BT2022 | FEB-MAY 2021 | QUIZ IV (Total = 40 marks)

Consider the given dataset (X, Y) obtained experimentally. The theoretical function $Y = f(X \mid a, b, c, d, h)$ seems to obey the following types of nonlinear ODEs whose solutions are not known.

NLS fit function nonlinear ODEs

Function 1.
$$\frac{df(X)}{dX} + a \ X \sin(h \ X \ f(X)) + b \ \sin(f(X))^2) + c \ X \ (f(X))^3 = d \ \sin(X)$$
 Function 2.
$$\frac{df(X)}{dX} + a \ \cos(h \ X \ f(X)) + b \ X \ (f(X))^2 + c \ \cos(X \ (f(X))^3) = d \ \cos(X)$$
 Function 3.
$$\frac{df(X)}{dX} + a \sin(h \ f(X)) + b \sin(X \ (f(X))^2) + c \sin((f(X))^3) = d \sin(X)$$
 Function 4.
$$\frac{df(X)}{dX} + a \cos(h \ f(X)) + b \sin(X \ (f(X))^2) + c \ ((f(X))^3) = d \sin(X)$$
 Function 5.
$$\frac{df(X)}{dX} + a \sinh(h \ f(X)) + b \ (f(X))^2 + c \ (f(X))^3 = d \cos(X)$$
 Function 6.
$$\frac{df(X)}{dX} + a \sin(h \ X \ f(X)) + b \cos(X \ (f(X))^2) + c \sin(X \ (f(X))^3) = d \sin(X^2)$$
 Function 7.
$$\frac{df(X)}{dX} + a \ X \cos(h \ f(X)) + b \ X \ (f(X))^2 + c \tan(X \ (f(X))^3) = d \sin(X^2)$$
 Function 8.
$$\frac{df(X)}{dX} + a \ \cosh(h \ f(X)) + b \ X \ (f(X))^2 + c \ \sin(X \ (f(X))^3) = d \cos(X^3)$$
 Function 9.
$$\frac{df(X)}{dX} + a \ \tanh(h \ f(X)) + b \cos(X \ (f(X))^2) + c \ X \ ((f(X))^3) = d \sin(X)$$
 Function 10.
$$\frac{df(X)}{dX} + a \ \cosh(h \ f(X)) + b \cos(X \ (f(X))^2) + c \ X \ ((f(X))^3) = d \sin(X)$$
 Function 11.
$$\frac{df(X)}{dX} + a \ \cosh(h \ f(X)) + b \ \sin(X) \ (f(X))^2 + c \ X \ ((f(X))^3) = d \sin(X)$$

Perform NLS fit using Marquart-Levenberg algorithm and obtain the estimates (5 X 2 = 10 marks), standard errors (5 X 2 = 10 marks) and p-values (5 X 1 = 5 marks) of the parameters (a, b, c, d, h) at alpha = 0.02563.

Use the Marquart-Levenberg settings as initial lambda = 50 with lower and higher limits (10^{-3} , 10^{8}), tuning parameter = 2, delta X to implement Euler differential = 10^{-6} , initial guess values (a, b, c, d, h) = (1.0, -1.7, 0.5, 1.0, 0.2) and tolerance limit to quit iteration = 10^{-12} .

Find out the total number of iterations (3 marks), final SSE (2 marks), final lambda (5 marks), calculated F value from ANOVA (3 marks), covariance of parameters (I, J) (2 marks).

Note: For the respective function number to be fitted and the parameter indices (I, J) for computing the covariance, check the data file.