CSE 598, Fall 2024, Homework #4

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Question 4(a) When Alice flips T and Bob flips T, the joint state is:

$$\begin{aligned}
|\psi\rangle &= (H \otimes H)\left(\frac{1}{3}|00\rangle + \frac{1}{3}|01\rangle + \frac{1}{3}|10\rangle\right) \\
&= \frac{1}{3}|+\rangle \otimes |+\rangle + \frac{1}{3}|+\rangle \otimes |-\rangle + \frac{1}{3}|-\rangle \otimes |+\rangle \\
&= \frac{1}{6}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) + \frac{1}{6}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) + \frac{1}{6}(|00\rangle + |01\rangle - |10\rangle - |11\rangle) \\
&= \frac{1}{\sqrt{12}}(3|00\rangle + |01\rangle + |10\rangle - |11\rangle)
\end{aligned}$$

Thus, it is possible for Alice and Bob to measure $|11\rangle$.

When Alice flips T and Bob flips H, since $H^2 = I$, the joint state is:

$$\begin{aligned}
(I \otimes H)|\psi\rangle &= (H \otimes I)\left(\frac{\sqrt{2}}{3}|+\rangle \otimes |0\rangle + \frac{1}{3}|01\rangle\right) \\
&= \frac{\sqrt{2}}{3}|00\rangle + \frac{1}{3\sqrt{2}}(|01\rangle + |11\rangle)
\end{aligned}$$

Thus, it is impossible to measure $|10\rangle$.

When Alice flips H and Bob flips T, since $H^2 = I$, the joint state is:

$$\begin{aligned}
(H \otimes I)|\psi\rangle &= (I \otimes H)\left(\frac{\sqrt{2}}{3}|0\rangle \otimes |+\rangle + \frac{1}{3}|10\rangle\right) \\
&= \frac{\sqrt{2}}{3}|00\rangle + \frac{1}{3\sqrt{2}}(|10\rangle + |11\rangle)
\end{aligned}$$

Thus, it is impossible to measure $|01\rangle$.

When Alice flips H and Bob flips H, since $H^2 = I$, the joint state is:

$$(H \otimes H)|\psi\rangle = \frac{1}{3}|00\rangle + \frac{1}{3}|01\rangle + \frac{1}{3}|10\rangle$$

Thus, it is impossible to measure $|11\rangle$.