**Collaborators :** None

**Sources** : Lecture Notes

**Q2**  **Projectors and reflections**

Let and be two unit vectors in . We will be interested in , which is a matrix, and can therefore be thought of as a transformation on dimensional vectors.

**(a)** Explicitly work out the matrix in the case and and also in the opposite case and

**Ans:**

**(b)** What does the transformation map the vector to, and what does it map every vector orthogonal to to?

**Ans:** As mapping vector after transformation , the resultant vector ()

Similarly, mapping orthogonal vector after transformation , the resultant vector ()

**(c)** Suppose now that Let Describe in (geometric) words the transformation

**Ans**: Let

Let

Similarly for n-dimensional vector, Here is a constant.

So, is a projection operator. Geometrically, it projects any vector onto the one-dimensional subspace spanned by

**(d)** Let denote the identity matrix in . Describe in (geometric) words the transformation I−2P. Your description should include the words “hyperplane perpendicular to”. Prove that this transformation is unitary.

**Ans**:

The resultant vector is a negative of reflection of with respect to subspace spanned by

To prove it's unitary, we need to show

Thus is unitary transformation.

**(e)** Suppose we are interested in the change-of-(orthonormal-)basis operation that takes the orthonormal basis to the orthonormal basis Show that can be written as.

**Ans**: we want to show that the change-of-basis operation UUU, which takes the orthonormal basis to the orthonormal basis , can be written as:

Let be any vector in . We can expand in the basis :

Applying to , we get: