

교과목 : 정보보호

9. Digital Signature

2023학년도 2학기
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- **참고자료**

- ✓ **[교재] William Stallings, Cryptography and Network Security, 7th edition**
- ✓ **[교재] Mark Stamp, Information Security: Principles and Practice, 2nd edition**
- ✓ **순천향대, 서울과학기술대 강의자료 참조**
- ✓ **<http://wiki.hash.kr/index.php/디지털서명>**

- **강의내용**

- ✓ **Digital Signature**
 - ✓ Digital Signature
 - ✓ Services of Digital Signature
 - ✓ Attacks on Digital Signature
 - ✓ Requirements of Digital Signature
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- ✓ **Digital Signature Algorithms**
 - ✓ Elgamal Digital Signature Scheme
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 - ✓ NIST Digital Signature Algorithm
 - ✓ RSA-PSS Digital Signature Scheme

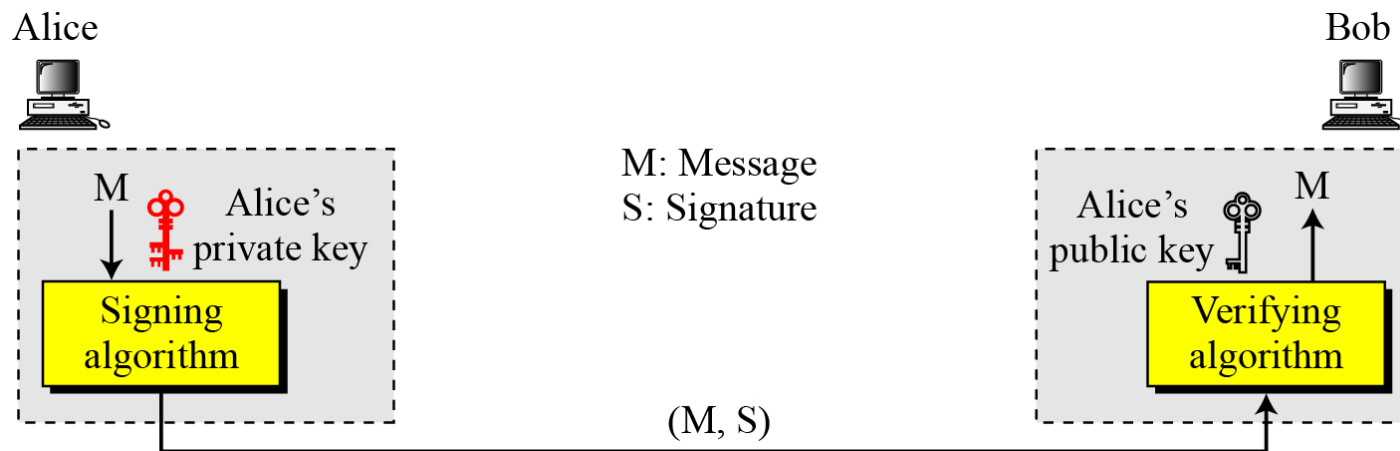
Definition of Digital Signature [Wikipedia, 해시넷 참조]

- A mathematical scheme for **verifying the authenticity (진위) of digital messages**.
 - 송신자(A)는 **A의 개인 키로 문서를 암호화**하여 수신자(B)에게 보내면, B는 **A의 공개키로 복호화**한 후, 그것이 정말 **A가 보낸 것이 맞는지 확인**하는 방식
 - **송신자는 디지털 서명을 통해 본인임을 인증**하고, 수신인은 **해당 메시지가 위, 변조 되지 않았음을 확인**한다.
 - ❖ **블록체인에 기록되는 데이터의 보안 및 무결성을 보장하는 주요 측면 중 하나이다.**
 - Digital Signature – Properties
 - It must verify the author and the date and time of the signature.
 - It must authenticate the contents at the time of the signature.
 - It must be verifiable by third parties, to resolve disputes.
- Thus, the digital signature function includes the authentication function.

Definition of Digital Signature [Wikipedia, 해시넷 참조]

- [Typically] 3개 알고리즘으로 구성
 - **Key Generation** : Selects a *private key* uniformly at random from a set of possible private keys. This algorithm outputs the *private key* and a corresponding *public key*.
 - **Signing** [서명 생성] : Given a message and a private key, produces a *signature*.
 - **Signature verifying** [서명 검증] : given the message, public key and signature, either accepts or rejects the message's claim to authenticity.
- Require Two Properties
 1. 고정된 메시지와 고정된 개인 키로부터 생성된 서명의 신뢰성은 대응하는 공개키를 사용하여 검증될 수 있다
 2. 당사자의 개인 키를 알고 있다 하더라도 유효한 서명을 생성하는 것은 불가능하다.

Definition of Digital Signature [Wikipedia, 해시넷 참조]



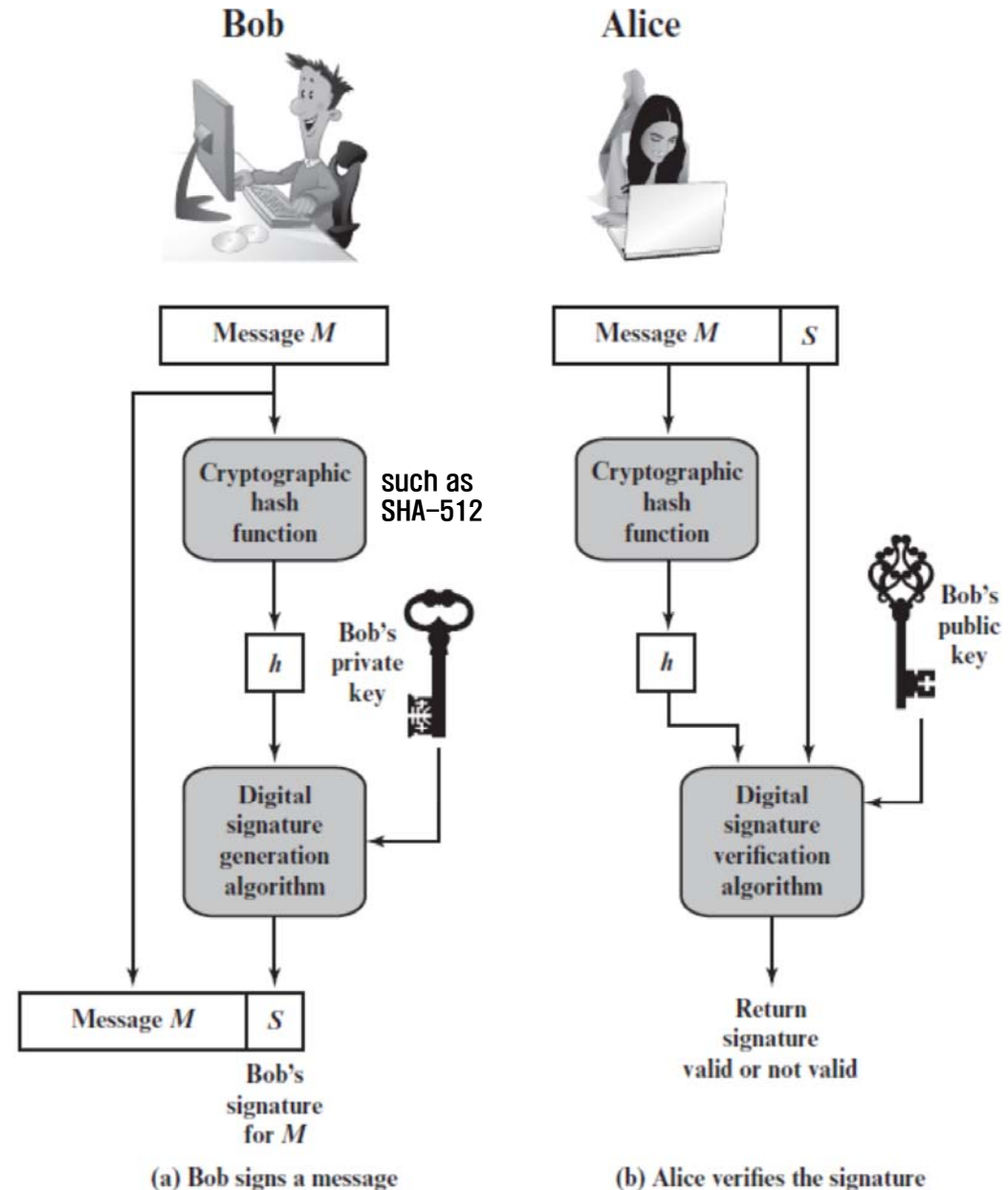
Generic Model of Digital Signature Process

