# Last time

## EPR pair measured in the standard basis

- Alice and Bob share EPR pair: $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
- A partial measurement conducted by Alice would produce the following readouts of her qubit: $|0\rangle$ with probability 1/2, $|1\rangle$ with probability 1/2.
- If, for example, the readout is $|0\rangle$, Alice instantly knows that Bob will read $|0\rangle$ when he measures his qubit.
- Question: Did Alice just transfer information to Bob faster than the speed of light?

# Last time

## EPR pair measured in the $|+\rangle, |-\rangle$ basis

- Alice and Bob share EPR pair: $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$
- A partial measurement conducted by Alice would produce the following readouts of her qubit: $|+\rangle$ with probability 1/2, $|-\rangle$ with probability 1/2.
- If, for example, the readout is $|+\rangle$, Alice instantly knows that Bob will read $|+\rangle$ when he measures his qubit.
- Check: $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = \frac{1}{\sqrt{2}}|u\rangle \otimes |u\rangle + \frac{1}{\sqrt{2}}|v\rangle \otimes |v\rangle$, for every orthonormal basis $|u\rangle, |v\rangle$ in $\mathbb{R}^2$

# Discriminating mixed states

## Can Alice communicate 1 bit of info by choosing a basis?

- Choice 1: Alice measures in $|0\rangle, |1\rangle$. Bob's qubit is $|0\rangle$ with probability 1/2 and $|1\rangle$ with probability of 1/2.
- Choice 2: Alice measures in $|+\rangle, |-\rangle$. Bob's qubit is $|+\rangle$ with probability 1/2 and $|-\rangle$ with probability of 1/2.
- We can describe the two scenarios as mixed states:
  - ρ1 = ?
  - ρ2 = ?

# Discriminating mixed states

## Can Bob discriminate between mixed states?

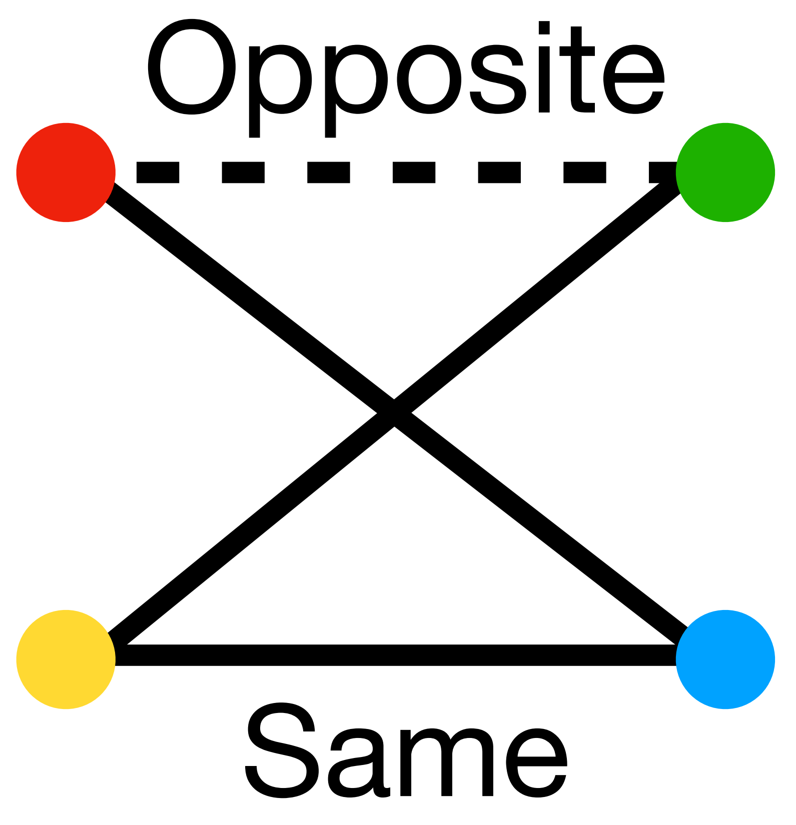
- $\rho_1 : |0\rangle$ with probability 1/2 and $|1\rangle$ with probability 1/2
- $\rho_2 : |+\rangle$ with probability 1/2 and $|-\rangle$ with probability 1/2
- What if Bob measures in $|0\rangle, |1\rangle$?

