Kinematic calculations for mobile robots using Omni/Meccanum wheels

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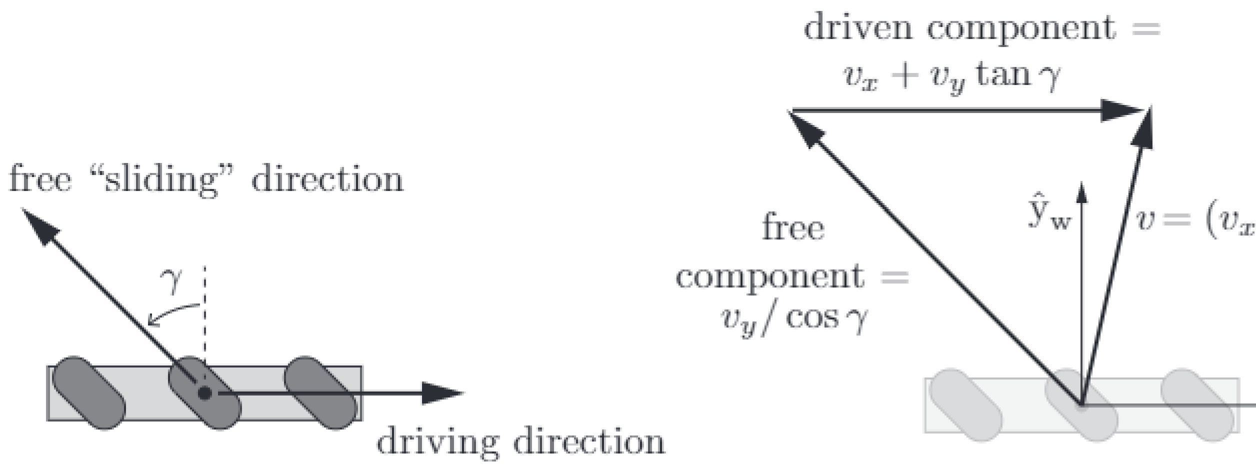
1 Generalized expression

This general expression is taken from the book "Modern robotics mecanics planning and control" by Kevin M Lynch, Frank C Park, page no 512, please refer this section for more details.



2 Three wheel omnidirectional robot

$$h_1(0)\mathcal{V}_b = \frac{1}{r_i}\begin{bmatrix} 1 & \tan\gamma_i \end{bmatrix}\begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix}\begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix}\mathcal{V}_b \tag{1}$$

$$\gamma_1 = 0, \beta_1 = 0$$

$$= \frac{1}{r_1}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \cos 0 & \sin 0 \\ -\sin 0 & \cos 0 \end{bmatrix}\begin{bmatrix} -y_1 & 1 & 0 \\ x_1 & 0 & 1 \end{bmatrix}\mathcal{V}_b \tag{2}$$

$$= \frac{1}{r_1}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -y_1 & 1 & 0 \\ x_1 & 0 & 1 \end{bmatrix}\mathcal{V}_b \tag{3}$$

$$= \frac{1}{r_1}\begin{bmatrix} -y_1 & 1 & 0 \end{bmatrix}\mathcal{V}_b \tag{4}$$

$$r_i = r_1 = r_2 = r_3 = r \tag{5}$$

$$h_1(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} -y_1 & 1 & 0 \end{bmatrix}\mathcal{V}_b \tag{6}$$

Similarly for the second wheel

$$h_2(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} \frac{1}{2}y_2 - \frac{\sqrt{3}}{2}x_2 & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}\mathcal{V}_b \tag{7}$$

Similarly for the third wheel

$$h_3(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} \frac{1}{2}y_3 + \frac{\sqrt{3}}{2}x_3 & \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\mathcal{V}_b \tag{8}$$

