# Discrete Mathematics Assignment

## Assignment 1

1. Let R be the set of all real numbers. Using the fact that every cubic equation with real coefficients has at least one real root, show that $x \to (x^3 - x)$ defines a mapping of R onto R. Also check whether this mapping is one-one or not.

→ let, $y \in \mathbb{R}$

s.t $f(x) = y$

Here, $f(x) = x^3 - x$

So, $x^3 - x = y$

$\Rightarrow x^3 - x - y = 0$

For, any real value of $y$ the equation $x^3 - x - y = 0$ ~~has~~ is a cubic equation with real coefficients has at least a ~~root~~ real root.

$\therefore$ The $f$ is onto.

$f(0) = 0$

$f(1) = 1 - 1 = 0$

$f(-1) = -1 + 1 = 0$

let, $x_1, x_2 \in \mathbb{R}$

$\exists\ f(x_1), f(x_2) \in \mathbb{R}$

s.t. $f(x_1) = f(x_2)$ and $x_1 \neq x_2$

$x_1(x_1^2 - 1)$

$\cancel{x_1(x_1-1)(x_1+1)}$

$\cancel{(x_1 - x_2)}$

$(x_1 - x_2)(x_1^2 - x_1 x_2 + x_2^2)$

$\Rightarrow x_1^3 - x_1 = x_2^3 - x_2$

$\Rightarrow x_1^3 - x_2^3 = x_1 - x_2$

$\Rightarrow \cancel{(x_1 - x_2)}(x_1^2 - x_1 x_2 + x_2^2) = \cancel{(x_1 - x_2)}$

$\Rightarrow x_1^2 - x_1 x_2 + x_2^2 = 1$ $\quad$ [as $x_1 \neq x_2$]

Here, for many value of $x \in \mathbb{R}$ ~~the~~ $\exists$ ~~$f(x)$~~ there exists same $f(x) \in \mathbb{R}$.

So, $f$ is not one to one.

2. Prove that cyclic group must be an abelian group.

→ Let, $G$ be a cyclic group and $a$ be a generator $G$ so that

$$G = \{a^n : n \in \mathbb{Z}\}$$

let, $g_1$ and $g_2$ are any two elements of $G$

$\exists\ r, s \in \mathbb{Z}$ s.t. $g_1 = a^r$ and $g_2 = a^s$

$\therefore\ g_1 g_2 = a^r \cdot a^s = a^{r+s}$

$= a^{s+r}$

$= a^s \cdot a^r = g_2 g_1 \quad \forall g_1, g_2 \in G$

$\therefore$ $G$ is abelian.

3. Find the edge connectivity of the complete graph with $n$ vertices.

→ The edge connectivity is the minimum number of edges that need to be removed to make the graph disconnected.

This is a complete graph with $n$ vertices (denoted by $K_n$) that means —

i. each vertex is connected to every other vertex.

ii. So, the degree of each vertex is $(n-1)$

So, we need to remove at least $(n-1)$ degrees.

So, the edge connectivity is $\lambda(K_n) = n-1$.

4. How many squares are there in a chess board? Explain your answer.

→ We know a chess board is 8×8 in size, so if we think that in 8×8 we have 64 squares.

So, we can partitioned the chess board, means 8 boxes in different parts.

Say

$1 \times 1 = 1$ box

$2 \times 2 = 4$ boxes

$3 \times 3 = 9$ boxes

We took 1 box as unit. So, with 3 unit we can form 9 boxes.

$4 \times 4 = 16$ boxes

$5 \times 5 = 25$ boxes

$6 \times 6 = 36$ boxes

$7 \times 7 = 49$ boxes

$8 \times 8 = 64$ boxes

We cannot go further because the chess board is of 8×8.

So, the sum is $= 1 + 4 + 9 + \cdots + 49 + 64$

$= 204$

∴ Total squares in a chess board is 204.