**Collaborators :** None

**Sources** : Lecture Notes

**Q5**   **Tensor Product Practice**

**(a)** Given an matrix , for every , show that the entry on the row and column of equals . Here represents the vector in the standard basis.

**Ans:**

* is a column vector with dimension where the element is , and all other elements are .
* is a similar column vector with dimension where the element is .

mx1

mx1

1x1

1x1

Thus, the entry on the row and column of, equals .

**(b)** Show that if and are invertible matrices, then so is , and in fact .

**Ans**: Given that both and are invertible, we know that and exist.

Multiplying with

As inverse is unique, thus

**(c)** Verify that

**Ans:** Consider the matrix element where and are the row and column indices in the tensor product matrix. This element can be expressed as:

Taking the conjugate transpose, we get:

This shows that the conjugate transpose of the tensor product is indeed the tensor product of the conjugate transposes:

**(d)** Suppose is an orthonormal basis for , and is an orthonormal basis for . Show that the collection (for all ) is an orthonormal basis for .

**Ans:**

If , then as is an orthonormal basis, ,

Else if ,

Similarly, if , then as is an orthonormal basis,

Else if ,

Thus, if ,

Therefore, the collection (for all ) is an orthonormal basis for .