Assignment 3

Graded

Student

Sujith Potineni

Total Points

26 / 28 pts

Question 1

Read me first

0 / 0 pts

- ✔ **+ 0 pts** Correct
- **+ 0 pts** Incorrect

Question 2

Indistinguishable States

6 / 8 pts

- **+ 8 pts** Correct
- ✔ **+ 2 pts** (a) Correct
- **+ 1 pt** (b) Did not explain clearly ths situation when a measurement is applied.
- **+ 2 pts** (b) Did not explain clearly the situation when a 1-qubit unitary is applied.
- ✔ **+ 3 pts** (b) Correct
- ✔ **+ 1 pt** (c) Did not explain clearly the situation when a 1-qubit unitary is applied.
- **+ 3 pts** (c) Correct
- **+ 0 pts** Incorrect

Question 3

Hadamard Transform I

6 / 6 pts

- ✔ **+ 6 pts** Correct
- **+ 4 pts** Partly correct
- **+ 2 pts** Mostly incorrect
- **+ 0 pts** Incorrect

Question 4

Quantum circuit exercise

6 / 6 pts

✔ **+ 6 pts** Correct

+ 2 pts (a) Mostly correct

+ 3 pts (a) Correct

+ 3 pts (b) Correct

+ 2 pts (b) Did not correctly normalize the collapsed state

+ 2.5 pts (b) Did not simplify the answer

+ 1 pt (b) Computed the correct probability distribution of the readout

+ 0 pts Incorrect

Question 5

Fun with gates

8 / 8 pts

✔ **+ 8 pts** Correct

+ 4 pts Correct circuit, but justification is incorrect.

+ 0 pts Incorrect

Q1 Read me first
0 Points

- Tests show that the people who get the most out of this assignment are those who read the "read me first" like this one.
- Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.
- Acknowledgment Requirements:

1. Acknowledge, individually for every problem at the beginning of each solution, a list of all collaborators and sources consulted other than the course notes. Examples include: names of people you discussed homework with, books, other notes, Wikipedia, and other websites.
2. If you consulted any online sources, please specify the exact webpages by including their links. Omission of links or any other required citations will result in a loss of grades and be considered a failure to acknowledge appropriately.
3. If no additional sources are consulted, you must write "sources consulted: none" or equivalent.
4. **Failure to acknowledge sources will lead to an automatic 1pt penalty.**

- Late policy: In general **no late homework** will be accepted unless there is a genuine emergency backed up by official documents.
- All steps should be justified.
- Formatting and Submission Requirements:

1. Separate Solutions: Ensure that solutions for each problem are separated clearly.
2. PDF Submissions: If you are submitting a LaTeX PDF, use the "fullpage" package to set the margins to 1 inch. Do not include additional information such as the title, date, your name, the problem statement, or any rough work—only include your final solution.
3. Typed Solutions: If typing directly in the provided textbox, please use LaTeX formatting for formulas.
   Images: Rotated images will not be graded. Ensure all images are properly oriented.
4. Scanning Quality: Use proper scanning software to scan your handwritten solutions. Avoid casual photos of your work.
5. **Failure to meet these formatting and submission requirements may result in up to a 2-point penalty for each problem.**

- You are encouraged to be **type in LaTeX**. To learn how to use LaTeX, I recommend the tutorials on Overleaf. It is ok to draw diagrams by hand and insert them as pictures in your TeX files.
- For each question below, upload a PDF file and/or type in the box (see Gradescope x LaTeX tutorial). Each submission should contain (1) the acknowledgement of all collaborators and sources consulted and (2) your solution.

Q2 Indistinguishable States

8 Points

(a) Let $|\psi\rangle$ and $|\psi^\perp\rangle$ be orthonormal real qubit states. Show

$$\frac{1}{\sqrt{2}}|\psi\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|\psi^\perp\rangle \otimes |\psi^\perp\rangle$$

is precisely equal to the Bell state,

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle.$$

(b) Let $|u\rangle \in \mathbb{C}^2$ be a qubit state and let $|v\rangle = c|u\rangle$, where c is a complex number of magnitude 1 (for example, $c = -1$ or $c = i$). (In this scenario, c is called a "global phase".)

