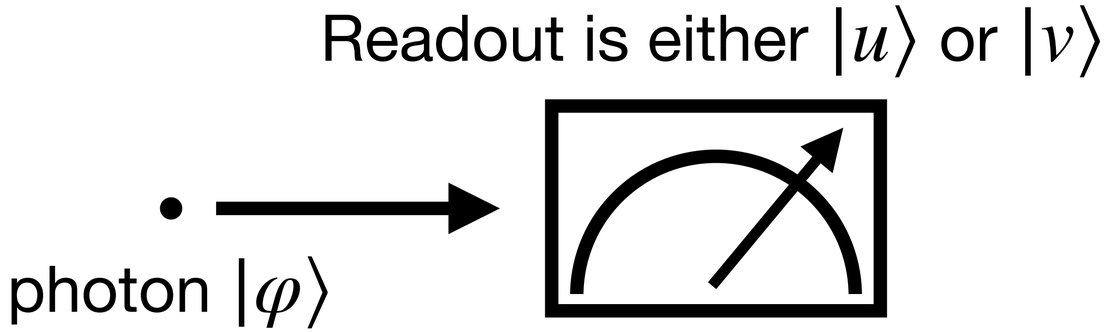
Measurement

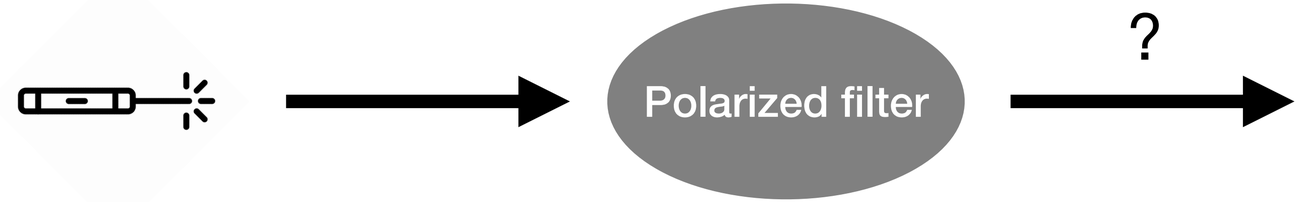
In basis $|u\rangle, |v\rangle$

- Standard measuring device measures in $|0\rangle, |1\rangle$ basis
- For any orthonormal basis $|u\rangle, |v\rangle$, can build a measuring device for this basis.



- The readout is $|u\rangle$ with probability $|\langle u \mid \varphi\rangle|^2$, and the readout is $|v\rangle$ with probability $|\langle v \mid \varphi\rangle|^2$.

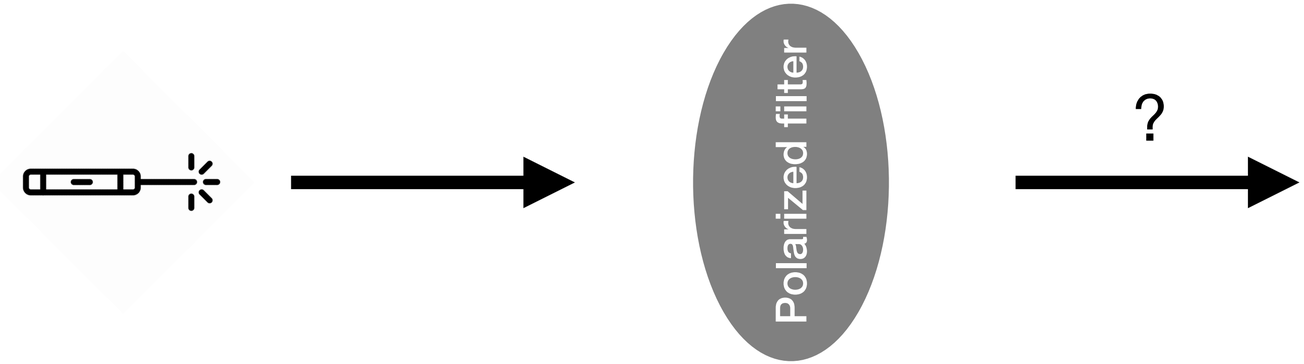
Examples



Polarize filter first measures photon's state,
if $|0\rangle$, the photon flies through;
if $|1\rangle$, photon is converted into heat.

