Collaborators : None

Sources : Lecture Notes

Q4): Hardy's Paradox

$$|\psi\rangle = (H \otimes H)\left(\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle\right)$$

$$= \frac{1}{\sqrt{3}}\left(|+\rangle + |+-\rangle + |-+\rangle\right)$$

$$= \frac{1}{\sqrt{3}}\left(|+\rangle \otimes \frac{\sqrt{2}}{\sqrt{2}}|0\rangle + |-+\rangle\right)$$

$$= \frac{1}{\sqrt{3}}\left(|10\rangle + |00\rangle + \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{3}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle\right)$$

a) Case-1: If Alice flips T & Bob flips T:

$\Pr(\text{measuring } |11\rangle) = \left(\frac{-1}{2\sqrt{3}}\right)^2 = \frac{1}{12}$

Case-2: If Alice flips H & Bob flips H:

$\Pr(\text{measuring } |11\rangle) = 0^2 = 0$ [No $|11\rangle$ in $\left(\frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle\right)$]

Case-3: If Alice flips H & Bob flips T:

Transformed $|\psi\rangle = \frac{1}{\sqrt{3}}|0+\rangle + \frac{1}{\sqrt{3}}|0-\rangle + \frac{1}{\sqrt{3}}|1+\rangle$

$= \frac{\sqrt{2}}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{6}}|10\rangle + \frac{1}{\sqrt{6}}|11\rangle$

$\Rightarrow \Pr(\text{measuring } |01\rangle) = 0^2 = 0$

Case-4: If Alice flips T & Bob flips H:

Here $|\psi\rangle$ is symmetric w.r.t. to 1st & 2nd qubits & swapping them doesn't change $|\psi\rangle$. $SWAP(|\psi\rangle) = |\psi\rangle$ & $SWAP(|01\rangle) = |10\rangle$

Thus, $\Pr(\text{measuring } |10\rangle) = 0^2 = 0$

∴ It's possible to measure 11 if Alice & Bob flips Tails. Else if Alice & Bob flips (TH) or (HT) or (HH), it's impossible to measure $|10\rangle$, $|01\rangle$ & $|11\rangle$ respectively.

Lucien's statements.

b) 1) "On one hand, consider the first statement in (a). Since it's possible that Alice will flip Tails & Bob will flip Tails, we conclude that prior to any coin flips/measuring, it's possible for Alice's qubit to register 1 after being directly measured".

↳ Correct. Prior to coin flips, there is indeed a possibility of both Alice & Bob flipping Tails later & the measurement of Alice's qubit being 1. (eg: $Pr(|1\rangle) = \frac{1}{12}$ is supporting statement). But this is a possibility in isolation but not definite.

2) "Now consider the second statement in (a). Since Alice's qubit is capable of generating a 1 when she flips Tails, it must be impossible for Bob's qubit to produce a 0, when he flips Heads, & consequently Hadamard's-then-measures."

↳ Incorrect. The impossibility of Bob measuring 0 when he flips Heads doesn't follow from Alice's ability to measure 1 when she flips Tails. These are independent events here with different probabilities. And when Bob flips heads & Alice flips Tails, the $Pr(|00\rangle) = \frac{2}{3}$. ⇒ There's possibility of Bob generating 0.

(Hypothetical outcomes without Actual measurements do not impose any restriction on other outcomes in QM)

3) "Let's repeat the previous 2 bullet points, interchanging 'Alice' & 'Bob'. By the first statement in (a), we conclude that prior to any coin flips/measuring, it's possible for Bob's qubit to register a 1 when directly measured. Hence by the third statement in (a), since Bob's qubit is capable of generating a 1 when directly measured, we conclude that it must be

impossible for Alice's qubit to produce a 0 when she Hadamards-then-measures.

↳ Incorrect. The same flawed reasoning in 2nd bullet point is applicable here. If both 1st & 2nd bullet points are, only then this will be true, but 2nd bullet point is incorrect.

(b) "We've concluded that in case of flipping Heads, for both Alice & Bob it's impossible for them to register a 0 when they Hadamard-&-measure, i.e, they must both register a 1 in this case. But this contradicts the fourth statement in (a).".

↳ Incorrect. This misinterprets the effect of Hadamard transformation & the entangled state. The state's structure ensures that it is indeed impossible to both measure 1,1 when both flips Heads & apply Hadamard gates. Lucien's conclusion is based on an incorrect assumption that previous measurement possibilities impose strict deterministic constraints on future measurements. This is an incorrect assumption.