In fact, the two mixed states are indistinguishable by *any* physical experiments because their “density matrices” are equal.

# CHSH game

## Clauser, Horne, Shimony, and Holt

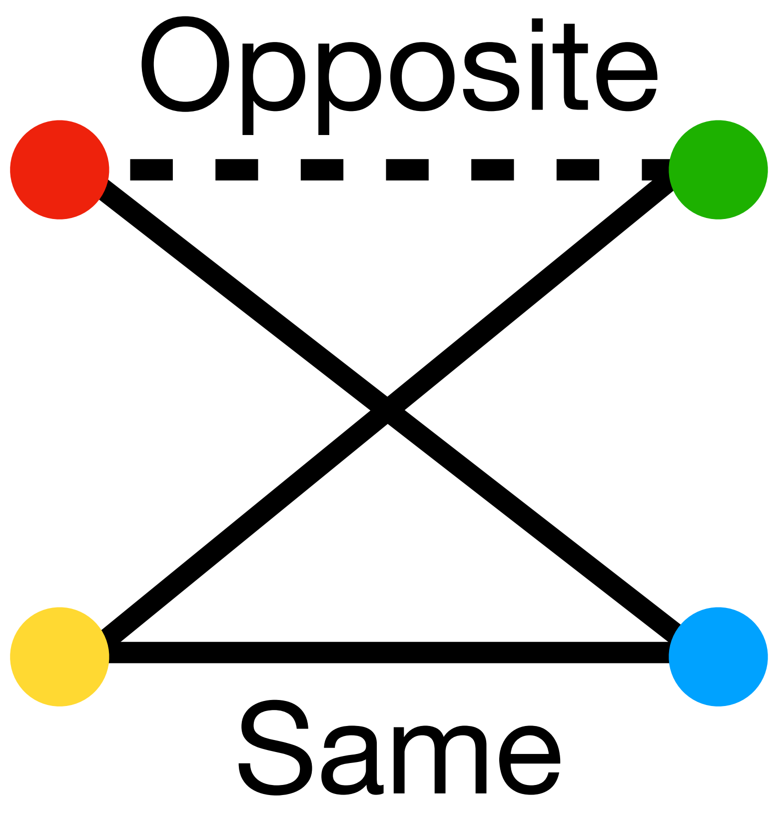
- Two referees A and B have synced watches
- Referee A goes to Alice on Mars and Referee B goes to Bob on Earth
- At the same time
  - Referee A flips a Red/Yellow coin, and challenges Alice with a color.
  - Referee B flips a Blue/Green coin, and challenges Bob with a color.
- Within 1 minute,
  - Alice answers with either 0 or 1 to Referee A
  - Bob answers with either 0 or 1 to Referee B
- Alice and Bob win the game if their answers following the conditions:
  - In the case where coins are Red and Green, Alice and Bob win if they offer opposite answers
  - In all other cases, Alice and Bob win if they offer the same answers



# CHSH game

## Classical strategies

- Suggestions?
  - Deterministic?
  - Random?