디지털 서명에서는 공개 키 시스템이 필요하다. : 서명자는 자신의 개인 키로 서명을 하고, 검증자는 서명자의 공개 키로 서명을 검증한다.

- 암호화 시스템 : 수신자의 개인 키와 공개 키가 활용
- 디지털 서명 : 송신자의 개인 키와 공개 키가 사용

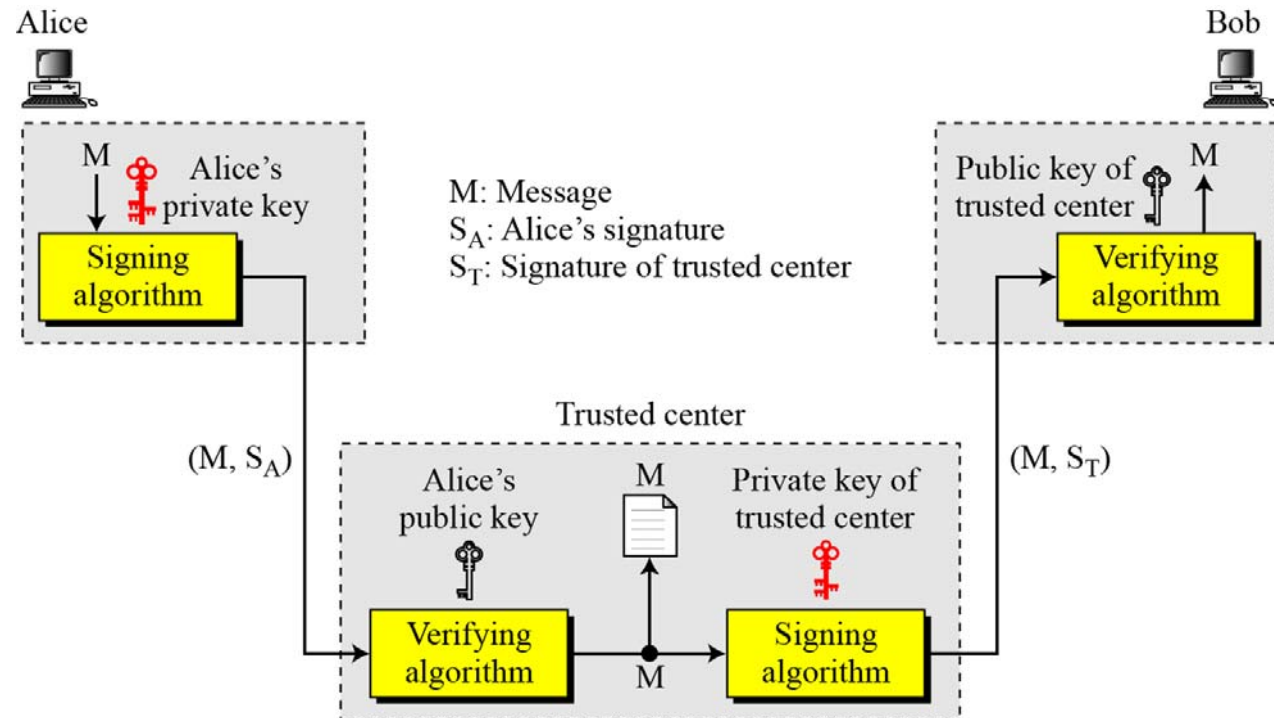
Digital Signature

- Simplified depiction of essential elements of digital signature process
 - ✓ Suppose that Bob wants to send a message to Alice. Although it is not important that the message be kept secret, **he wants Alice to be certain that the message is indeed from him.**
 - ✓ If the signature is valid, Alice is assured that the message must have been signed by Bob. **No one else has Bob's private key and therefore no one else could have created a signature that could be verified for this message with Bob's public key.**
 - ✓ In addition, it is **impossible to alter the message without access to Bob's private key**, so the message is authenticated both in terms of source and in terms of data integrity.



❖ Digital Signature 통해 기밀성(Confidentiality)은 보장할 수 없지만, 메시지 인증(Message Authentication), 메시지 무결성(Message Integrity), 부인봉쇄(Nonrepudiation)는 보장함. 기밀성도 보장하려면 암호화/복호화가 필요

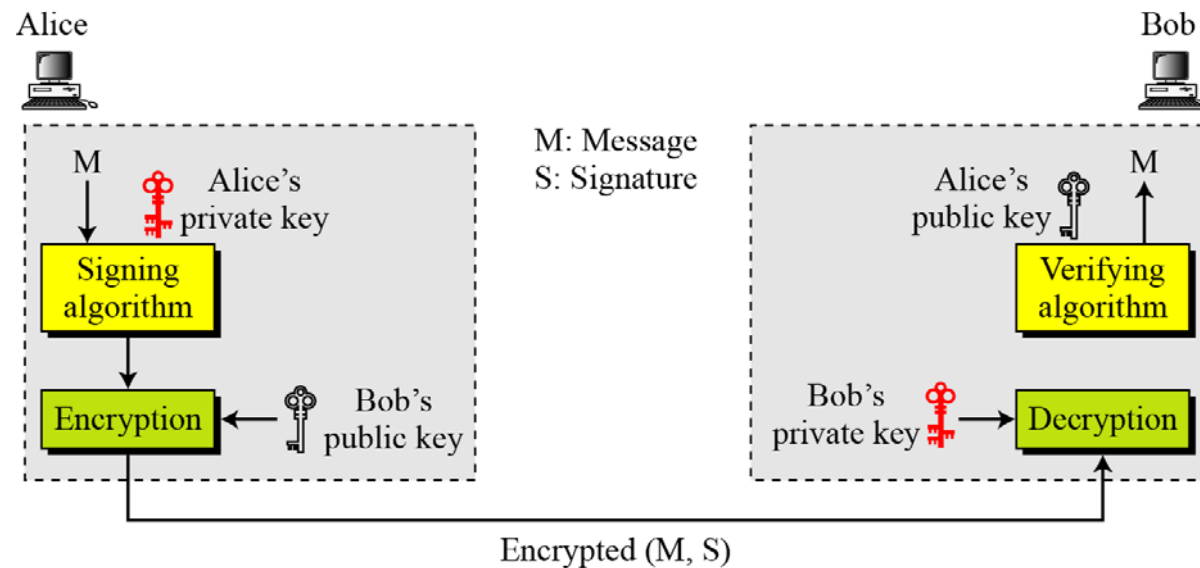
- **Message Authentication**
 - ✓ 디지털 서명 구조는 메시지 인증(데이터 근원 인증) 보장
- **Message Integrity**
 - ✓ 메시지가 변경되면 서명이 달라짐
- **Nonrepudiation**
 - ✓ 신뢰받는 제 3자를 이용하면 부인봉쇄를 할 수 있다.



