal and there is still some time left to back in the game by scoring a goal. Lewandowski is your main striker and if you can pass the ball to him he will give the goal for sure. Currently the ball is in your goalkeeper’s (Stegen) hand. So, your main target should be Passing the ball in minimum steps from Stegen to Lewandowski as the time left is very low.

Stegen can pass the ball to Alba or Roberto  
 Alba can pass the ball to Torres  
 Roberto can pass the ball to Busquets or Jong  
 Busquets can pass the ball to Torres or Gavi  
 Jong can pass the ball to Gavi  
 Gavi can pass the ball to Lewandowski  
 Torres can pass the ball to Lewandowski

Now answer the following questions.

- c)** Using a suitable algorithm, **find** the minimum number of passes your team will give to reach the ball from Stegen to Lewandowski. **Mention** the name of the algorithm, **Show** each step of the simulation properly to find out the minimum number of passes. **07**
- d)** Find out the players who are required for this minimum number of passes scenario. **03**

**Answer only one of 4, 4(or)**

**4** Suppose the company, "Sparkling Ideas," has been new in the market for the past few years and you work as a financial analyst hired for this company.

You have been provided a list of the company’s monthly incomes over the last year. However, the incomes are mixed with positive and negative figures, which indicates the company has faced some difficulties during certain months. The company's CFO has a special skill to identify the trend in the company's income details, which helps her make informed decisions. She believes that there is a particular set of consecutive months where the company earns the most, and she wants you to find out which one(s) it is.

Your task is to find the maximum earnings from the given monthly incomes and determine in which part of the year the company earned the most.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
17	-15	9	-4	12	-18	3	11	16	-2	-16	10

The figures are in millions of USD.

- CO4 c) Try to think of an efficient algorithm to solve this problem and provide the result in terms of the starting and ending months, along with the corresponding total income. **Show** a simulation of your algorithm. 07
- CO4 d) **Calculate** the time complexity of your algorithm. Show proper mathematical logic. 03

- 4 or Inspired by Karatsuba's algorithm for multiplying integers, your friend has come up with the following divide-and-conquer algorithm for squaring  $n$ -digit numbers. Pseudocode for the algorithm is given below. Your job is to fill in the details. You may assume that  $n$  is a power of two.

Algorithm 1: KARAT-SQUARE( $x$ )	
<b>Input</b> : An $n$ -digit number $x$	
<b>Output</b> : $x^2$	
1 <b>if</b> $n = 1$ <b>then</b>	
2       <b>return</b> $x^2$	
3 <b>end</b>	
4 $a \leftarrow$ the leftmost $n/2$ digits of $x$ ;	
5 $b \leftarrow$ the rightmost $n/2$ digits of $x$ ;	
6 $S_1 \leftarrow$ KARAT-SQUARE( $a$ ) ;	/* $S_1$ equals $a^2$ */
7 $S_2 \leftarrow$ KARAT-SQUARE( $b$ ) ;	/* $S_2$ equals $b^2$ */
8 $S_3 \leftarrow$ KARAT-SQUARE(_____);	/* fill in the blank */
9 $P \leftarrow$ _____;	/* $P$ should equal $2ab$ */
10 <b>return</b> _____ $\times 10^n + P \times 10^{n/2} +$ _____;	

Now carefully read the algorithm above, and answer the following questions.

- viii) Write  $x$  in terms of  $a$  and  $b$ . Looking at lines 4 and 5 of the algorithm should help. 0.5
- ix) Square your answer from (i). The formula  $(m + n)^2 = m^2 + 2mn + n^2$  should help. 0.5
- x) Fill in the blank in line 8. Your answer should involve  $a$  and  $b$  in some way. The idea is to compute a square that, along with  $a^2$  and  $b^2$ , helps you compute  $2ab$ . You may want to think about questions (iii) and (iv) at the same time. 02
- CO4 xi) Fill in the blank in line 9. Your answer should involve  $S_1$ ,  $S_2$ , and  $S_3$  in such a way that  $P$  equals  $2ab$ . 03
- xii) Fill in the blank in line 10. 01
- xiii) Write the running time of your algorithm above. 01
- xiv) Say you have another algorithm KARAT-SQUARE-2( $x$ ) that takes an  $n$ -digit number and squares it by recursively squaring six  $n/3$ -digit numbers, and combining them in  $O(n)$  time. Which algorithm is asymptotically faster: KARAT-SQUARE( $x$ ) or KARAT-SQUARE-2( $x$ )? 02