BITMAP INDEXES - CSE510

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Chee-Yong Chan and Yannis E. Ioannidis. 1998. Bitmap index design and evaluation. SIGMOD Rec. 27, 2 (June 1998), 355-366. https://doi.org/10.1145/276305.276336

Bitmap Index – Equality based

$\pi_A(R)$	B^8	B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
3	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0
2	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	1
7	0	1	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0

- Integer
- Domain: 0 to 8

Advantage: highly compressible!

Bitmap Index – Equality based

$\pi_A(R)$	B^8	B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
3	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	1	0
2	0	0	0	0	0	0	1	0	0
8	1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0
2	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	1
7	0	1	0	0	0	0	0	0	0
5	0	0	0	$\boxed{1}$	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
4	0	0	0	0	1	0	0	0	0

$B^5 = 1$

Equality search (A = 5)

Bitmap Index – Equality based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B^8	B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0

$$(B^5 = 1) \vee (B^7 = 1)$$

bitwise-or

Equality search (A = 5) or (A = 7)

Bitmap Index – Equality based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B^8	B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0

bitwise-or

$$(A = 5) \vee (A = 6) \vee (A = 7) \vee (A = 8)$$

Range search (A >= 5)

Bitmap Index – Range based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0

Bitmap Index – Range based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0

$$(B^5 = 1) \wedge (B^4 = 0)$$
$$\equiv \quad (B^5 = 1) \wedge (\neg B^4 = 1)$$

Equality search (A = 5)

Bitmap Index – Range based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
1	1	1	1	1	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	0
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0

$$(B^4 = 0) \quad \equiv \quad (\neg B^4 = 1)$$

Range search (A >= 5)

Bitmap Index – Equality and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$\xrightarrow{1\times3+0}$	0	1	0	0	0	1
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
1	$\xrightarrow{0\times3+1}$	0	0	1	0	1	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
8	$\xrightarrow{2\times3+2}$	1	0	0	1	0	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
0	$\xrightarrow{0\times3+0}$	0	0	1	0	0	1
7	$\xrightarrow{2\times3+1}$	1	0	0	0	1	0
5	$\xrightarrow{1\times3+2}$	0	$\boxed{1}$	0	$\boxed{1}$	0	0
6	$\xrightarrow{2\times3+0}$	1	0	0	0	0	1
4	$\xrightarrow{1\times3+1}$	0	1	0	0	1	0

Advantage: Further reduced space!

Bitmap Index – Equality and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$1\times3+0$	0	1	0	0	0	1
2	$0\times3+2$	0	0	1	1	0	0
1	$0\times3+1$	0	0	1	0	1	0
2	$0\times3+2$	0	0	1	1	0	0
8	$2\times3+2$	1	0	0	1	0	0
2	$0\times3+2$	0	0	1	1	0	0
2	$0\times3+2$	0	0	1	1	0	0
0	$0\times3+0$	0	0	1	0	0	1
7	$2\times3+1$	1	0	0	0	1	0
5	$1\times3+2$	0	1	0	1	0	0
6	$2\times3+0$	1	0	0	0	0	1
4	$1\times3+1$	0	1	0	0	1	0

$$\left((B_2^1 = 1) \wedge (B_1^2 = 1)\right)$$

Equality search (A = 5)

Bitmap Index – Equality and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$1\times3+0$	0	1	0	0	0	1
2	$0\times3+2$	0	0	1	1	0	0
1	$0\times3+1$	0	0	1	0	1	0
2	$0\times3+2$	0	0	1	1	0	0
8	$2\times3+2$	1	0	0	1	0	0
2	$0\times3+2$	0	0	1	1	0	0
2	$0\times3+2$	0	0	1	1	0	0
0	$0\times3+0$	0	0	1	0	0	1
7	$2\times3+1$	1	0	0	0	1	0
5	$1\times3+2$	0	1	0	1	0	0
6	$2\times3+0$	1	0	0	0	0	1
4	$1\times3+1$	0	1	0	0	1	0

$$\underbrace{\left((B_2^1=1)\wedge(B_1^2=1)\right)}_{5}\vee\underbrace{\left((B_2^2=1)\wedge(B_1^0=1)\right)}_{6}\vee\underbrace{\left((B_2^2=1)\wedge(B_1^1=1)\right)}_{7}\vee\underbrace{\left((B_2^2=1)\wedge(B_1^2=1)\right)}_{8}$$

$$(A=5)\vee(A=6)\vee(A=7)\vee(A=8)$$

Range search (A >= 5)