부인 방지를 위한 Trusted center 활용

- Confidentiality

- ✓ Digital Signature은 Privacy를 보장해주지 못한다.
- ✓ Privacy가 필요하다면 암호화/복호화를 할 수 있는 또 다른 수단이 적용되어야 한다.



Digital Signature 구조에 기밀성 추가

Attacks :

- ✓ A : user whose signature method is being attacked
- ✓ C : attacker

- **Key-only attack** : C only knows A' s public key.
- **Known message attack** : C is given access to a set of messages and their signatures.
- **Generic chosen message attack** : C chooses a list of messages before attempting to breaks A' s signature scheme, independent of A' s public key. *C then obtains from A valid signatures for the chosen messages.* The attack is generic, because it does not depend on A' s public key.
- **Directed chosen message attack** : Similar to the generic attack, except that the list of messages to be signed is chosen after C knows A' s public key but before any signatures are seen.
- **Adaptive chosen message attack** : C is allowed to use A as an “oracle.” This means that C may request from A signatures of messages that depend on previously obtained message-signature pairs.

Forgery :

- ✓ C can do any of the following with a non-negligible probability.
- **Total break** : C determines A' s private key.
- **Universal forgery** : C finds an efficient signing algorithm that provides an equivalent way of constructing signatures on arbitrary messages.
- **Selective forgery** : C forges a signature for a particular message chosen by C.
- **Existential forgery** : C forges a signature for at least one message. C has no control over the message. Consequently, this forgery may only be a minor nuisance to A.

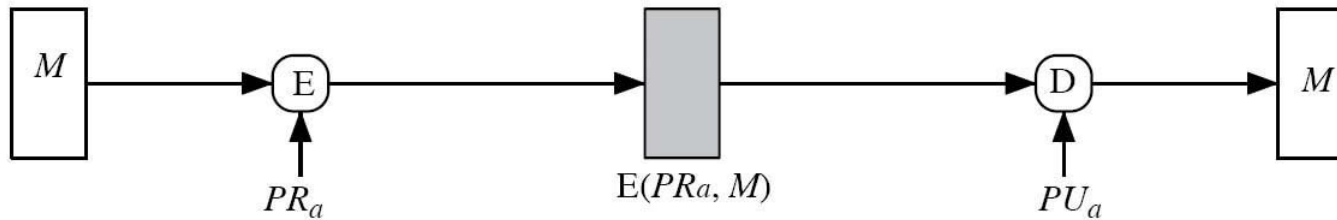
Requirements

- The signature must be a bit pattern that depends on the message being signed.
 - The signature must use some information only known to the sender to prevent both forgery and denial.
 - It must be relatively easy to produce the digital signature.
 - It must be relatively easy to recognize and verify the digital signature.
 - It must be computationally infeasible to forge a digital signature, either by constructing a new message for an existing digital signature or by constructing a fraudulent digital signature for a given message.
 - It must be practical to retain a copy of the digital signature in storage.
- Secure hash function provides a basis for satisfying these requirements.

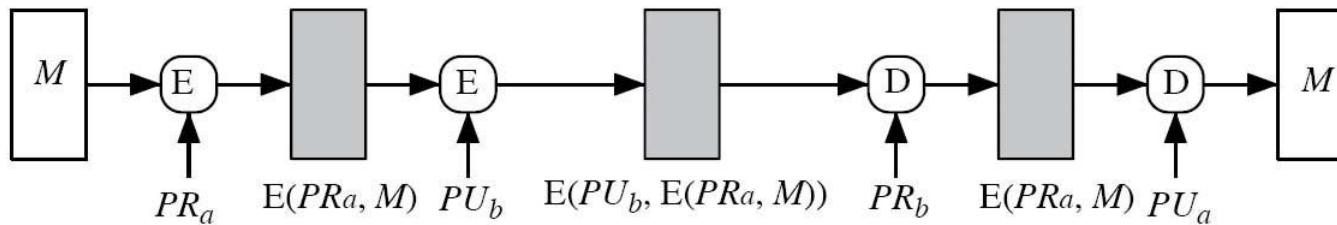
Direct Digital Signature

- 공개키 암호 알고리즘과 hash function을 사용한 모델
 - ✓ Involve only Sender & Receiver
 - ✓ Assumed receiver has sender's public-key, digital signature made by sender signing entire message or hash with private-key
 - ✓ Can encrypt using receiver's public-key
 - ✓ important that sign first then encrypt message & signature
 - ✓ 대표적인 예 : ElGamal Digital Signature, Schnoor Digital Signature
- Weakness
 - ✓ Sender의 private-key 안전성에 따라 유효성이 달라질수 있음
 - ✓ Sender가 private-key를 분실하거나 도난당했다고 거짓주장 하거나, 실제로 Sender의 private-key가 도난 당할 수 있으며, 제3자의 개입이 필요하다는 문제점이 있음
 - 독립적인 검증 프로세스가 없기 때문에 발신자와 수신자 간의 신뢰가 필요함.
 - 이 프로세스에서는 보낸 사람에게 private-key가 있어야 하고 받는 사람에게만 public-key가 있어야 함

Direct Digital Signature



(c) Public-key encryption: authentication and signature



(d) Public-key encryption: confidentiality, authentication, and signature

Weakness: Security depends on sender's private-key

Arbitrated Digital Signature

- Involves use of arbiter A
 - ✓ validates any signed message
 - ✓ then dated and sent to recipient
- Requires suitable level of trust in arbiter
- Can be implemented with either private or public-key algorithms
- Arbiter may or may not see message
- Arbiter이 one-way이나 상대방에 대한 편견을 보여줄 수 있는 가능성이 있음

Signature

Signature

(1) $X \rightarrow A: ID_X \parallel E(PR_X, [ID_X \parallel E(PU_Y, E(PR_X, M))])$
(2) $A \rightarrow Y: E(PR_A, [ID_X \parallel E(PU_Y, E(PR_X, M)) \parallel T])$

