

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
 - Write all answers sequentially.
 - Start answering a question (not the part of the question) from the top of a new page.
 - Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
 - Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.
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1. Consider a function $f(x) = x^3 + x^2 - 4x - 4$.
 - (a) (3 marks) Compute the minimum number of iterations required to find the root if the machine epsilon (error bound) is 1×10^{-2} and the interval is $[-10, -1.5]$.
 - (b) (4 marks) Show 5 iterations using the Bisection Method to find the root of the above function within the interval $[-10, -1.5]$.
 - (c) (2 marks) State the exact roots of $f(x)$ and construct two different fixed point functions $g(x)$ such that $f(x) = 0$.
 - (d) (3 marks) Compute the convergence rate of each fixed point function $g(x)$ obtained in the previous part, and state which root it is converging to or diverging.
2. Consider the following function: $f(x) = xe^x - 1$.
 - (a) (3 marks) Find solution of $f(x) = 0$ up to 5 iterations using Newton's method starting with $x_0 = 1.5$. Keep up to four significant figures.
 - (b) (4 marks) Consider the fixed point function, $g(x) = \frac{2x+1}{\sqrt{x+1}}$. Show that to be super-linearly convergent, the root must satisfy $x^* = \frac{-3}{2}$.
3.
 - (a) (3 marks) Consider the function $f(x) = x^2 - x + 1$ and starting point $x_0 = 0$. Show that the sequence using Newton's method x_1, x_2, \dots fails to approach a root of $f(x)$.
 - (b) (4 marks) Consider the function $f(x) = \cos(2x) - \sin x$. Compute the solution of the function, such that $f(x) = 0$, using Newton's method with Aitken's acceleration and starting point, $x_0 = 0$. Consider up to five decimal places. [Error bound is 1×10^{-3}]