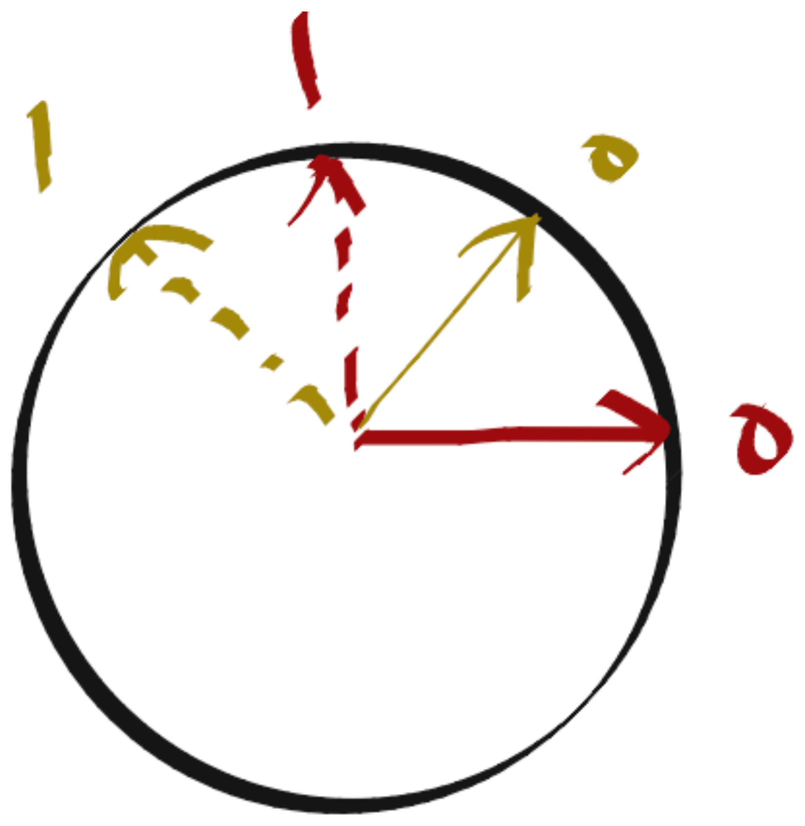



- Shared randomness: In an attempt to increase their winning probability, Alice and Bob together generate a random *k*-digit binary number *n*.

