Assignment 5

Graded

Student

Sujith Potineni

Total Points

28 / 28 pts

Question 1

Read me first

0 / 0 pts

✔ **+ 0 pts** Correct

+ 0 pts Incorrect

Question 2

Borromean entanglement vs. non-Borromean entanglement

8 / 8 pts

✔ **+ 8 pts** Correct

+ 2 pts (a)

+ 1.5 pts (a) Only showed that the GHZ cannot be written as the tensor of three qubits

+ 2 pts (b)

+ 4 pts (c)

+ 2 pts (c) part 1

+ 1.5 pts (c) part 1, only showed the state cannot be decomposed into the tensor of three qubits

+ 2 pts (c) part 2

+ 1.5 pts (c) part 2, the probability is wrong

+ 0 pts Incorrect

Question 3

Quantum Money Attacks

6 / 6 pts

✔ **+ 6 pts** Correct

+ 0 pts Incorrect

Question 4

Entanglement swapping

6 / 6 pts

✔ **+ 6 pts** Correct

+ 4 pts Showed the right protocol without sufficient justification

+ 0 pts Incorrect

Question 5

Product probability spaces

8 / 8 pts

✔ **+ 8 pts** Correct

+ 0 pts Incorrect

Q1 Read me first
0 Points

- Tests show that the people who get the most out of this assignment are those who read the "read me first" like this one.
- Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.
- Acknowledgment Requirements:

1. Acknowledge, individually for every problem at the beginning of each solution, a list of all collaborators and sources consulted other than the course notes. Examples include: names of people you discussed homework with, books, other notes, Wikipedia, and other websites.
2. If you consulted any online sources, please specify the exact webpages by including their links. Omission of links or any other required citations will result in a loss of grades and be considered a failure to acknowledge appropriately.
3. If no additional sources are consulted, you must write "sources consulted: none" or equivalent.
4. **Failure to acknowledge sources will lead to an automatic 1pt penalty.**

- Late policy: In general **no late homework** will be accepted unless there is a genuine emergency backed up by official documents.
- All steps should be justified.
- Formatting and Submission Requirements:

1. Separate Solutions: Ensure that solutions for each problem are separated clearly.
2. PDF Submissions: If you are submitting a LaTeX PDF, use the "fullpage" package to set the margins to 1 inch. Do not include additional information such as the title, date, your name, the problem statement, or any rough work—only include your final solution.
3. Typed Solutions: If typing directly in the provided textbox, please use LaTeX formatting for formulas.
   Images: Rotated images will not be graded. Ensure all images are properly oriented.
4. Scanning Quality: Use proper scanning software to scan your handwritten solutions. Avoid casual photos of your work.
5. **Failure to meet these formatting and submission requirements may result in up to a 2-point penalty for each problem.**

- You are encouraged to be **type in LaTeX**. To learn how to use LaTeX, I recommend the tutorials on Overleaf. It is ok to draw diagrams by hand and insert them as pictures in your TeX files.
- For each question below, upload a PDF file and/or type in the box (see Gradescope x LaTeX tutorial). Each submission should contain (1) the acknowledgement of all collaborators and sources consulted and (2) your solution.

Q2 Borromean entanglement vs. non-Borromean entanglement

8 Points

(a) The "GHZ state" is $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$. Prove that this state is entangled (i.e., it is not unentangled).

(b) Suppose Alice, Bob, and Charlie each hold one qubit of a GHZ state. Suppose Charlie measures her bit. Prove that with 100% probability, Alice and Bob's qubits become unentangled.

Remarks: Had Charlie first taken her qubit to Jupiter and Alice and Bob never really hear from her again, then they would have no way of distinguishing whether or not Charlie actually does measure her qubit. Thus the "mixed" state that Alice and Bob's two qubits are in is said to be "unentangled" either way. By symmetry of the GHZ state $\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle$, we therefore have a funny situation: Any two of the three qubits are not entangled, but all three of them are entangled.

(c) The 3-qubit "W state" is defined to be $\frac{1}{\sqrt{3}}|001\rangle + \frac{1}{\sqrt{3}}|010\rangle + \frac{1}{\sqrt{3}}|100\rangle$. Prove that this state is entangled. Furthermore, prove that if Charlie measures one of the three qubits, there is a positive probability that the remaining two qubits are still entangled.