Brightness 50%

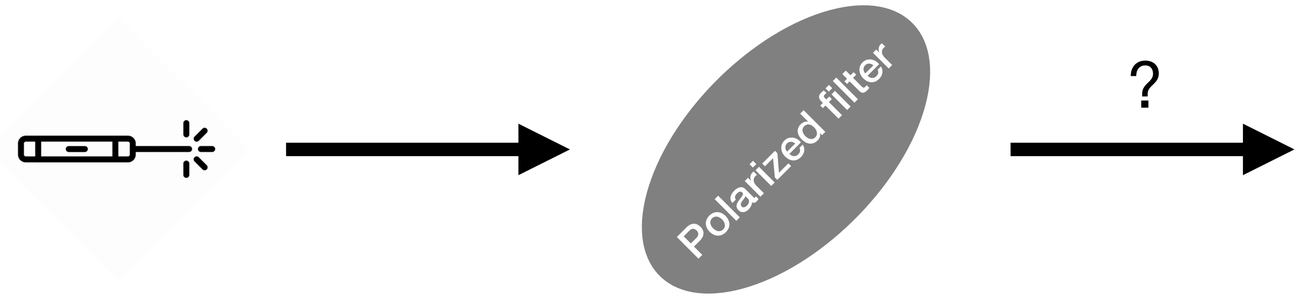
Examples



Polarize filter rotated 90° first measures photon's state,
if $|1\rangle$, the photon flies through;
if $|0\rangle$, photon is converted into heat.

Brightness 50%

Examples



Polarize filter rotated 45° first measures photon's state in

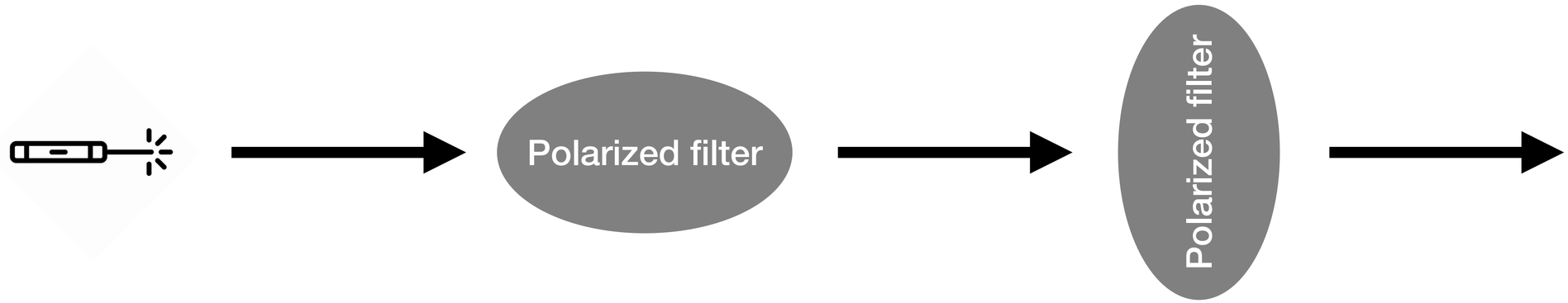
$|+\rangle, |-\rangle,$

if $|+\rangle$, the photon flies through;

if $|-\rangle$, photon is converted into heat.

