Collaborators : None

Sources : Lecture Notes; https://en.wikipedia.org/wiki/Quantum_Zeno_effect

Q3 Quantum Anti-Zeno Effect

Assume you have a single qubit that you know is in the state $|0\rangle$. You wish to change its state to $|1\rangle$. You have the ability to build any measurement device, and use it as many times as you want. How can you almost surely get the qbit's state changed to $|1\rangle$?

Remark: More specifically, given $\varepsilon > 0$, build a quantum circuit that outputs a qbit $|1\rangle$ with probability at least $1-\varepsilon$. You are not allowed to apply gates (or rotations) to your qubit.

Ans: To change the state of a single qubit from $|0\rangle\ to\ |1\rangle$ with a probability of at least $1-\epsilon$, without using quantum gates, you can employ a probabilistic measurement strategy. Here's a step-by-step approach:

Strategy for Changing Qubit State

1. **Measurement Setup**: Design 2 measurement devices that measures the qubit in the $\{|+\rangle\ and\ |-\rangle\}$ basis ; and $\{|0\rangle$ and $|1\rangle\}$ basis respectively.
2. **Initial Measurement**: Measure the qubit in the $|+\rangle, |-\rangle$ basis and then in $|0\rangle$ and $|1\rangle$ basis:
   - Whatever the result is whether $|+\rangle\ or\ |-\rangle$ initially, do nothing and then measure the intermediate output in $|0\rangle$ and $|1\rangle$ basis.
   - The input $|0\rangle$ has equal chance of outputting $|+\rangle$ and $|-\rangle$ in the first measurement. Be it $|+\rangle$ or $|-\rangle$, the next measurement in $|0\rangle$ and $|1\rangle$ basis also has equal chance of outputting $|0\rangle$ and $|1\rangle$.
   - So, the

     $Probability(Single\ Measurement\ Final\ output = |1\rangle) = Probability(|0\rangle\ turning\ |+\rangle) * Probability(|+\rangle\ turning\ |1\rangle) + Probability(|0\rangle\ turning\ |-\rangle) * Probability(|-\rangle\ turning\ |1\rangle) = \frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{1}{2} = \frac{1}{2}$
3. **Repeat Measurements**: Repeat the measurement process after getting $|0\rangle$. Each time you get a result of $|0\rangle$, you have a 50% chance that the qubit is in state $|1\rangle$ after the next measurement.

$$Probability(\ Final\ output = |1\rangle\ after\ atmost\ N\ measurements) = 1 - \frac{1}{2^N}$$

4. **Stopping Criterion**: Continue this process until we are confident that the probability of the qubit being in state $|1\rangle$ is at least $1-\epsilon$.

$$1 - \frac{1}{2^N} \geq 1 - \epsilon \quad \Longrightarrow \quad N \leq -\log_2 \epsilon$$

After repeating this measurement for sufficiently large number of times, we can almost be sure to get $|\,1\rangle$ as output.