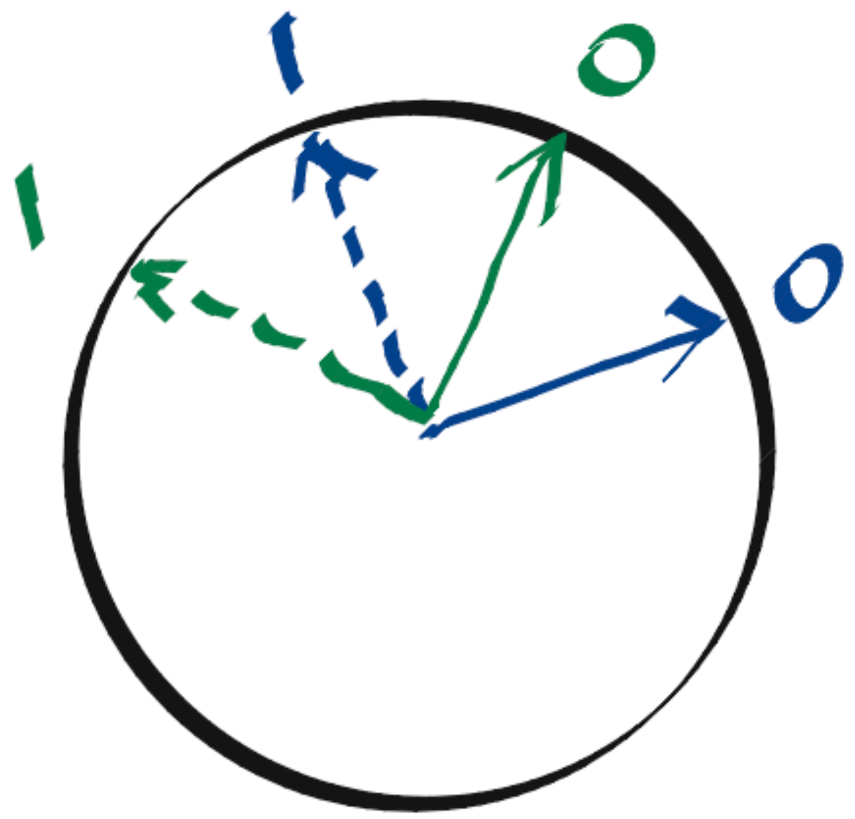
# Quantum strategy

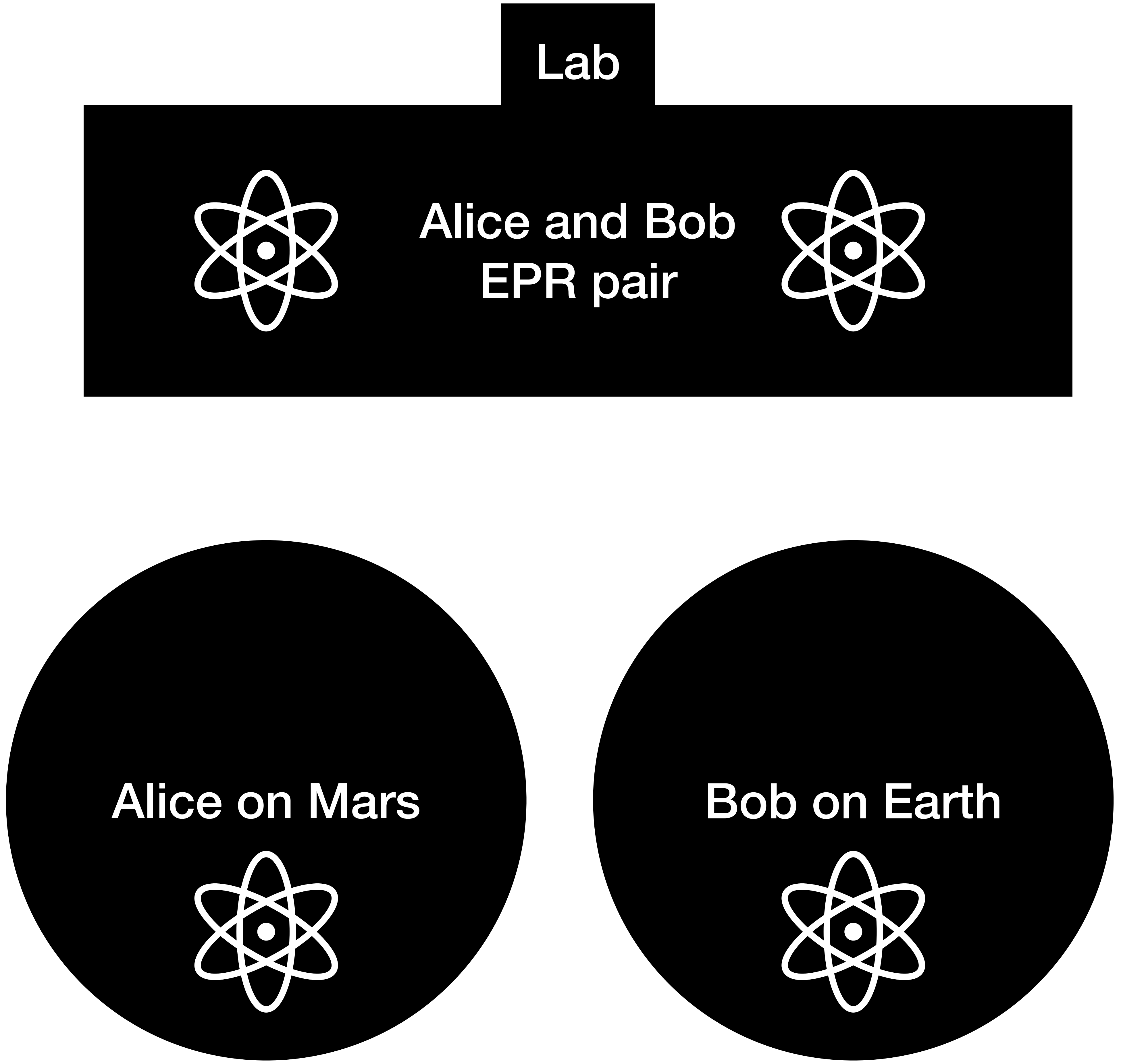
## Using entanglement

- Alice and Bob share an EPR pair
- Alice measures in Red/Yellow basis, and answers with the readout