Bitmap Index – Equality and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$\xrightarrow{1\times3+0}$	0	1	0	0	0	1
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
1	$\xrightarrow{0\times3+1}$	0	0	1	0	1	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
8	$\xrightarrow{2\times3+2}$	1	0	0	1	0	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
2	$\xrightarrow{0\times3+2}$	0	0	1	1	0	0
0	$\xrightarrow{0\times3+0}$	0	0	1	0	0	1
7	$\xrightarrow{2\times3+1}$	1	0	0	0	1	0
5	$\xrightarrow{1\times3+2}$	0	1	0	1	0	0
6	$\xrightarrow{2\times3+0}$	1	0	0	0	0	1
4	$\xrightarrow{1\times3+1}$	0	1	0	0	1	0

$$\left((B_2^1 = 1) \wedge (B_1^2 = 1)\right) \vee ((B_2^2 = 1)$$

$$(A = 5) \vee (A = 6) \vee (A = 7) \vee (A = 8)$$

Range search (A >= 5)

Bitmap Index – Range and Component based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

Bitmap Index – Range ~~and Component~~ based

$\pi_A(R)$	B^7	B^6	B^5	B^4	B^3	B^2	B^1	B^0
3	1	1	1	1	1	0	0	0
2	1	1	1	1	1	1	0	0
1	1	1	1	1	1	1	1	0
2	1	1	1	1	1	1	0	0
8	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	0	0
2	1	1	1	1	1	1	0	0
0	1	1	1	1	1	1	1	1
7	1	0	0	0	0	0	0	0
5	1	1	1	0	0	0	0	0
6	1	1	0	0	0	0	0	0
4	1	1	1	1	0	0	0	0

Bitmap Index – ~~Range and~~ Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$1 \times 3 + 0$	0	1	0	0	0	1
2	$0 \times 3 + 2$	0	0	1	1	0	0
1	$0 \times 3 + 1$	0	0	1	0	1	0
2	$0 \times 3 + 2$	0	0	1	1	0	0
8	$2 \times 3 + 2$	1	0	0	1	0	0
2	$0 \times 3 + 2$	0	0	1	1	0	0
2	$0 \times 3 + 2$	0	0	1	1	0	0
0	$0 \times 3 + 0$	0	0	1	0	0	1
7	$2 \times 3 + 1$	1	0	0	0	1	0
5	$1 \times 3 + 2$	0	$\boxed{1}$	0	$\boxed{1}$	0	0
6	$2 \times 3 + 0$	1	0	0	0	0	1
4	$1 \times 3 + 1$	0	1	0	0	1	0

Bitmap Index – Range and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$\xrightarrow{1\times3+0}$	1	1	0	1	1	1
2	$\xrightarrow{0\times3+2}$	1	1	1	1	0	0
1	$\xrightarrow{0\times3+1}$	1	1	1	1	1	0
2	$\xrightarrow{0\times3+2}$	1	1	1	1	0	0
8	$\xrightarrow{2\times3+2}$	1	0	0	1	0	0
2	$\xrightarrow{0\times3+2}$	1	1	1	1	0	0
2	$\xrightarrow{0\times3+2}$	1	1	1	1	0	0
0	$\xrightarrow{0\times3+0}$	1	1	1	1	1	1
7	$\xrightarrow{2\times3+1}$	1	0	0	1	1	0
5	$\xrightarrow{1\times3+2}$	1	1	0	1	0	0
6	$\xrightarrow{2\times3+0}$	1	0	0	1	1	1
4	$\xrightarrow{1\times3+1}$	1	1	0	1	1	0