We know that

$$y_2 = y_3, x_2 = -x_3 \tag{9}$$

The final expression for the three wheel omnidirectional robot is

$$H(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} -y_1 & 1 & 0 \\ \frac{1}{2}y_2 - \frac{\sqrt{3}}{2}x_2 & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{2}y_3 + \frac{\sqrt{3}}{2}x_3 & \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \omega_{bz} \\ \upsilon_{bx} \\ \upsilon_{by} \end{bmatrix} \tag{10}$$

To find $\begin{bmatrix} \omega_{bz} \\ \upsilon_{bx} \\ \upsilon_{by} \end{bmatrix}$

Lets insert the symbol for the above expression

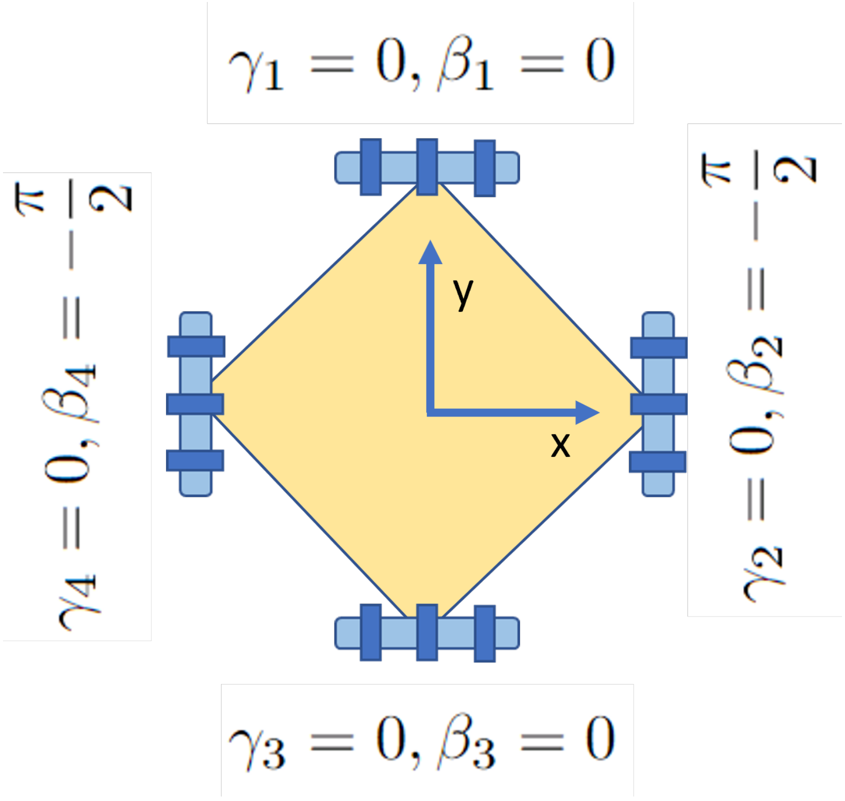
$$\mathcal{A} = \begin{bmatrix} -y_1 & 1 & 0 \\ \frac{1}{2}y_2 - \frac{\sqrt{3}}{2}x_2 & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{2}y_3 + \frac{\sqrt{3}}{2}x_3 & \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{11}$$

Solving the above will lead to the following expression

$$\mathcal{A} = \begin{bmatrix} -d & 1 & 0 \\ -d & \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ -d & \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \tag{12}$$

To find $\begin{bmatrix} \omega_{bz} \\ \upsilon_{bx} \\ \upsilon_{by} \end{bmatrix} = \mathcal{A}^{-1}\mathcal{V}_b$