(c) Public-Key Encryption, Arbiter Does Not See Message

Notations:

X=sender

M=message

Y=recipient

T=time stamp

A=Arbiter

PR_X =X's private key

ID_X =ID of X

PU_Y =Y's public key

PR_A =A's private key

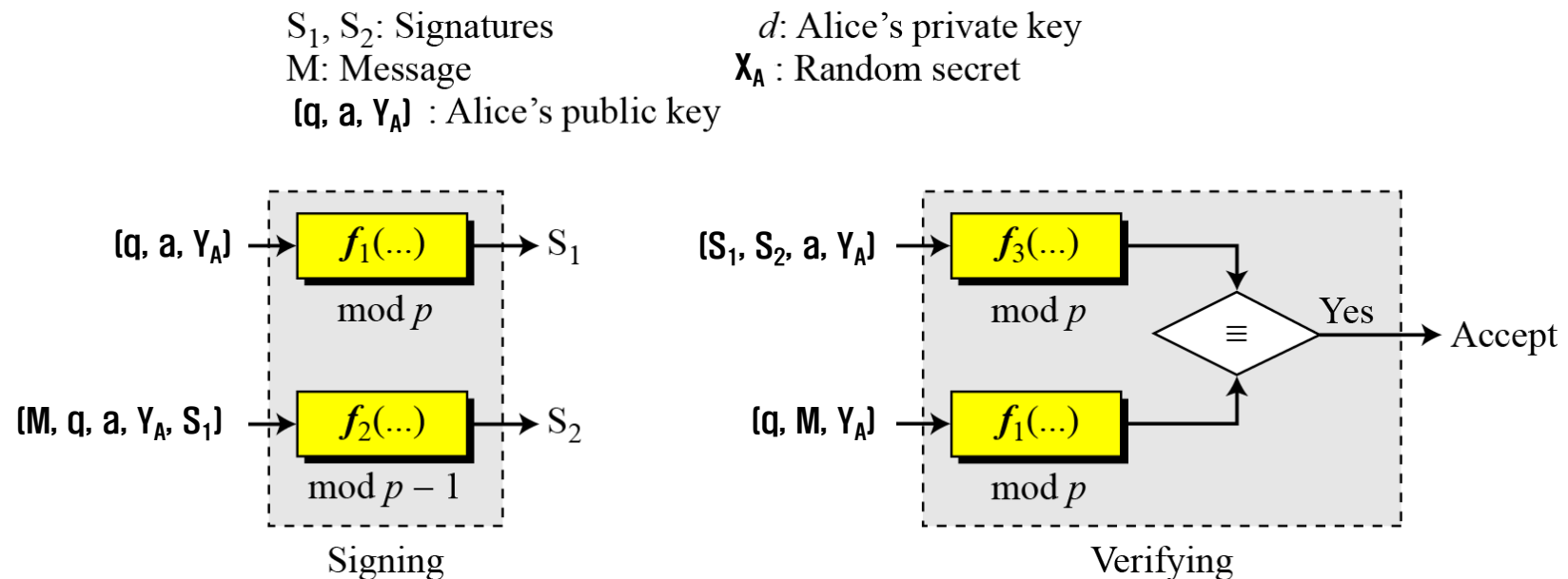
Weakness: twice public-key encryptions on the message

Digital Signature Algorithms

1. Elgamal Digital Signature Scheme

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- Elgamal signature is designed to enable **encryption by user's private key**, and **decryption by the user's public key**
 - ✓ involves the use of **the private key for digital signature generation** and **the public key for digital signature verification**



Elgamal Digital Signature에 대한 일반적 아이디어

1. Elgamal Digital Signature Scheme

2023년 2학기

- First, there are prime number q and its primitive root a .
- **Key generation**
 - ✓ Generate a random integer X_A , such that $1 < X_A < q-1$.
 - ✓ Compute $Y_A = a^{X_A} \bmod q$
 - ✓ A 's private key is X_A ; A 's public key is $\{q, a, Y_A\}$
- **(User A) form digital signature as**
 - ✓ Compute the hash $m=H(M)$, which m is an integer in $0 \leq m \leq q-1$
 - ✓ Choose a random integer K such that $1 \leq K \leq q-1$ and $\gcd(K, q-1) = 1$. That is, K is relatively prime to $q-1$.
 - ✓ Compute $S_1 = a^K \bmod q$. Note that this is the same as the computation of C_1 for Elgamal encryption.
 - ✓ Compute $K^{-1} \bmod (q-1)$. That is, compute the inverse of K modulo $q-1$.
 - ✓ Compute $S_2 = K^{-1} (m - X_A S_1) \bmod (q-1)$.
 - ✓ The signature consists of the pair $[S_1, S_2]$.

- (User B) Verify the signature as

- ✓ Compute $V_1 = a^m \bmod q$
- ✓ Compute $V_2 = (Y_A)^{S_1}(S_1)^{S_2} \bmod q$

- The signature is valid if $V_1 = V_2$

❖ Let us demonstrate that this is so. Assume that the equality is true. Then we have

$$\alpha^m \bmod q = (Y_A)^{S_1}(S_1)^{S_2} \bmod q$$

$$\alpha^m \bmod q = \alpha^{X_A S_1} \alpha^{K S_2} \bmod q$$

$$\alpha^{m - X_A S_1} \bmod q = \alpha^{K S_2} \bmod q$$

$$m - X_A S_1 \equiv K S_2 \bmod (q - 1)$$

$$m - X_A S_1 \equiv K K^{-1} (m - X_A S_1) \bmod (q - 1)$$

assume $V_1 = V_2$

substituting for Y_A and S_1

rearranging terms

property of primitive roots

substituting for S_2

- **Example**

- ✓ Prime field $GF(19)$; $q=19$. It has primitive roots $\{2, 3, 10, 13, 14, 15\}$.
- ✓ We choose $a=10$.

Alice generates a key pair as follows:

1. Alice chooses $X_A = 16$.
2. Then $Y_A = \alpha^{X_A} \bmod q = 10^{16} \bmod 19 = 4$.
3. Alice's private key is 16; Alice's public key is $\{q, \alpha, Y_A\} = \{19, 10, 4\}$.

Suppose Alice wants to sign a message with hash value $m = 14$.

1. Alice chooses $K = 5$, which is relatively prime to $q - 1 = 18$.
2. $S_1 = \alpha^K \bmod q = 10^5 \bmod 19 = 3$ (see Table 2.7).
3. $K^{-1} \bmod (q - 1) = 5^{-1} \bmod 18 = 11$.
4. $S_2 = K^{-1} (m - X_A S_1) \bmod (q - 1) = 11 (14 - (16)(3)) \bmod 18 = -374 \bmod 18 = 4$.

Bob can verify the signature as follows.