Bitmap Index – Range and Component based

$\pi_A(R)$		B_2^2	B_2^1	B_2^0	B_1^2	B_1^1	B_1^0
3	$\xrightarrow{1\times3+0}$		1	0		1	1
2	$\xrightarrow{0\times3+2}$		1	1		0	0
1	$\xrightarrow{0\times3+1}$		1	1		1	0
2	$\xrightarrow{0\times3+2}$		1	1		0	0
8	$\xrightarrow{2\times3+2}$		0	0		0	0
2	$\xrightarrow{0\times3+2}$		1	1		0	0
2	$\xrightarrow{0\times3+2}$		1	1		0	0
0	$\xrightarrow{0\times3+0}$		1	1		1	1
7	$\xrightarrow{2\times3+1}$		0	0		1	0
5	$\xrightarrow{1\times3+2}$		1	0		0	0
6	$\xrightarrow{2\times3+0}$		0	0		1	1
4	$\xrightarrow{1\times3+1}$		1	0		1	0

Bitmap Index – Range and Component based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

B_2^1	B_2^0
1	0
1	1
1	1
1	1
0	0
1	1
1	1
1	1
0	0
1	0
0	0
1	0

B_1^1	B_1^0
1	1
0	0
1	0
0	0
0	0
0	0
0	0
1	1
1	0
0	0
1	1
1	0

Bitmap Index – Range and Component based

$\pi_A(R)$
3
2
1
2
8
2
2
0
7
5
6
4

$1 \times 3 + 2$

B_2^1	B_2^0
1	0
1	1
1	1
1	1
0	0
1	1
1	1
1	1
0	0
1	0
0	0
1	0

1

B_1^1	B_1^0
1	1
0	0
1	0
0	0
0	0
0	0
0	0
1	1
1	0
0	0
1	1
1	0

Equality search (A = 5)

Bitmap Index – Range and Component based

Evaluation Algorithms for Selection Queries Using Range-Encoded Bitmap Indexes.

Input: n is the number of components in the range-encoded index.
$< b_n, b_{n-1}, \ldots, b_1 >$ is the base of the index.
op is the predicate operator, $op \in \{<, >, \leq, \geq, =, \neq\}$.
v is the predicate value.
B_{nn} is a bitmap representing the set of records with non-null values for the indexed attribute.

Output: A bitmap representation of the set of records that satisfies the predicate "A op v".

Algorithm `RangeEval`

$B_{GT} = B_{LT} = B_0$;
$B_{EQ} = B_{nn}$;
let $v = v_n v_{n-1} \ldots v_1$;
for i = n **downto** 1 **do**
    if $(v_i > 0)$ **then**
        $B_{LT} = B_{LT} \vee (B_{EQ} \wedge B_i^{v_i - 1})$;
        if $(v_i < b_i - 1)$ **then**
            $B_{GT} = B_{GT} \vee (B_{EQ} \wedge \overline{B_i^{v_i}})$;
            $B_{EQ} = B_{EQ} \wedge (B_i^{v_i} \oplus B_i^{v_i - 1})$;
        else
            $B_{EQ} = B_{EQ} \wedge \overline{B_i^{b_i - 2}}$;
    else
        $B_{GT} = B_{GT} \vee (B_{EQ} \wedge \overline{B_i^0})$;
        $B_{EQ} = B_{EQ} \wedge B_i^0$;
$B_{NE} = \overline{B_{EQ}} \wedge B_{nn}$;
$B_{LE} = B_{LT} \vee B_{EQ}$; $B_{GE} = B_{GT} \vee B_{EQ}$;
return B_{op};

Algorithm `RangeEval-Opt`

$B = B_1$;
if $(op \in \{<, \geq\})$ **then** $v = v - 1$;
let $v = v_n v_{n-1} \ldots v_1$;
if $(op \in \{<, >, \leq, \geq\})$ **then**
    if $(v_1 < b_1 - 1)$ **then** $B = B_1^{v_1}$;
    for i = 2 **to** n **do**
        if $(v_i \neq b_i - 1)$ **then** $B = B \wedge B_i^{v_i}$;
        if $(v_i \neq 0)$ **then** $B = B \vee B_i^{v_i - 1}$;
else
    for i = 1 **to** n **do**
        if $(v_i = 0)$ **then** $B = B \wedge B_i^0$;
        else if $(v_i = b_i - 1)$ **then** $B = B \wedge \overline{B_i^{b_i - 2}}$;
        else $B = B \wedge (B_i^{v_i} \oplus B_i^{v_i - 1})$;
if $(op \in \{>, \geq, \neq\})$ **then**
    return $\overline{B} \wedge B_{nn}$;
else
    return $B \wedge B_{nn}$;