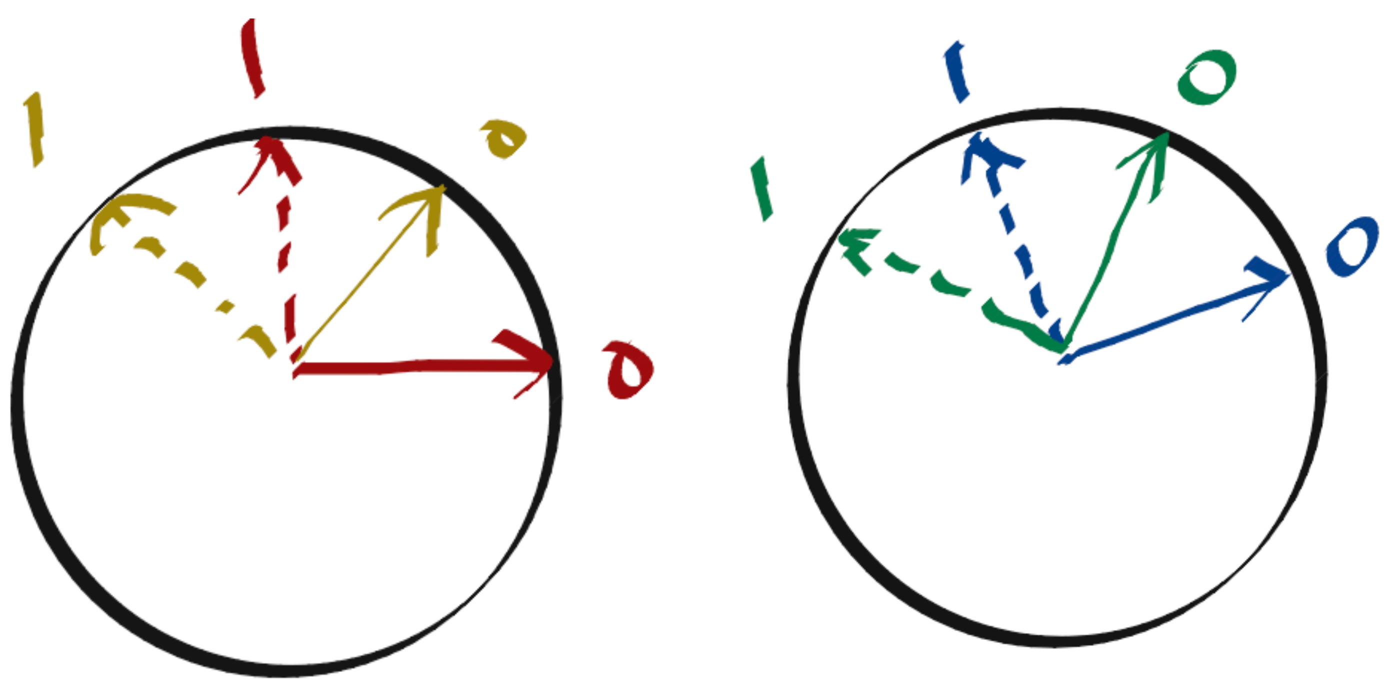



- Bob measures in Blue/Green basis, and answers with the readout



- Alice and Bob share an EPR pair
- Alice measures in Red/Yellow basis, and answers with the readout
- Bob measures in Blue/Green basis, and answers with the readout




- Analysis:
  - Case 1: Challenges are Red + Blue
  - Case 2: Challenges are Red + Green
  - Case 3: Challenges are Yellow + Blue
  - Case 4: Challenges are Yellow + Green