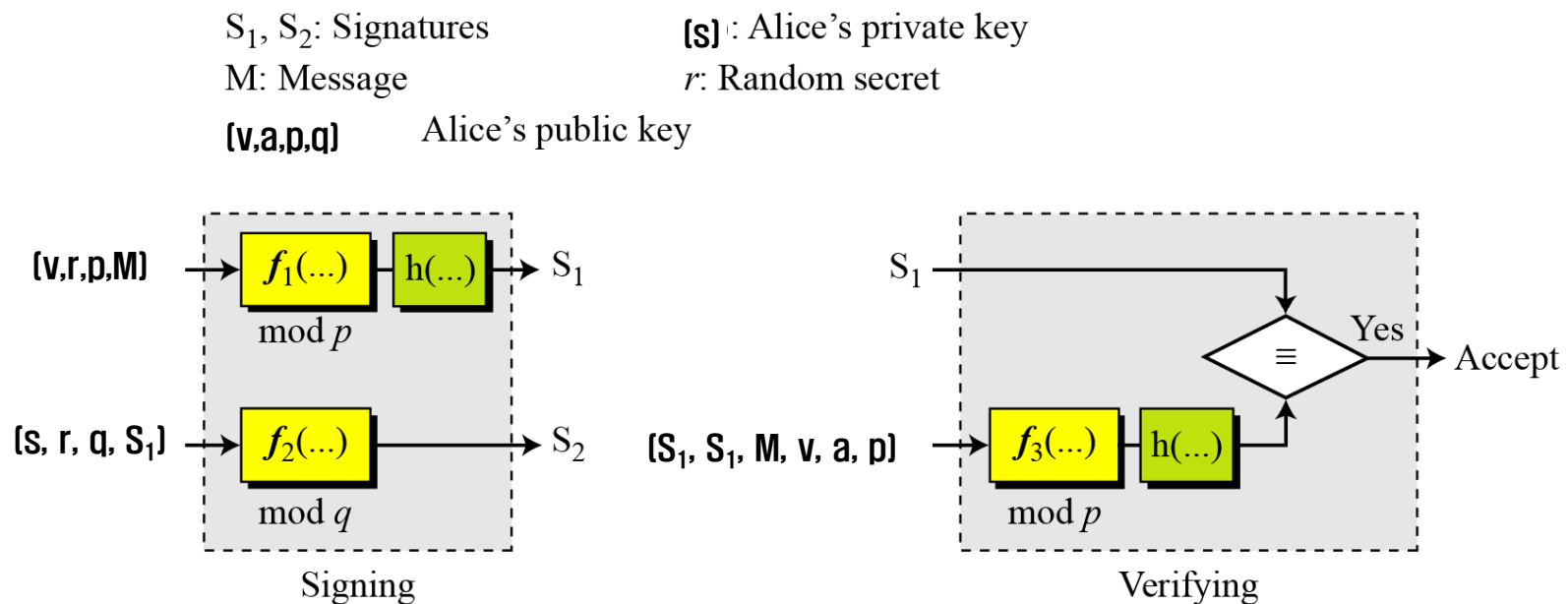
1. $V_1 = \alpha^m \bmod q = 10^{14} \bmod 19 = 16$.
2. $V_2 = (Y_A)^{S_1} (S_1)^{S_2} \bmod q = (4^3)(3^4) \bmod 19 = 5184 \bmod 19 = 16$.

Thus, the signature is valid because $V_1 = V_2$.

2. Schnorr Digital Signature Scheme

2023년 2학기

- It is based on discrete logarithms
- Minimizes the message-dependent amount of computation required to generate a signature



Schnorr Digital Signature에 대한 일반적 아이디어

2. Schnorr Digital Signature Scheme

2023년 2학기

- (private/public) Key generation

- ✓ Choose primes p and q , such as q is a prime factor of $p-1$
 - p is 1024 bit , q is 160bit num (as SHA-1)
- ✓ Choose an integer a , such that $aq = 1 \bmod p$. the values a , p , and q comprise a global public key that can be common to a group of user
- ✓ Choose a random integer s with $0 < s < q$. this is the user s private key
- ✓ Calculate $v = a^{-s} \bmod p$, this is the user public key.
- ✓ a, p, q : global public key
- ✓ s : user s private key
- ✓ v : user s public key

- **Signature generation**

- ✓ Choose a random integer r with $0 < r < q$ and compute $x = a^r \bmod p$.
 - This computation is a preprocessing state independent of the message M to be signed
- ✓ Concatenate the message M with x and hash the result to compute the value e : $e = H(M||x)$
- ✓ Compute $y = (r + se) \bmod q$ the signature consists of the pair (e, y) .

- **Signature verification**

- ✓ Compute $x' = a^y v^e \bmod p$.
- ✓ Verify that $e = H(M||x')$

- To see the verification works, observe that

- ✓ $x' \equiv a^y v^e \equiv a^y a^{-se} \equiv a^{y-se} \equiv a^r \equiv x \pmod{p}$
- ✓ Hence, $H(M||x') = H(M||x)$

- DSA is designed to provide only the digital signature by the National Institute of Standards and Technology. DSA make use of the Secure Hash Algorithm (SHA).
- Unlike RSA, it cannot be used for encryption or key exchange. Nevertheless, It is a public-key technique.
- It is FIPS 186 proposed in 1991 and revised in 1993 → FIPS 186-2 in 2000, → FIPS 186-3 in 2009 and FIPS 186-4 in 2013 (based on RSA and elliptic curve cryptography)
- DSA is based on the difficulty of computing DLP (Discrete Logarithm Problem)
- Also it is based on the Elgamal and Schnorr scheme.
- It' s main processes are independent on Message.