Suppose someone hands you a qubit $|\psi\rangle$ and promises you that $|\psi\rangle$ is either $|u\rangle$ or $|v\rangle$. (You know, mathematically, exactly what $|u\rangle$ and $|v\rangle$ are; but you do not know whether $|\psi\rangle$ is $|u\rangle$ or $|v\rangle$.) Show, to the best of your abilities, that there is nothing you can possibly do to tell the difference. You should at least show that applying 1-qubit unitaries and 1-qubit measurements in any combination does not help. (If you want to be even more sophisticated, show that it doesn't help even if you introduce additional qubits in known states, and then apply unitaries and measurements to this larger-dimensional system.)

(c) Suppose someone hands you a qubit $|\psi\rangle$ and promises you that they prepared it according to one of the following two scenarios:

Scenario 1: They flipped a fair coin, and set $|\psi\rangle = |0\rangle$ if the result was Heads and set $|\psi\rangle = |1\rangle$ if the result was Tails.

Scenario 2: They flipped a fair coin, and set $|\psi\rangle = |+\rangle$ if the result was Heads and set $|\psi\rangle = |-\rangle$ if the result was Tails.

Show, to the best of your abilities, that there is nothing you can possibly do to tell whether they employed Scenario 1 or Scenario 2. (Same comments as in (b) about what you should at least do, and what you can further strive to do.)

Sources consulted:

Lecture Notes;

Solution:

HW3_Q2 (1).pdf

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Q3 Hadamard Transform I

6 Points

Suppose we start with n qubits in the state $|000\cdots0\rangle$. Then, for a certain subset $S \subseteq \{1, 2, \ldots, n\}$, suppose we apply the Hadamard gate H to qubit i for each $i \in S$. Describe the resulting state in the most succinct/compelling way that you can. You might want to introduce the "indicator-string" $y \in \{0,1\}^n$ for the set S, and/or use the word "XOR", in your description.

Sources consulted:

Lecture Notes;

Solution:

HW3_Q3.pdf

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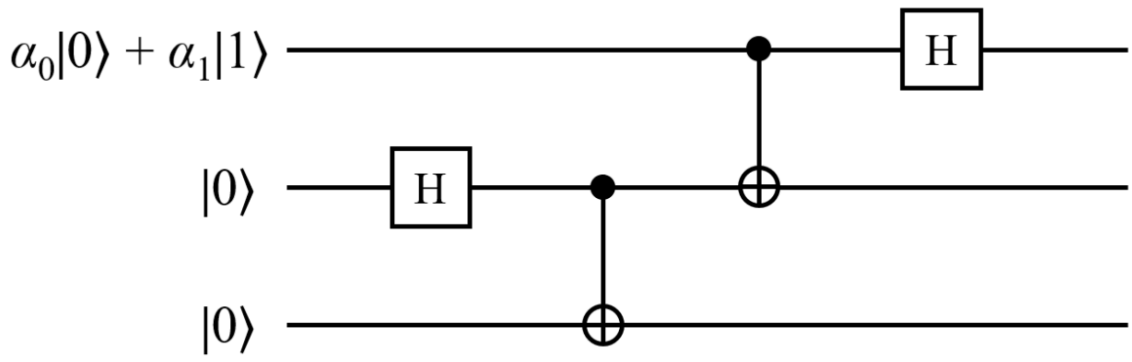
Q4 Quantum circuit exercise
6 Points

Consider the following quantum circuit operating on 3 qbits:



(a) Determine with proof the state of the three qbits at the end of the circuit's operation.

(b) Suppose the top two qbits are measured (after the last Hadamard gate). Determine the probabilities of the possible outcomes, and what state the third qbit collapses to in each of the four cases.

Convention: By default, all the measurements are in the standard basis.

Sources consulted:

Lecture Notes

Solution:

HW3_Q4.pdf

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Q5 Fun with gates

8 Points

The following question concerns a 2-qubit circuit. We designate the 2 qubits “A” and “B”. Suppose that (for some physical reason) you are able to build a device that effects the CNOT operation with the A qubit as the ‘control’ and the B qubit as the ‘target’; yet, you aren’t able to build a device that does the CNOT the other way around. Show how to nevertheless implement a CNOT with the A qubit as the ‘target’ and the B qubit as the ‘control’, assuming you can also build and use Hadamard gates.

Sources consulted:

Lecture Notes;
https://en.wikipedia.org/wiki/Controlled_NOT_gate

Solution:

HW3_Q5.pdf

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