# Summary of various strategies

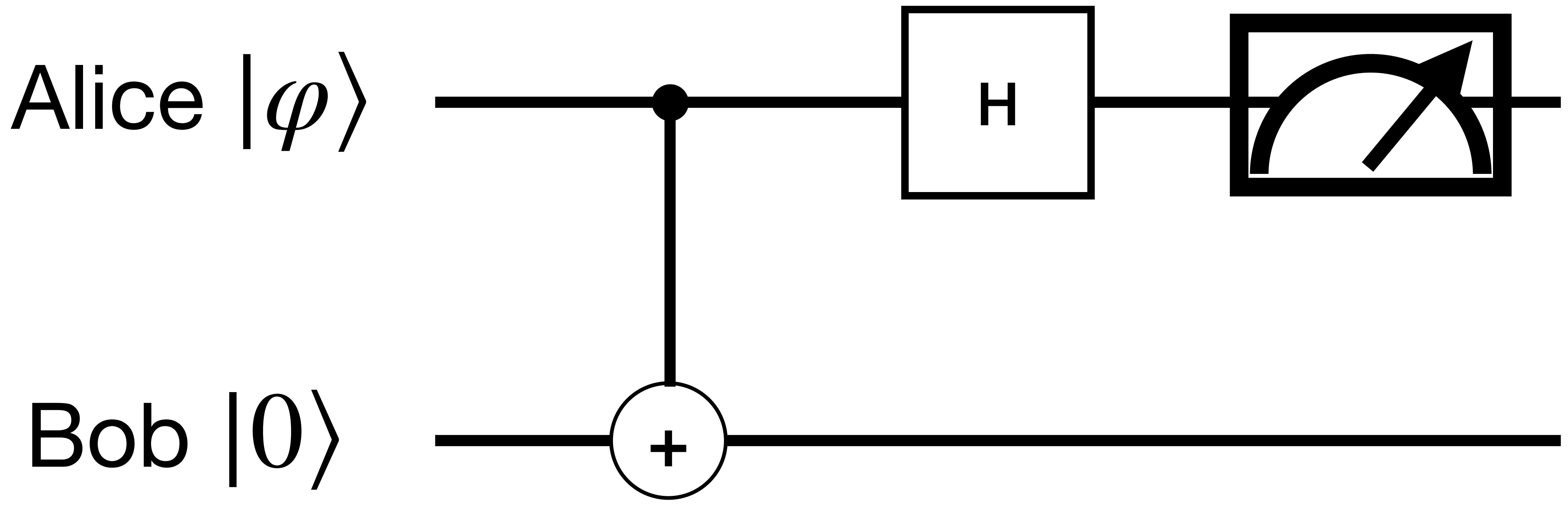
## Classical vs quantum

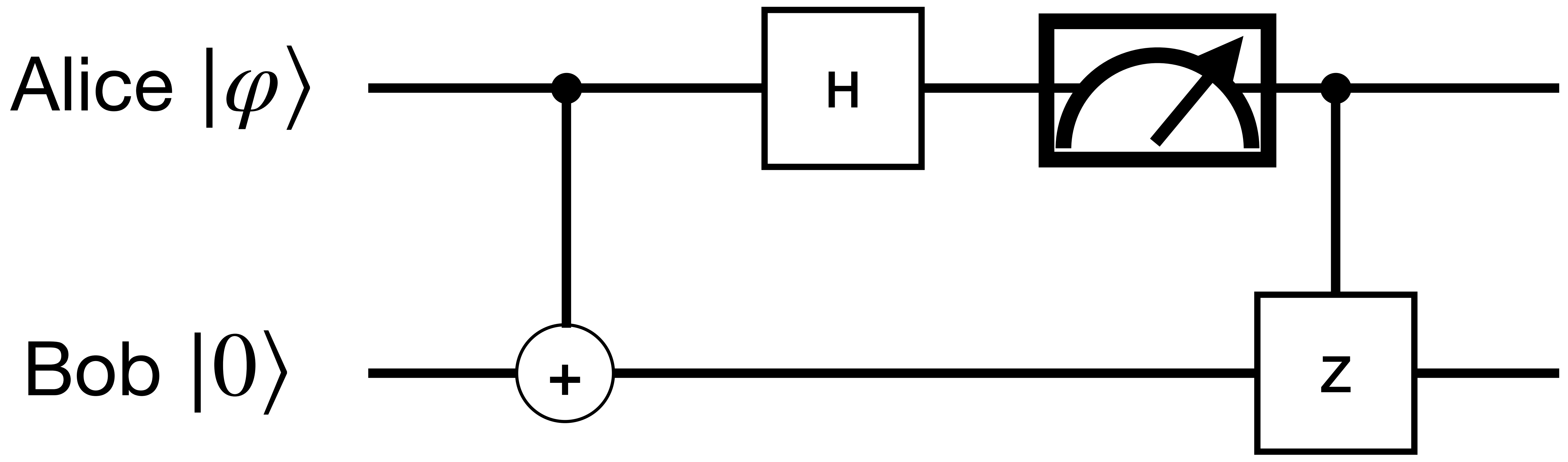
- Deterministic / shared randomness, winning probability at most 75%
- Quantum entanglement 85%
  - Violation of Bell's inequality
  - Refutes local realism
- CHSH done in practice
  - Early 80s, A. Aspect et al.
  - 2014, R. Hanson lab

# Quantum teleportation

## Part I

- Alice wants to send Bob a qubit $|\varphi\rangle$ (doesn't want to visit Bob in person)
- There is a CNOT gate between their offices
- Say $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$
    - Joint states?
- If Alice's readout is $|1\rangle$, then send "Apply Z gate" to Bob
- Needs 1 classical bit of communication






The controlled Z gate is done by messaging

The ctrl-Z gate is equivalent to