3. NIST Digital Signature Scheme (DSA)

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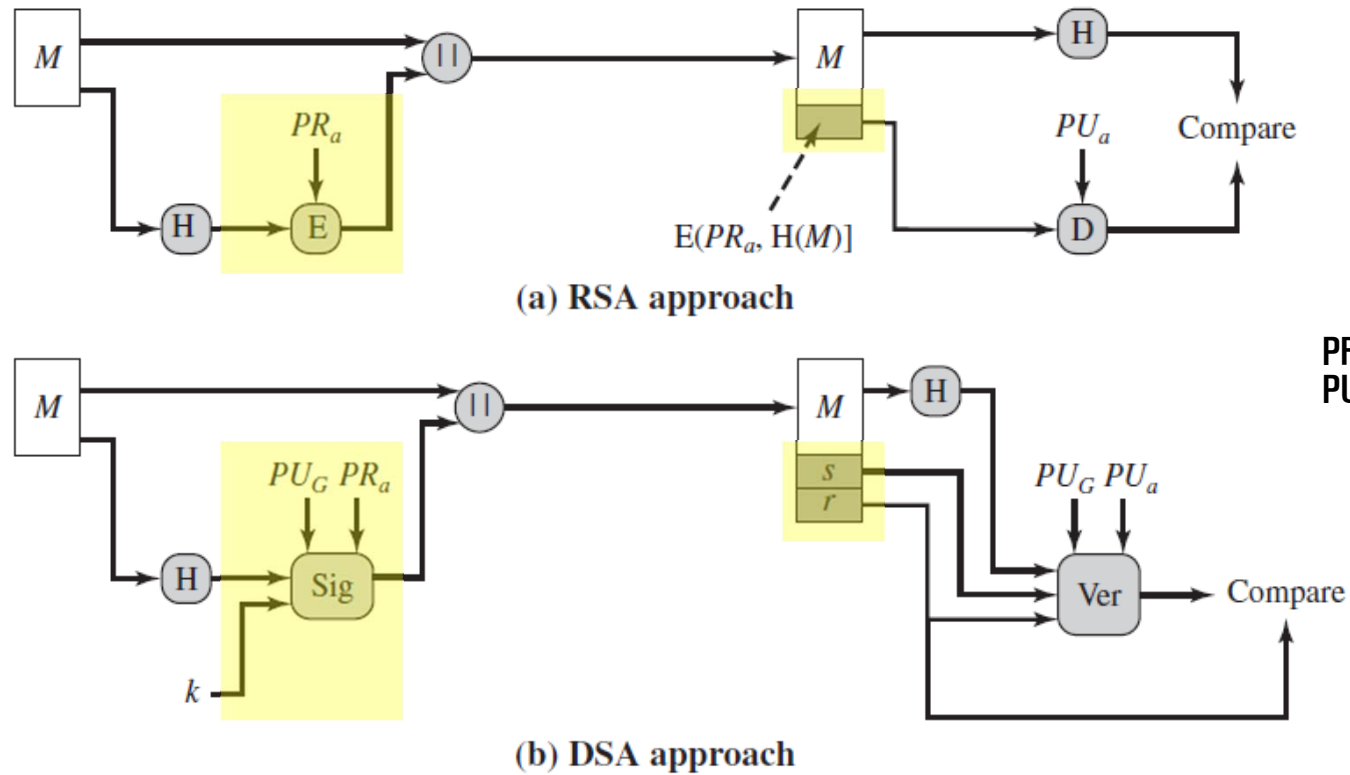


Figure 13.3 Two Approaches to Digital Signatures

3. NIST Digital Signature Scheme (DSA)

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- Key generation

Global Public-Key Components

- p prime number where $2^{L-1} < p < 2^L$
for $512 \leq L \leq 1024$ and L a multiple of 64;
i.e., bit length of between 512 and 1024 bits
in increments of 64 bits
- q prime divisor of $(p - 1)$, where $2^{N-1} < q < 2^N$
i.e., bit length of N bits
- $g = h(p - 1)/q \bmod p$,
where h is any integer with $1 < h < (p - 1)$
such that $h^{(p-1)/q} \bmod p > 1$

User's Private Key

- x random or pseudorandom integer with $0 < x < q$

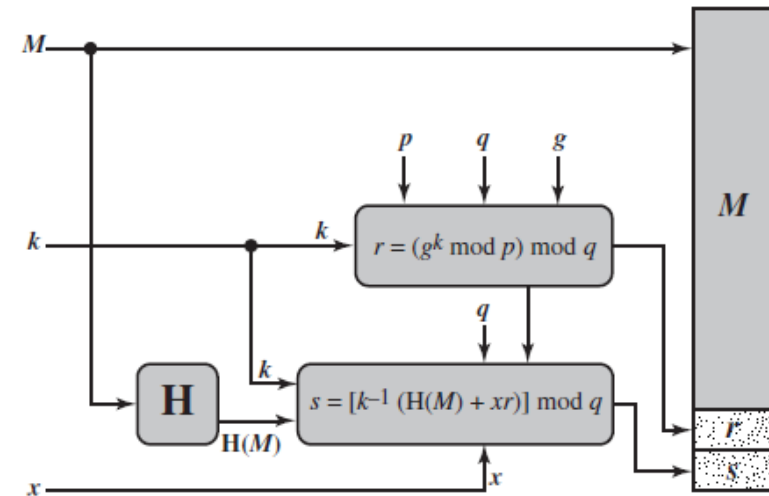
User's Public Key

$$y = g^x \bmod p$$

User's Per-Message Secret Number

- k random or pseudorandom integer with $0 < k < q$

- Signature generation



(a) Signing

Signing

$$r = (g^k \bmod p) \bmod q$$

$$s = [k^{-1} (H(M) + xr)] \bmod q$$

$$\text{Signature} = (r, s)$$

M = message to be signed

$H(M)$ = hash of M using SHA-1

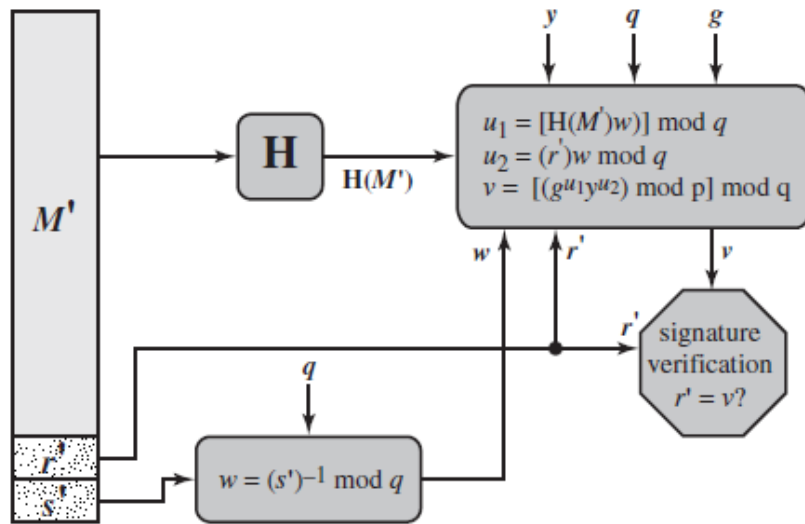
M', r', s' = received versions of M, r, s

Figure 13.4 The Digital Signature Algorithm (DSA)

3. NIST Digital Signature Scheme (DSA)

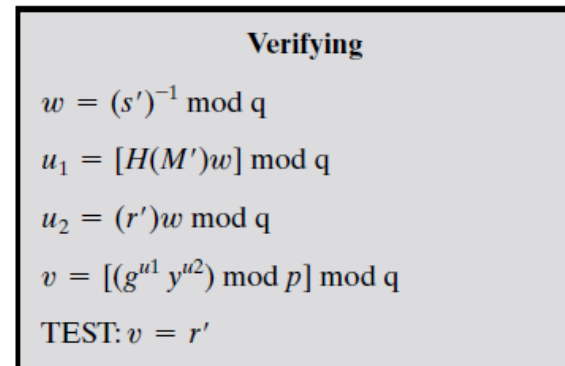
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- Signature Verification



(b) Verifying

Figure 13.5 DSA Signing and Verifying



M = message to be signed

$H(M)$ = hash of M using SHA-1

M', r', s' = received versions of M, r, s

4. RSA-PSS Digital Signature Scheme

2023년 2학기

- **RSA-PSS (RSA Probabilistic Signature Scheme)**, which is the latest of the RSA schemes and the one that RSA Laboratories recommends as the **most secure of the RSA schemes**.
- RSA-based schemes differ mainly in the **padding format**, and in **how the verification operation determines that the hash and message representative are consistent**
- Before PSS, there was not been possible to develop mathematical proof that scheme is secure as RSA encryption/decryption.