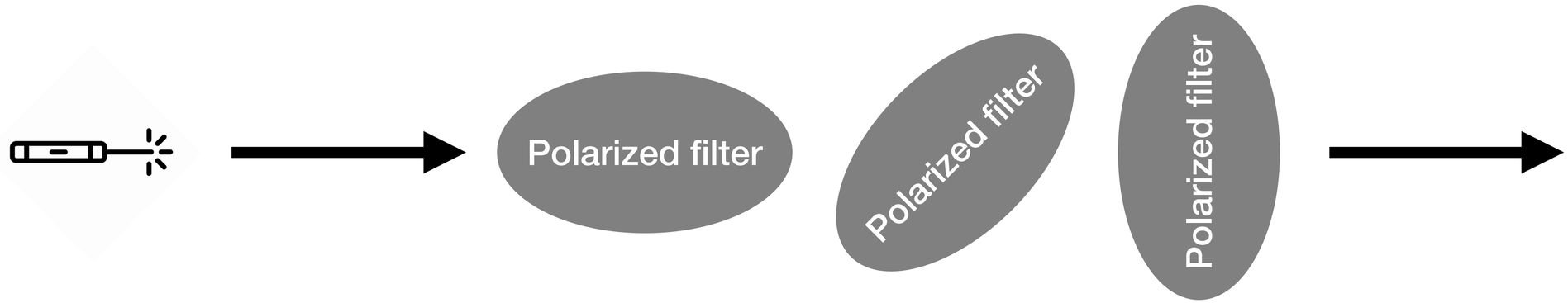
Brightness 50%

What happens if…



Brightness 0%

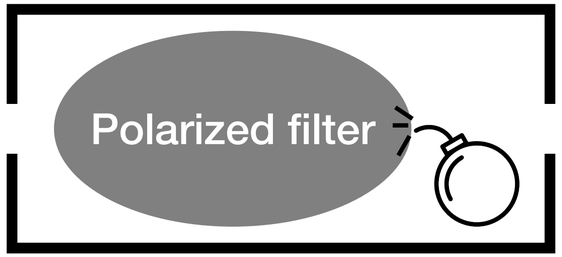
What happens if…



Brightness 12.5%

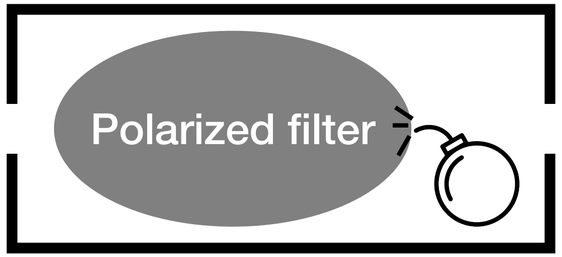

Light or
Dark?

Elitzur–Vaidman Bomb



- Inside, either NOTHING or BOMB fuse attached to a horizontal polarized lens
- Task: try to determine NOTHING or BOMB (explosion-free)
- Option 1: Send in $|0\rangle$ and measure in the standard basis, no information
- Option 2: Send in $|1\rangle$, explodes 100% if BOMB.
- Option 3: Send in $|+\rangle$, explodes 50% if BOMB.

Elitzur–Vaidman Bomb



- Question: Is it possible to boost the detection rate to 99%?
- Fact: can build physical device that rotate a unit (real) vector by θ counterclockwise.
- Observation: this physical device performs a linear transformation $|\varphi\rangle \mapsto R_\theta|\varphi\rangle$.
- Question: What is R_θ in matrix form?

Better Elitzur–Vaidman bomb detector

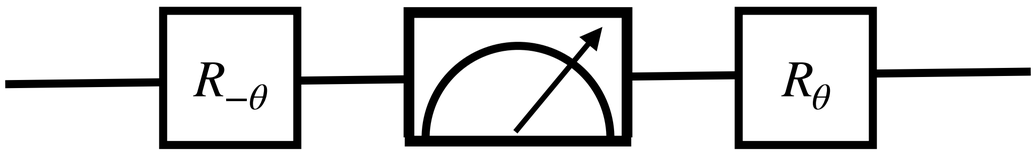
“Rotate, compute, rotate”

- Start with $|0\rangle$. Choose $n \approx 1000$.
- Repeat n times
  - Apply R_θ, where $\theta = \pi/2n$
  - Send in the photon
- Measure in the standard basis
- Analysis
  - If NOTHING, readout is $|1\rangle$
  - If BOMB, explodes with probability $\leq n \sin^2\theta \leq \pi^2/(4n)$, otherwise, readout is $|0\rangle$

Measure in other basis

Using rotation

- You can use $R_{-\theta}$ and R_θ and the measurement in standard basis to simulate a measurement in the $R_\theta|0\rangle, R_\theta|1\rangle$ basis



More transformations

Reflection

- Fact: can build physical device that reflects across a line
- Example 1: Reflection across x = y. The matrix form is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- Example 2: Reflection that maps $|0\rangle, |1\rangle$ to $|+\rangle, |-\rangle$. The matrix form is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- The last transformation is important, and is called Hadamard transformation.

Quantum mechanics

Law #3

- A qubit state (in general, a joint state) can be changed by any linear transformation that preserves lengths.
- Unitary transformation
  - A linear transformation U is unitary if it preserves lengths.
  - Proposition: U is unitary if and only if $U^\dagger U = I$
  - Theorem: U is unitary if and only if $U^\dagger = U^{-1}$ if and only if $UU^\dagger = I$
  - Corollary: Columns of U form an orthonormal basis. Same holds for rows.
  - Theorem: Unitary transformations preserve inner products