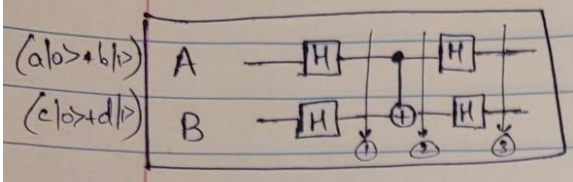
Collaborators : None

Sources : Lecture Notes ; https://en.wikipedia.org/wiki/Controlled_NOT_gate

(Q5) Fun with Gates



In the above circuit,

At ①, J.S = $(a|+\rangle + b|-\rangle) \otimes (c|+\rangle + d|-\rangle)$

$= \left[\left(\frac{a+b}{\sqrt{2}}\right)|0\rangle + \left(\frac{a-b}{\sqrt{2}}\right)|1\rangle\right] \otimes \left[\left(\frac{c+d}{\sqrt{2}}\right)|0\rangle + \left(\frac{c-d}{\sqrt{2}}\right)|1\rangle\right]$

At ②, J.S. $= \left(\frac{a+b}{\sqrt{2}}\right)|0\rangle \otimes \left[\left(\frac{c+d}{\sqrt{2}}\right)|0\rangle + \left(\frac{c-d}{\sqrt{2}}\right)\right] + \left(\frac{a-b}{\sqrt{2}}\right)|1\rangle \otimes \left[\left(\frac{c-d}{\sqrt{2}}\right)|0\rangle + \left(\frac{c+d}{\sqrt{2}}\right)|1\rangle\right]$

$= \left(\frac{a+b}{\sqrt{2}}\right)|0\rangle \otimes \left[c|+\rangle + d|-\rangle\right] + \left(\frac{a-b}{\sqrt{2}}\right)|1\rangle \otimes \left[c|+\rangle - d|-\rangle\right]$

$= \left(\frac{a+b}{2}\right)\left[|+\rangle + |-\rangle\right] \otimes \left[c|+\rangle + d|-\rangle\right] + \left(\frac{a-b}{2}\right)\left[|+\rangle - |-\rangle\right] \otimes \left[c|+\rangle - d|-\rangle\right]$

At ③, J.S. $= \left(\frac{a+b}{2}\right)\left[|0\rangle + |1\rangle\right] \otimes \left[c|0\rangle + d|1\rangle\right] + \left(\frac{a-b}{2}\right)\left[|0\rangle - |1\rangle\right] \otimes \left[c|0\rangle - d|1\rangle\right]$

$= ac|00\rangle + bd|01\rangle + bc|10\rangle + ad|11\rangle$

$= (a|0\rangle + b|1\rangle) \otimes c|0\rangle + (b|0\rangle + a|1\rangle) \otimes d|1\rangle$

$(CNOT = CNOT^{-1})$ [control = B, target = A]

$(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$

So, ~~we get CNOT~~ this circuit implements a CNOT with the A qubit as target & B qubit as control.

This works because:

The first set of Hadamard gates transforms the computational basis states into Hadamard basis ($|+\rangle$ & $|-\rangle$)

In Hadamard basis, the CNOT gate behaves differently. It applies a phase flip (Z) to control qubit if target qubit is in $|-\rangle$ state. This is equivalent to a controlled-Z operation with the roles of control & target reversed.

The final set of Hadamard gates transform the qubits back to the computational basis, completing the reversed CNOT operation.