Mask Generation Function (MGF)

- MGF is used as a **building fixed length output**.
- **MGF(X, maskLen)** : A pseudo random function that has as the input of a string X in any length and desired length L (maskLen) in octets of the output.
- Typically based on hash function as SHA-1
- In RSA-PSS, MGF1 is used with the parameters

Option	Hash:	Hash function with output hLen octets
Input	X; maskLen:	Octet string to be masked Length in octets of mask
Output	mask:	An octet string of length maskLen

➤ MGF1 is defined as follows:

- Initialize variables

T = empty string

$K = \lceil \text{maskLen} / \text{hLen} \rceil - 1$

- Calculate intermediate values

for *counter* = 0 to k

Represent *counter* as a 32-bit string C

$T = T \parallel \text{Hash}(X \parallel C)$

- Output results

Mask = the leading maskLen octets of T

- If $\text{maskLen} = \text{hLen}$,
the output is the hash of the X concatenated with 32-bit counter value of 0.
- If $\text{maskLen} \geq \text{hLen}$,
MGF1 iterate by hashing X concatenated with the counter and appending that to the current string T .
- output : $\text{Hash}(X \parallel 0) \parallel \text{Hash}(X \parallel 1) \parallel \dots \parallel \text{Hash}(X \parallel K)$
- This is repeated until T is greater or equal with maskLen, at which point the output is the first maskLen octets of T

The Signing Operation

➤ Message Encoding

- Generate from a message M a fixed-length message digest, (encoded message, EM)

Definitions of parameters and functions

Options	Hash	hash function with output $hLen$ octets. The current preferred alternative is SHA-1, which produces a 20-octet hash value.
	MGF	mask generation function. The current specification calls for MGF1.
	$sLen$	length in octets of the salt. Typically $sLen = hLen$, which for the current version is 20 octets.
Input	M	message to be encoded for signing.
	$emBits$	This value is one less than the length in bits of the RSA modulus n .
Output	EM	encoded message. This is the message digest that will be encrypted to form the digital signature.
Parameters	$emLen$	length of EM in octets = $\lceil emBits/8 \rceil$.
	padding ₁	hexadecimal string 00 00 00 00 00 00 00 00; that is, a string of 64 zero bits.
	padding ₂	hexadecimal string of 00 octets with a length $(emLen - sLen - hLen - 2)$ octets, followed by the hexadecimal octet with value 01.
	$salt$	a pseudorandom number.
	bc	the hexadecimal value BC.

4. RSA-PSS Digital Signature Scheme

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➤ Message Encoding Process

1. Generate the hash value of M :
 $mHash = Hash(M)$
2. Generate a pseudorandom octet string $salt$ and form block M' :
 $M' = padding_1 || mHash || salt$
3. Generate the hash value of M' :
 $H = Hash(M')$
4. Form data block DB :
 $DB = padding_2 || salt$
5. Calculate the MGF value of H :
 $dbMask = MGF(H, emLen - hLen - 1)$
6. Calculate maskedDB:
 $maskedDB = DB \oplus dbMask$
7. Set the leftmost $8emLen - emBits$ bits of the leftmost octet in masked DB to 0
8. $EM = maskedDB || H || bc$

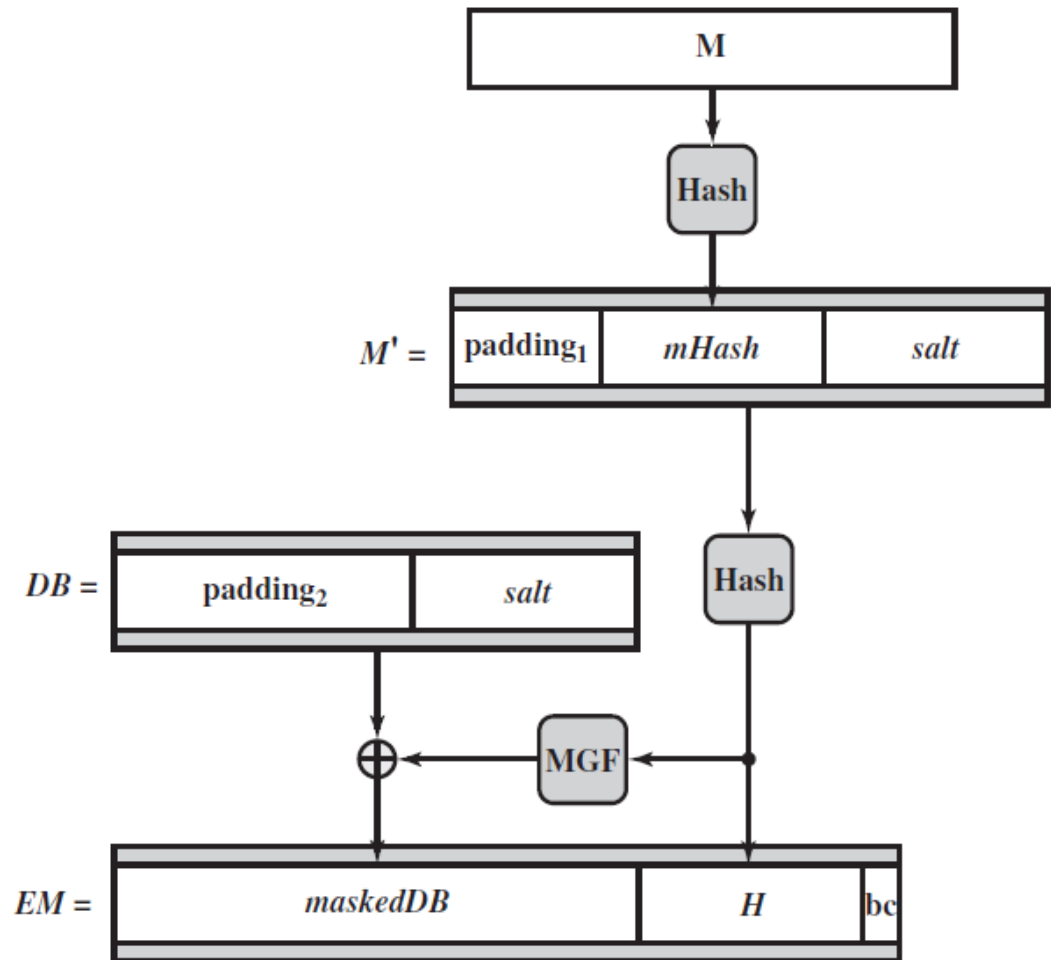


Figure 13.7 RSA-PSS Encoding

➤ Forming the Signature

- Private key $\{d, n\}$, public key $\{e, n\}$ – (RSA algorithm)
- Treat the octet string EM as an unsigned, nonnegative binary integer m . the signature s is formed by encrypting octet string S of length k octets.

$$s = m^d \bmod n$$

- Let k be the length in octets of the RSA modulus n .
- If key size is 2048bits, then $k = 2048/8=256$.
- Then convert the signature value s into the octet string S of length k octets.

