0/6 Questions Answered

Assignment 2

Q1 Read me first
0 Points

- Tests show that the people who get the most out of this assignment are those who read the "read me first" like this one.
- Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.
- Acknowledgment Requirements:

1. Acknowledge, individually for every problem at the beginning of each solution, a list of all collaborators and sources consulted other than the course notes. Examples include: names of people you discussed homework with, books, other notes, Wikipedia, and other websites.
2. If you consulted any online sources, please specify the exact webpages by including their links. Omission of links or any other required citations will result in a loss of grades and be considered a failure to acknowledge appropriately.
3. If no additional sources are consulted, you must write "sources consulted: none" or equivalent.
4. **Failure to acknowledge sources will lead to an automatic 1pt penalty.**

- Late policy: In general **no late homework** will be accepted unless there is a genuine emergency backed up by official documents.
- All steps should be justified.
- Formatting and Submission Requirements:

1. Separate Solutions: Ensure that solutions for each problem are separated clearly.
2. PDF Submissions: If you are submitting a LaTeX PDF, use the "fullpage" package to set the margins to 1 inch. Do not include additional information such as the title, date, your name, the problem statement, or any rough work—only include your final solution.
3. Typed Solutions: If typing directly in the provided textbox, please use LaTeX formatting for formulas.
   Images: Rotated images will not be graded. Ensure all images are properly oriented.
4. Scanning Quality: Use proper scanning software to scan your handwritten solutions. Avoid casual photos of your work.
5. **Failure to meet these formatting and submission requirements may result in up to a 2-point penalty for each problem.**

- You are encouraged to be **type in LaTeX**. To learn how to use LaTeX, I recommend the [tutorials on Overleaf]. It is ok to draw diagrams by hand and insert them as pictures in your TeX files.
- For each question below, upload a PDF file and/or type in the box (see [Gradescope x LaTeX tutorial]). Each submission should contain (1) the

acknowledgement of all collaborators and sources consulted and (2) your solution.

Save Answer

Q2 Projectors and reflections

5 Points

Let $|\psi\rangle$ and $|\phi\rangle$ be two unit vectors in $\mathbb{C}^d$. We will be interested in $Q = |\phi\rangle\langle\psi|$, which is a $d \times d$ matrix, and can therefore be thought of as a transformation on d-dimensional vectors.

(a) Explicitly work out the matrix Q in the case $|\psi\rangle = |0\rangle$ and $|\phi\rangle = |+\rangle$, and also in the opposite case $|\psi\rangle = |+\rangle$ and $|\phi\rangle = |0\rangle$.

(b) What does the transformation Q map the vector $|\psi\rangle$ to, and what does it map every vector orthogonal to $|\psi\rangle$ to?

(c) Suppose now that $|\psi\rangle = |\phi\rangle$. Let $P = |\psi\rangle\langle\psi|$. Describe in (geometric) words the transformation P.

(d) Let I denote the identity matrix in $\mathbb{R}^d$. Describe in (geometric) words the transformation $I - 2P$. Your description should include the words “hyperplane perpendicular to”. Prove that this transformation is unitary.

(e) Suppose we are interested in the change-of-(orthonormal-)basis operation U that takes the orthonormal basis $|\psi_1\rangle, \ldots, |\psi_d\rangle$ to the orthonormal basis $|\phi_1\rangle, \ldots, |\phi_d\rangle$. Show that U can be written as

$$U = |\phi_1\rangle\langle\psi_1| + \cdots + |\phi_d\rangle\langle\psi_d|.$$

Sources consulted:

Solution:

Please select file(s) Select file(s)

Save Answer

Q3 Quantum Anti-Zeno Effect
4 Points

Assume you have a single qubit that you know is in the state $|0\rangle$. You wish to change its state to $|1\rangle$. You have the ability to build any measurement device, and use it as many times as you want. How can you almost surely get the qbit's state changed to $|1\rangle$?

Remark: More specifically, given $\varepsilon > 0$, build a quantum circuit that outputs a qbit $|1\rangle$ with probability at least $1 - \varepsilon$. You are not allowed to apply gates (or rotations) to your qubit.

Sources consulted:

Solution:

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Save Answer

Q4 Perfect Powers

4 Points

(a) Give pseudocode for an algorithm that takes as input a positive integer A and determines whether or not A is a perfect square. If it is, your algorithm should also determine the number B such that $B^2 = A$. If A is n binary digits long, your algorithm should take $O(nM(\ n))$ steps, where $M(n)$ is the number of steps required to multiply two numbers of at most n binary digits. Thus, your algorithm should take $O(n^3)$ with the usual grade school multiplication algorithm; or, it would be $O(n^2 \log n \log\log n)$ steps with the sophisticated Schönhage–Strassen multiplication algorithm. (Hint: binary search.)

(b) Give pseudocode for an algorithm that takes as input a positive integer A and determines whether or not A is a perfect power (i.e., a perfect square, cube, fourth power, etc.). If it is, your algorithm should also determine numbers B and $C > 1$ such that $B^C = A$. When A is an n-bit number, justify that your algorithm takes at most $O(n^d)$ steps for some constant d (such as $d = 5$).

Sources consulted:

Solution:

Please select file(s) Select file(s)

Save Answer

Q5 Tensor Product Practice

4 Points

(a) Given an $m \times n$ matrix A, for every $1 \leq i \leq m, 1 \leq j \leq n$, show that the entry on the ith row and jth column of A equals $\langle i|A|j\rangle$. Here $|i\rangle$ represents the i-th vector in the standard basis.

(b) Show that if A and B are invertible matrices, then so is $A \otimes B$, and in fact $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$.

(c) Verify that $(A \otimes B)^{\dagger} = A^{\dagger} \otimes B^{\dagger}$.

(d) Suppose $|u_1\rangle, \ldots, |u_d\rangle$ is an orthonormal basis for $\mathbb{C}^d$, and $|v_1\rangle, \ldots, |v_e\rangle$ is an orthonormal basis for $\mathbb{C}^e$. Show that the collection $|u_i\rangle \otimes |v_j\rangle$ (for all $1 \leq i \leq d, 1 \leq j \leq e$) is an orthonormal basis for $\mathbb{C}^{de}$.

Sources consulted:

Solution:

Please select file(s) Select file(s)

Save Answer

Q6 1 ebit + 1 qubit ≥ 2 bits

4 Points

(a) Alice and Bob prepare an EPR pair (that is, two qubits in the state $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$).

They each take one qubit home. Suddenly, Alice decides she wishes to convey one of 4 messages to Bob; in other words, she wants to convey a classical string $uv \in \{0,1\}^2$ to Bob.

Alice does the following in the privacy of her own home: First, if $u = 1$, she applies a NOT gate to her qubit (else if $u = 0$ she does nothing here). Next, if $v = 1$, she applies a "Z" gate,

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

to her qubit (else if $v = 0$, she does nothing here). Finally, she walks to Bob's house and silently hands him her qubit.

Show that by measuring in an appropriate basis, Bob can exactly determine Alice's message $uv \in \{0,1\}^2$.

(b) Work out a circuit using only CNOT gates, 1-qubit gates, and "standard" measurement gates, which actually outputs Alice's message with 100% probability.

Sources consulted:

Solution:

Please select file(s) Select file(s)

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Save All Answers

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