Collaborators : None

Sources : Lecture Notes

Q5 Dirac notation and measurement exercises

5-a)

Let $|\phi\rangle = 3|0\rangle - 5i|1\rangle$

Then, $\langle\phi|\phi\rangle = 3*3 + (-5i)*(-5i) = 9 - 25 = -16$

5-b)

What number, C, should $|\phi\rangle$ be divided by to make it a "normalized" state; i.e., a unit vector? For future reference, define $|\psi\rangle = C^{-1}|\phi\rangle$ to be this state vector.

$C = ||\phi|| = (3^2 + |-5i|^2)^{\frac{1}{2}} = (9+25)^{\frac{1}{2}} = 34^{\frac{1}{2}} \approx 5.8309$

5-c)

What are the possible outcomes and associated probabilities if $|\psi\rangle$ is measured in the standard $\{|0\rangle, |1\rangle\}$ basis?

For readout as $|0\rangle$, the associated probability = $\frac{3^2}{||\phi||^2} = \frac{9}{34} \approx 0.2647$

For readout as $|1\rangle$, the associated probability = $\frac{|-5i|^2}{||\phi||^2} = \frac{25}{34} \approx 0.7353$

5-d)

Measurement in $\{|+\rangle, |-\rangle\}$ basis:

Equation-1 : $|\phi\rangle = 3|0\rangle - 5i|1\rangle = \langle+|\phi\rangle\,|+\rangle + \langle-|\phi\rangle\,|-\rangle$

$\langle+|\phi\rangle = \frac{1}{\sqrt{2}}*3 + \frac{1}{\sqrt{2}}*(-5i) = \frac{3-5i}{\sqrt{2}}$

Similarly, $\langle-|\phi\rangle = \frac{1}{\sqrt{2}}*3 + \frac{(-1)}{\sqrt{2}}*(-5i) = \frac{3+5i}{\sqrt{2}}$

For readout in $|+\rangle$, the associated probability = $\frac{|\frac{3-5i}{\sqrt{2}}|^2}{||\phi||^2} = \frac{|\frac{3-5i}{\sqrt{2}}|^2}{34} = \frac{1}{2} = 0.5$

For readout in $|-\rangle$, the associated probability = $\frac{|\frac{3+5i}{\sqrt{2}}|^2}{||\phi||^2} = \frac{|\frac{3+5i}{\sqrt{2}}|^2}{34} = \frac{1}{2} = 0.5$

5-e)

Verify that $\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ and $\frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$ form an orthonormal basis for $\mathbb{C}^2$. (These two vectors are sometimes called |i□ and |−*i*□.) Then do the prior question for measuring in the {|*i*□,|−*i*□} basis.

Measurement in $\{|i\rangle, |-i\rangle\}$ basis:

Equation-1 : $|\phi\rangle = 3|0\rangle - 5i|1\rangle = \langle i|\phi\rangle\ |i\rangle + \langle -i|\phi\rangle\ |-i\rangle$

$\langle i|\phi\rangle = \frac{1}{\sqrt{2}} * 3 + \frac{i}{\sqrt{2}} * (-5i) = \frac{3+5}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

Similarly, $\langle -|\phi\rangle = \frac{1}{\sqrt{2}} * 3 + \frac{(-i)}{\sqrt{2}} * (-5i) = \frac{3-5}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

For readout in $|+\rangle$, the associated probability = $\frac{|4\sqrt{2}|^2}{||\phi||^2} = \frac{32}{34} \approx 0.9412$

For readout in $|-\rangle$, the associated probability = $\frac{|-\sqrt{2}|^2}{||\phi||^2} = \frac{2}{34} \approx 0.0588$