The Signing Verification

➤ Decryption

- The message digest m is recovered by decrypting s

$$m = s^e \bmod n$$

- Then, convert m to EM of length $emLen = \lceil (modBits - 1)/8 \rceil$ octets, where $modBits$ is the length in bits of the modulus n .

4. RSA-PSS Digital Signature Scheme

2023년 2학기

The Signing Verification

➤ EM Verification

1. Generate the hash value of M : $mHash = Hash(M)$
2. If $emLen < hLen + sLen + 2$, output “inconsistent” and stop
3. If the rightmost octet of EM does not have hexadecimal value BC, output “inconsistent” and stop
4. Let maskedDB be the leftmost $emLen - hLen - 1$ octets of EM, and let H be the next $hLen$ octets
5. If the leftmost $8emLen - emBits$ bits of the leftmost octet in maskedDB are not all equal to zero, output “inconsistent” and stop
6. Calculate $dbMask = MGF(H, emLen - hLen - 1)$

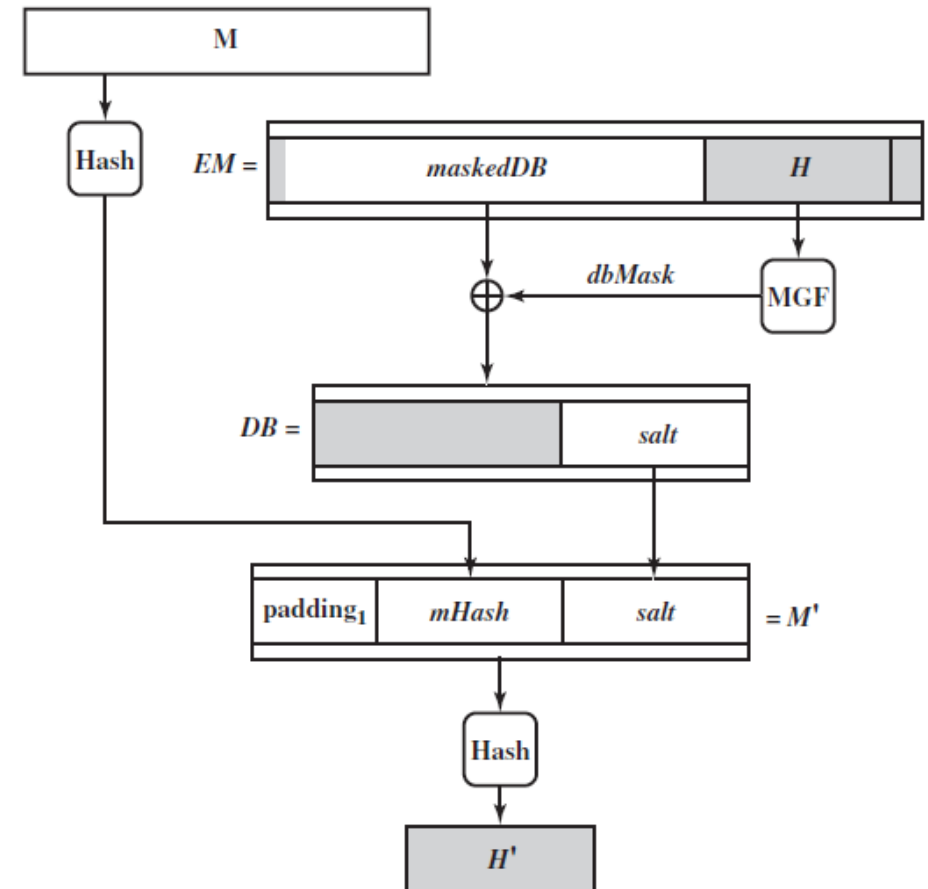


Figure 13.8 RSA-PSS EM Verification

The Signing Verification

➤ EM Verification

7. Calculate **DB = maskedDB XOR dbMsk**
8. Set the leftmost (8emLen-emBits) bits of the leftmost octet in DB to zero
9. If the leftmost (emLen - hLen - sLen - 1) octets of DB are not equal to padding₂, output “inconsistent” and stop
10. Let salt be the last sLen octets of DB
11. Form block **M' = padding₁ || mHash || salt**
12. Generate the hash value of M' : **H' = Hash(M')**
13. If $H = H'$, output “consistent.” Otherwise, output “inconsistent”

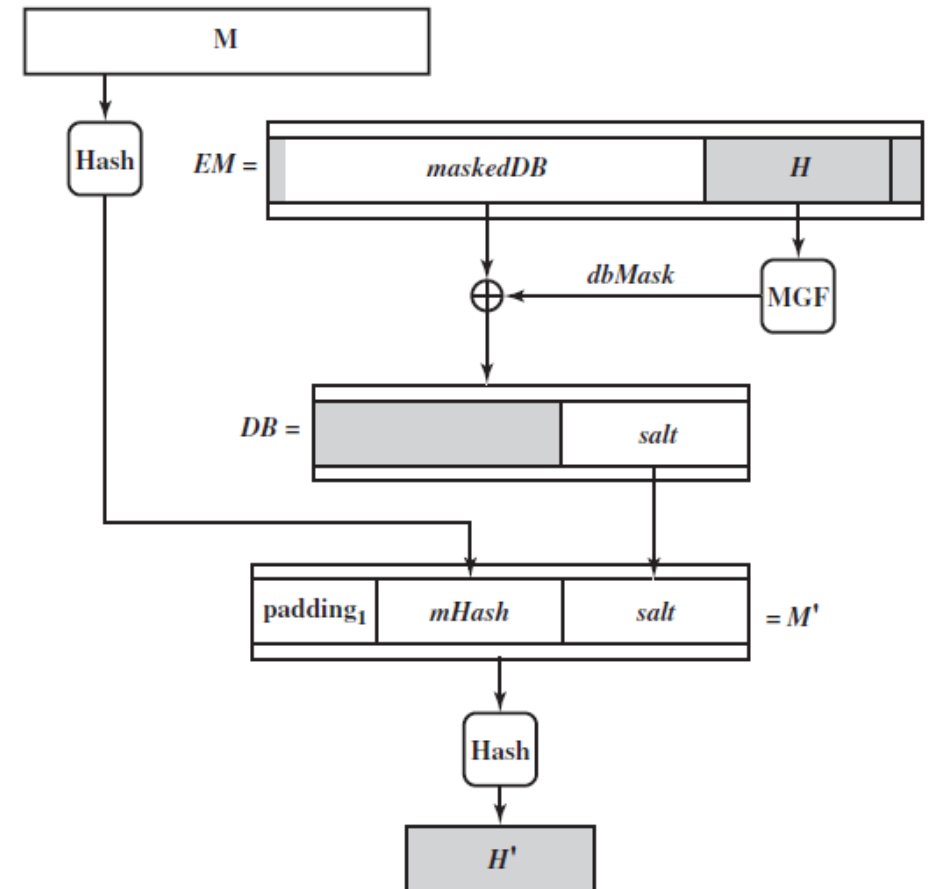


Figure 13.8 RSA-PSS EM Verification