1. Suppose the set S_n contains all the bit strings of length i, where $i = 0, 1, 2, \ldots, n$

Again ~~length~~ let $P_i = S_i - S_{i-1}$, $i = 1, 2, 3, \ldots, n$.

Then show that the sets P_i, $i = 1, 2, \ldots, n$ form the partition of the set S_n.

S_i contains all the bit strings of length $\leq i$.

That means, as example,

$$S_2 = \{"", "0", "1", "00", "01", "10", "11"\}$$

$$\therefore \quad S_n = \bigcup_{i=0}^{n} S_i$$

And each $P_i = S_i - S_{i-1}$

That means, as example

$$P_2 = S_2 - S_1$$

$$= \{"00", "01", "10", "11"\}$$

~~Also~~ So, P_i contains all the bit strings of exactly length i

Now, A collection of sets A_i, $i = 1, 2, \ldots, n$ can be a ~~partition~~ partition of set A if

i) $\bigcup_{i=1}^{k} A_i = A$

ii) $A_i \cap A_j = \phi \;\; \forall\, i \neq j$

Now, S_n contains all the bit strings of length $\leq n$

Now
each P_i contains all the bit strings of only length i

So, $P_i \cap P_j = \phi \ \forall \ i \neq j$

But
$\bigcup_{i=1}^{n} P_i \neq S_n$. as there is no P_0.

Absence of P_0 means there is no bitstrings length zero, in $\bigcup_{i=1}^{n} P_i$. But S_n does contain all bit strings of length zero.

So, that violates the condition of being a partition of S_n.

$\therefore$ The sets P_i, $i=1,2,\ldots,n$ cannot be partition of set S_n.

2. Prove that K_5 is non-planar graph.

K_5 (Kuratowski's first graph) is a complete graph which has 5 vertices and each vertex is connected to all the other vertices. So, it has ${}^5C_2 = 10$ edges.

Now, let $n = 5$ and $e = 10$.

Now, a necessary condition for a graph to be planar is that it should follow $e \leq 3n-6$ where e is number of edges and n is number of vertices in the graph.

So, for K_5

$$3n-6 = 3\times5-6 = 9 < 10 = e$$

$$\therefore \quad 3n-6 < e.$$

which means K_5 is a non-planar graph.

3. How many reflexive relations are possible on a set A with n elements?

A reflexive relation R on set A with n elements must include all (a,a) ordered pairs for all $a \in A$. and ~~a~~ also must be a subset of $A \times A$.

Now there are total n number of such pairs. which will all be included in the reflexive relation on A.

So, there remains $n^2 - n$ ordered pairs in $A \times A$ ~~as~~ which has (a,b) where $a \neq b$. as $A \times A$ has total n^2 elements/ordered pairs.

Now, each of these (a,b) pairs may or may not be included in the relation with n. (a,a) ordered pairs,

which gives us a total of $2^{(n^2-n)}$ combinations of such relations.

$$2^{n^2-n} = 2^{n(n-1)}$$

$\therefore$ ~~As~~ There are $2^{n(n-1)}$ relations which are reflexive on a set A with n elements.

4. Suppose a person takes minimum one egg in everyday, if he took 50 eggs in a month, then show that he took exactly 9 eggs in consecutive days.

Let,

There be 30 days in the month.

$a_1, a_2, \ldots a_{30}$ be numbers of eggs taken in the i-th day. $i = 1, \ldots, 30$.

where $a_i \geq 1$

and $\sum_{i=1}^{30} a_i = 50$.

~~Now,~~

~~Let,~~

Now,

Let, $S_0, \ldots S_{30}$ be the cumulative numbers of eggs taken upto the ith day. $i = 0, 1, \ldots, 30$.

and set $S = \{S_0, S_1, \ldots, S_{30}\}$.

Also, let take a set T, such that

$T = \{S_0 + 9, S_1 + 9, \ldots, S_{30} + 9\}$

Set S Range = $[0, 50]$

T Range = $[9, 59]$

Both has, 31 numbers of elements.

So, total 62 elements.

But, the ~~it~~ elements itself values in $[0, 59]$ which are 60 values.

So, by Pigeon hole principle

Among 62 values and only 60 possible values we can say that,

~~60~~ $62 - 60 = 2$, atleast 2 of them are equal.

$\therefore$ There exists a ~~s_i from S and~~ s_i and $s_j + 9$ such that

$$s_i = s_j + 9 \Rightarrow s_i - s_j = 9$$

$\therefore$ ~~E~~ There are ~~@~~ exactly 9 eggs taken in the consecutive days from $j+1$ to i.

$\therefore$ There exists a sequence of consecutive days when the person took exactly 9 eggs.