3 Four wheel omnidirectional robot



For wheel 1

$$\gamma_1 = 0, \beta_1 = 0$$

For wheel 2

$$\gamma_2 = 0, \beta_2 = -\frac{\pi}{2}$$

For wheel 3

$$\gamma_3 = 0, \beta_3 = 0$$

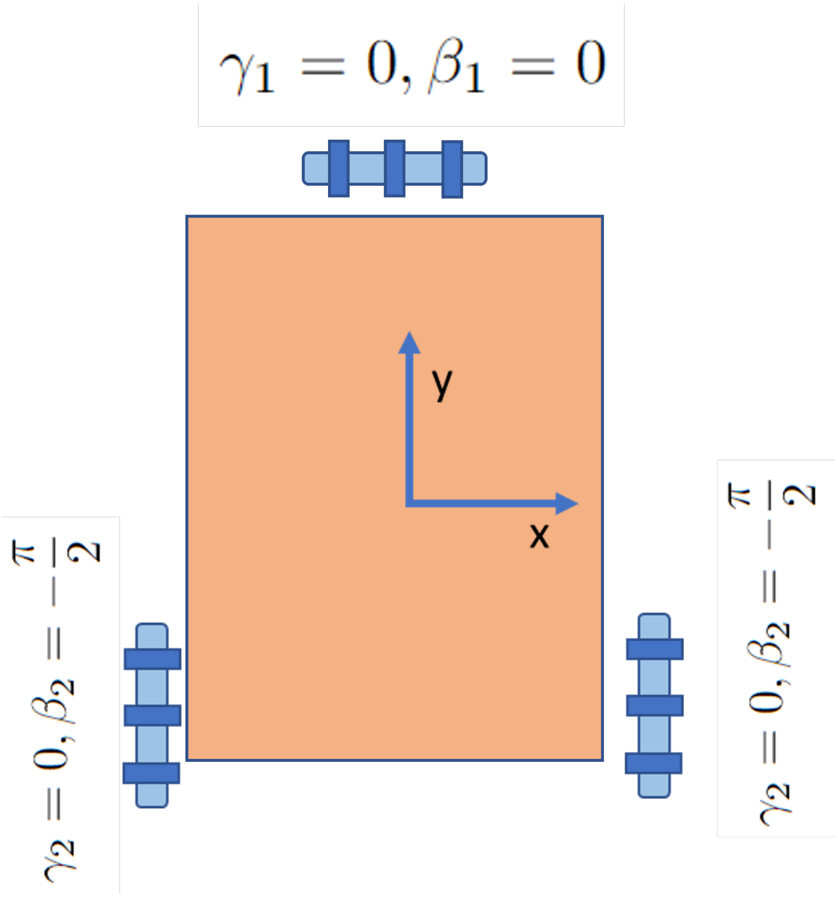
For wheel 4

$$\gamma_4 = 0, \beta_4 = -\frac{\pi}{2}$$

$$H(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} -y_1 & 1 & 0 \\ -x_2 & 0 & -1 \\ -y_3 & 1 & 0 \\ -x_3 & 0 & -1 \end{bmatrix}\begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} \tag{13}$$

$$\mathcal{A} = \begin{bmatrix} -y_1 & 1 & 0 \\ -x_2 & 0 & -1 \\ -y_3 & 1 & 0 \\ -x_3 & 0 & -1 \end{bmatrix} \tag{14}$$

4 For custom skateboard which is in our lab



This is the same case of four wheel omnidirectional robot, the resulant matrix will be

$$\mathcal{A} = \begin{bmatrix} -y_1 & 1 & 0 \\ -x_2 & 0 & -1 \\ -x_3 & 0 & -1 \end{bmatrix} \tag{15}$$

$$H(0)\mathcal{V}_b = \frac{1}{r} \begin{bmatrix} -y_1 & 1 & 0 \\ -x_2 & 0 & -1 \\ -x_3 & 0 & -1 \end{bmatrix} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} \tag{16}$$

In case of $y_1 = y, x_2 = -x_3$ the resultant matrix will be

$$H(0)\mathcal{V}_b = \frac{1}{r}\begin{bmatrix} -y & 1 & 0 \\ -x & 0 & -1 \\ x & 0 & -1 \end{bmatrix}\begin{bmatrix} \omega_{bz} \\ \upsilon_{bx} \\ \upsilon_{by} \end{bmatrix} \tag{17}$$