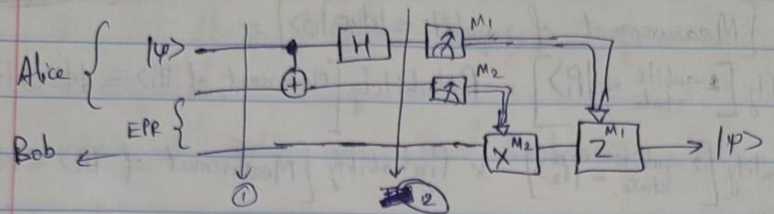
Collaborators : None

Sources : Lecture Notes

Q4) Entanglement Swapping

Standard Teleportation Protocol:



Let $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

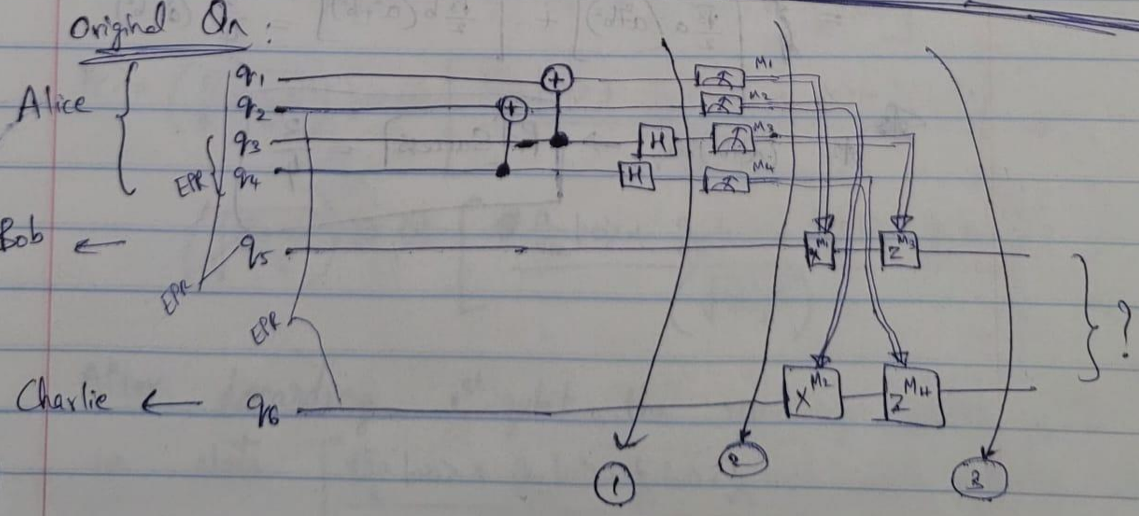
At ①, J.S. $= (\alpha|0\rangle + \beta|1\rangle) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$.

At ②, J.S. $= \frac{1}{2}\Big[|00\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) + |01\rangle \otimes (\alpha|1\rangle + \beta|0\rangle) + |10\rangle \otimes (\alpha|0\rangle - \beta|1\rangle)$
$+ |11\rangle \otimes (\alpha|1\rangle - \beta|0\rangle)\Big]$

If $M_1M_2 = 00$: Apply Identity Transformation on Bob's qubit
If $M_1M_2 = 01$: " X (bit flip) " " "
If $M_1M_2 = 10$: " Z (phase flip) " " "
If $M_1M_2 = 11$: Apply ZX (phase & bit flip) " " "

The 3rd qubit then equals original 1st qubit i.e, $|\psi\rangle$.

Original Qn:



Given:

(q_1 & q_5) are EPR pair shared by Alice & Bob.

(q_2 & q_6) are EPR pair shared by Alice & Charlie

(q_3 & q_4) is EPR pair prepared by Alice.

As $|q_3 q_4\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$

$$|q_3 q_4 q_1 q_5 q_2 q_6\rangle = \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)$$

$$= \underbrace{\left[\frac{|00\rangle}{\sqrt{2}} \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)\right]}_{\text{Part-1}} + \underbrace{\left[\frac{|11\rangle}{\sqrt{2}} \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|00\rangle + |11\rangle}{\sqrt{2}}\right)\right]}_{\text{Part-2}}$$

~~At ①, J.S. = $\left[\frac{1}{2\sqrt{2}}\left[|001\rangle + |010\rangle - |101\rangle - |110\rangle\right]\right] + \left[\frac{1}{2\sqrt{2}}\left[|000\rangle + |011\rangle + |100\rangle + |111\rangle\right]\right]$~~

At ①, J.S. $= \frac{1}{\sqrt{2}}\left[\underbrace{\frac{1}{2}\left[|001\rangle + |010\rangle - |101\rangle - |110\rangle\right] \otimes \{\text{itself}\}}\right] +$

ordering ($q_3 q_1 q_5 q_4 q_2 q_6$)

$$\frac{1}{\sqrt{2}}\left[\left(\frac{1}{2}\left[|000\rangle + |011\rangle + |100\rangle + |111\rangle\right]\right) \otimes \{\text{itself}\}\right]$$

ordering ($q_3 q_1 q_5 q_4 q_2 q_6$)

Even after measurements & ~~by computing~~ X, Z transformations, as, we are computing individual parts of expression from Part-1 & Part-2. Hence, at ③, [Consider only q_5 & q_6]

At ③, $|q_5 q_6\rangle = \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$

As the original state $|\phi\rangle$ was maximally entangled;
Quantum teleportation preserves quantum state, including entanglement because teleportation involves only local operations & classical communication which cannot create or increase entanglement b/w distant parties.