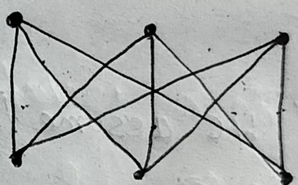
1. Do you think complete bipartite graph K_{nn}, where $n \geq 3$ is Planar or not? Explain your Answer.

→ Complete bipartite graph K_{nn} where $n \geq 3$ is a non-Planar.

We know that, In graph Theory, a Planar graph is a graph that can be draw in a Plane without any edges crossing each other, and from the kuratowski theorem we say that a graph is non-Planner if it contains a subgraph that is a subdivision of $K_{3,3}$ or K_5.

Ex → $K_{3,3}$ is non-Planar



($K_{3,3}$)

Proof → $K_{3,3}$ is a complete bipartite graph on 6 verties and 9 edges

In a bipartite graph there is no odd cycles.

∴ $K_{3,3}$ has no triangle.

If G is a Plane graph on $n \geq 3$ vertices and m edges with no triangles, $m \leq 2n-4$

n = no of verties

∴ $2n-4 = 2\times6-4$

$= 12-4 = 8$

m = no of edges

$m \leq 2n-4$

$9 \nleq 8$

Hence, $K_{3,3}$ is not a Planar graph.

② How many bit strings of length four do not have two Consecutive 1s?

→ Let $f(n)$ be the number of bit strings of length n with no two consecutive 1s.

Now the initial condition.

① for 1 bit strings like 0, 1

$\therefore f(1) = 2$

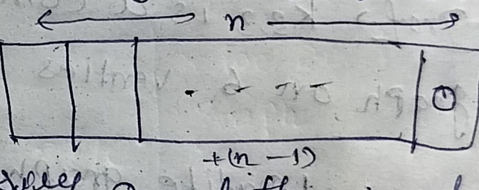
and

② for 2 bit strings where do not have two consecutive 1s. like = 00, 10, 01

$\therefore f(2) = 3$

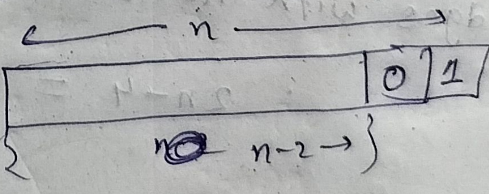
Now,

we assume that n length bit strings where last bit is 0



So, last bit is 0 so, we can easily count the no of bit strings ~~do not have two~~ that of length $(n-1)$ there do not have two consecutive 1s.

Again if the n length bit strings where last bit is 1 then because we do not want consecutive 1 then second last bit always 0, now



no of bit strings of length $(n-2)$ there do not have two consecutive 1s.

$\therefore$ ~~The Relation is~~, $\therefore$ The Recurrence Relation →

~~f(n) = f~~ $\boxed{f(n) = f(n-1) + f(n-2)}$

$\therefore f(1) = 2$

$f(2) = 3$

$\therefore f(3) = f(2) + f(1) = 3+2 = 5$

$f(4) = f(3) + f(2) = 5+3 = 8$

$\therefore$ There are 8 bit strings of length 4 that do not contain consecutive 1s.

3. How many simple labelled graphs are possible with n vertices? Explain your answer.

→ We know that A simple graph means there are no loops means edges from a vertex to itself and no multiple edges between the same pair of vertices and labeled graph means each vertex is distinctly labeled.

Now in a simple ~~undirected~~ undirected graph with n vertices Each pair of verties can either be connected or not connected,

so the number of edges in ~~any~~ a undirected graph with n verties is $\frac{n(n-1)}{2}$

Each of these pairs (edges) can independently exist or not exist so there are 2 choices.

and $\frac{n(n-1)}{2}$ edges can be independently chosen present or absent.

$\therefore$ Total number of simple labeled graphs

$= \cancel{2^{n(n-1)}} \; 2^{\frac{n(n-1)}{2}}$

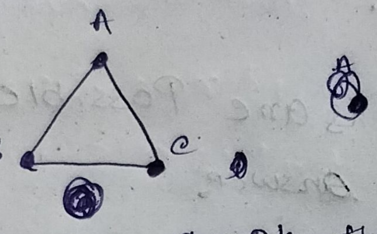
EX → now take a Example when no of ventiees$(n) = 3$

∴ Number of Possible edges $= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = \frac{3 \times 2}{2} = 3$

and Total simple labled graphs

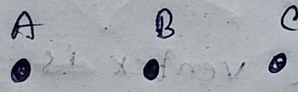
$\Rightarrow 2^{\frac{n(n-1)}{2}} \Rightarrow 2^{3} = 8$

Draw the graph

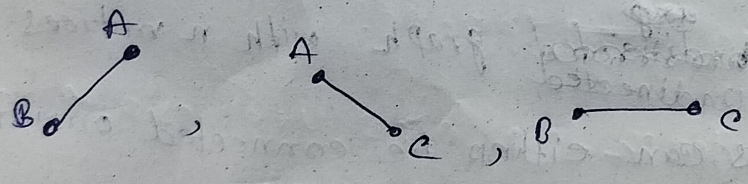


For this graph the ~~total~~ simple Labled graphs are

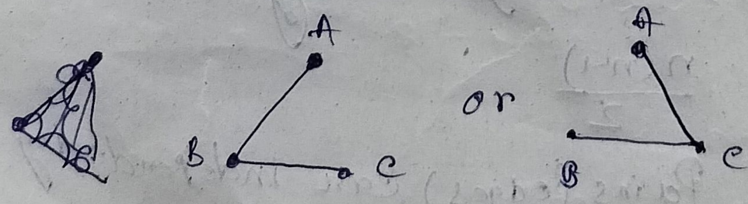
① Three isolated venties



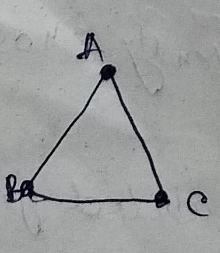
② Two venties with one ~~edg~~ edge



③ Three venties and 2 two edge.



④ Three venties and 3 edge



* For Directed graph

(i) The no of edges in directed graph

$\Rightarrow n(n-1)$

and

(ii) Total no of simple labeled graph

$\Rightarrow 2^{n(n-1)}$

④ Determine the mean and standred deviation at the probability distribution at the upper face of a unbiased-dice be thrown.

→ We know that the upper face of a unbiased dice outcomes are 1, 2, 3, 4, 5, 6

∴ Each number has equal probability $(P(x)) = \frac{1}{6}$

∴ mean (Expected value) of the distribution (μ)

$$\mu = E(x) = \sum_{i=1}^{6} x_i \cdot P(x_i) = \frac{1+2+3+4+5+6}{6}$$

$$\Rightarrow \frac{21}{6} \Rightarrow 3.5$$

∴ variance $(\sigma^2) = \sum_{i=1}^{6} P(x_i) \cdot (x_i - \mu)^2$

$$\therefore \sigma^2 = \frac{1}{6}\{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2\}$$

$$= \frac{1}{6}(6.25 + 2.25 + 0.25 + 0.25 + 2.25 + 6.25)$$

$$\Rightarrow \frac{17.5}{6} \simeq 2.91667$$

∴ $\sigma^2 = 2.91667$

$\sigma = \sqrt{2.91667}$

$\simeq 1.7078$

∴ standard deviation $(\sigma) = 1.7078$