Sources consulted:

Lecture Notes

Solution:

HW5_Q2.pdf

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Q3 Quantum Money Attacks

6 Points

Suppose you're a quantum money counterfeiter, trying to forge a banknote in Wiesner's scheme. You're given a qubit that's $|0\rangle, |1\rangle, |+\rangle$ or $|-\rangle$, each with equal probability $\frac{1}{4}$. You can apply any quantum circuit you like to the qubit to produce a two-qubit state. Then, both of your output qubits will separately be given back to the bank for verification. (I.e., if the original qubit was $|0\rangle$ or $|1\rangle$ then the bank will measure both output qubits in the $\{|0\rangle, |1\rangle\}$ basis and accept if and only if both outcomes match the original qubit, and likewise if the original qubit was $|+\rangle$ or $|-\rangle$ the bank will measure and check in the $\{|+\rangle, |-\rangle\}$ basis.) Your goal is to maximize the probability that the bank accepts.

Now consider the following procedure. Among 3 qubits, initialize the first two qubits to $|0\rangle$ and let the third qubit be the qubit from the original banknote to be counterfeited. Then apply a 3 qubit unitary transformation whose effect is the following mapping:

$$|000\rangle \to \frac{\sqrt{3}}{2}|000\rangle + \frac{1}{\sqrt{12}}|110\rangle + \frac{1}{\sqrt{12}}|101\rangle + \frac{1}{\sqrt{12}}|011\rangle$$

$$|001\rangle \to \frac{\sqrt{3}}{2}|111\rangle + \frac{1}{\sqrt{12}}|001\rangle + \frac{1}{\sqrt{12}}|010\rangle + \frac{1}{\sqrt{12}}|100\rangle$$

Finally, discard (perform partial trace over) the first qubit and output the state given by the second two qubits. Show that the probability of success for this procedure is $3/4$.

Note: This procedure actually turns out to be the optimal one.

Sources consulted:

Lecture Notes

Solution:

HW5_Q3.pdf

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Q4 Entanglement swapping

6 Points

Consider the following scenario involving entangled states:

Alice and Bob share an EPR pair, and Alice and Charlie also share an EPR pair. Alice prepares a third EPR pair and teleports one half to Bob and the other half to Charlie. In the end, Bob and Charlie will hold halves of an entangled EPR pair despite never physically interacting.

Your task is to prove that this procedure works. Provide a detailed explanation of the steps involved in the teleportation process and the resulting entanglement between Bob and Charlie.

Sources consulted:

Lecture Notes

Solution:

HW5_Q4.pdf

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Q5 Product probability spaces

8 Points

(a) Let $p \in \mathbb{R}^d$ be a probability distribution on $[d] = \{1, 2, \ldots, d\}$. Let $q \in \mathbb{R}^e$ be a probability distribution on $[e] = \{1, 2, \ldots, e\}$. Prove that the Kronecker product $p \otimes q$ (which is a vector naturally indexed by the set $[d] \times [e]$) is the associated "product probability distribution" on $[d] \times [e] = \{(i, j) : 1 \leq i \leq d,\ 1 \leq j \leq e\}$; i.e., it's the distribution gotten by drawing i from p and j from q independently.

(b) Let $(p_1, |\psi_1\rangle), \ldots, (p_m, |\psi_m\rangle)$ be the mixed state of a d-dimensional particle (meaning we have probability p_i of pure state $|\psi_i\rangle \in \mathbb{C}^d$, $i = 1, \ldots, m$). Similarly, let $(q_1, |\phi_1\rangle), \ldots, (q_n, |\phi_n\rangle)$ be the mixed state of an e-dimensional particle. Write $\rho \in \mathbb{C}^{d \times d}$ for the density matrix of the first mixed state and $\sigma \in \mathbb{C}^{e \times e}$ for the density matrix of the second. Suppose the particles were created completely separately and independently, but we now decide to view them as a joint de-dimensional state. Recalling the rules of how to do this for pure states, show that the resulting de-dimensional mixed state has density matrix $\rho \otimes \sigma$, the Kronecker product of ρ and σ.

Sources consulted:

Lecture Notes

Solution:

HW5_Q5.pdf

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