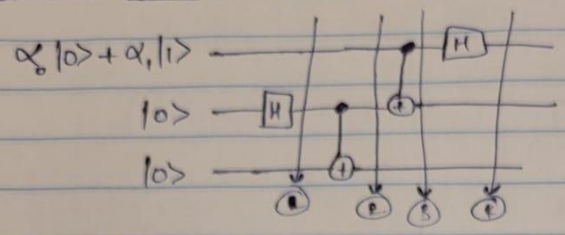
Collaborators : None

Sources : Lecture Notes

Q4 Quantum circuit exercise:



a)

At ①, J.S. $= (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes |+\rangle \otimes |0\rangle$

$= (\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |0\rangle$

$= \frac{\alpha_0}{\sqrt{2}}|000\rangle + \frac{\alpha_0}{\sqrt{2}}|010\rangle + \frac{\alpha_1}{\sqrt{2}}|100\rangle + \frac{\alpha_1}{\sqrt{2}}|110\rangle$

At ②, J.S. $= \frac{\alpha_0}{\sqrt{2}}|000\rangle + \frac{\alpha_0}{\sqrt{2}}|010\rangle + \frac{\alpha_1}{\sqrt{2}}|100\rangle + \frac{\alpha_1}{\sqrt{2}}|110\rangle$

At ③, J.S. $= \frac{\alpha_0}{\sqrt{2}}|000\rangle + \frac{\alpha_0}{\sqrt{2}}|010\rangle + \frac{\alpha_1}{\sqrt{2}}|110\rangle + \frac{\alpha_1}{\sqrt{2}}|101\rangle$

At ④, J.S. $=$

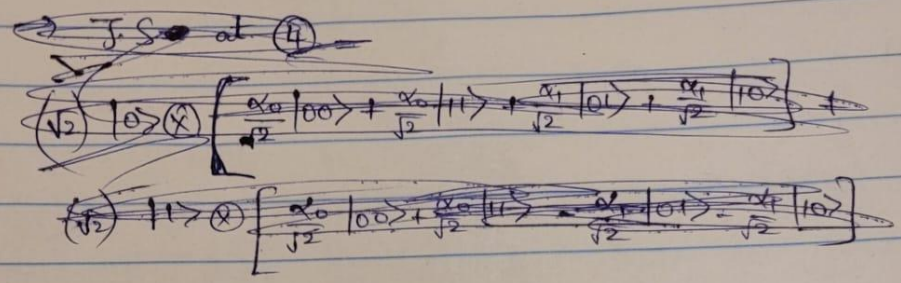
$\left(\frac{|0\rangle}{\sqrt{2}} + \frac{|1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{\alpha_0}{\sqrt{2}}|00\rangle + \frac{\alpha_0}{\sqrt{2}}|11\rangle\right) + \left(\frac{|0\rangle}{\sqrt{2}} - \frac{|1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{\alpha_1}{\sqrt{2}}|10\rangle + \frac{\alpha_1}{\sqrt{2}}|01\rangle\right)$

$= \frac{\alpha_0}{2}|000\rangle + \frac{\alpha_1}{2}|001\rangle + \frac{\alpha_1}{2}|010\rangle + \frac{\alpha_0}{2}|011\rangle + \frac{\alpha_0}{2}|100\rangle - \frac{\alpha_1}{2}|101\rangle$

$- \frac{\alpha_1}{2}|110\rangle + \frac{\alpha_0}{\sqrt{2}}|111\rangle$

$= \frac{\alpha_0}{2}\left(|000\rangle + |011\rangle + |100\rangle + |111\rangle\right) + \frac{\alpha_1}{2}\left(|001\rangle + |010\rangle - |101\rangle - |110\rangle\right)$

b) Suppose 1st 2-qubits are measured.



After measurement

$$\Rightarrow \text{J.S. at } (4) =$$

$$|00\rangle \otimes \left[\frac{\alpha_0}{2}|0\rangle + \frac{\alpha_1}{2}|1\rangle\right] + |01\rangle \otimes \left[\frac{\alpha_0}{2}|1\rangle + \frac{\alpha_1}{2}|0\rangle\right] +$$

$$|10\rangle \otimes \left[\frac{\alpha_0}{2}|0\rangle - \frac{\alpha_1}{2}|1\rangle\right] + |11\rangle \otimes \left[\frac{\alpha_0}{2}|1\rangle - \frac{\alpha_1}{2}|0\rangle\right]$$

$$= \frac{1}{2}|00\rangle \otimes \left[\alpha_0|0\rangle + \alpha_1|1\rangle\right] + \frac{1}{2}|01\rangle \otimes \left[\alpha_1|0\rangle + \alpha_0|1\rangle\right] + \frac{1}{2}|10\rangle \otimes \left[\alpha_0|0\rangle - \alpha_1|1\rangle\right] +$$

$$\frac{1}{2}|11\rangle \otimes \left[-\alpha_1|0\rangle + \alpha_0|1\rangle\right]$$

After measurement,

$$\text{Readout} = \begin{cases} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{cases} \text{ with probabilities } 1/4$$

$$3^{rd} \text{ qubit} = \begin{cases} \alpha_0|0\rangle + \alpha_1|1\rangle & \text{if readout} = |00\rangle \\ \alpha_1|0\rangle + \alpha_0|1\rangle & \text{if readout} = |01\rangle \\ \alpha_0|0\rangle - \alpha_1|1\rangle & \text{if readout} = |10\rangle \\ -\alpha_1|0\rangle + \alpha_0|1\rangle & \text{if readout} = |11\rangle \end{cases}$$