Name → Harshadeep Bhaduri - 312 - UG-II
Discrete Mathematics Assignment
Semester - IV

Q1) Let $S = \{a, b, c\}$, Show that $(P(S), \supseteq)$ forms a poset, where $P(S)$ is power set of S. Also, draw Hasse Diagram representing this poset.

Given $S = \{a, b, c\}$
$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$

(i) Reflexivity :- ($a \forall a \in S$, aRa)
Since every set is a subset of itself,
we can say, $\forall a \in S$, ~~$a \subseteq a$ or~~ $a \supseteq a$.
∴ Each element of this powerset satisfies this condition

∴ It is reflexive —— (i)

(ii) Anti-Symmetric :- ($\forall x, y \in S$, xRy & yRx implies $x = y$)
So, we know if $a \subseteq b$ & $b \subseteq a$, then $a = b$,
so by this property, we can say that,
$\forall a, b \in S$, ~~a~~ if $a \subseteq b$ & $b \subseteq a$, then $a = b$

∴ It is anti symmetric ~~reflexive~~ —— (ii)

(iii) Transitive :- ($\forall x, y, z \in S$, xRy & yRz, then xRz)
We know if $x \subseteq y$ & $y \subseteq z$, then $x \subseteq z$
due to its property
$\forall a, b, c \in S$, where $a \subseteq b$ & $b \subseteq c$ then $a \subseteq c$.
∴ It is transitive —— (iii)

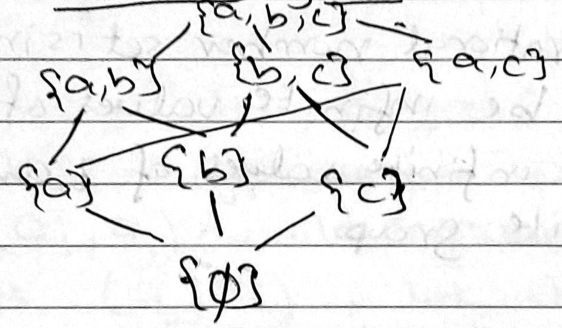
__/__/__

From (i), (ii) & (iii), we can say $(P(S), \supseteq)$ forms a poset.

*

Q2) Prove that the set $G = \{(\cos\theta + i\sin\theta)$: θ runs over all rational number$\}$ forms an infinite abelian group with respect to ordinary multiplication.

* Hasse Diagram



Solution(2) → $G = \{(\cos\theta + i\sin\theta) : \theta \in \mathbb{Q}\}$

Using euler's formula, $\cos\theta + i\sin\theta$ can be written as $e^{i\theta}$.

$\therefore G = \{e^{i\theta} : \theta \in \mathbb{Q}\}$

Now the group is $(G, \times)$.

for any $\theta_1, \theta_2 \in \mathbb{Q}$, $z_1 = e^{i\theta_1}$ and $z_2 = e^{i\theta_2}$ where $z_1, z_2 \in G$.

$\therefore z_1 \times z_2 = e^{i\theta_1} \times e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}$

Similarly

$z_2 \times z_1 = e^{i\theta_2} \times e^{i\theta_1} = e^{i(\theta_2 + \theta_1)}$

__/__/__

Now $\theta_2+\theta_1$ also belongs to Q,

$\therefore$ z_1z_2 and z_2z_1 also belongs to G.

$e^{i(\theta_1+\theta_2)} = e^{i(\theta_2+\theta_1)}$, then

$z_1z_2 = z_2z_1$

$\therefore$ The grap is abelian, as it follows commutative law.

Now, since the rational number set is infinite, then there will be infinite values of θ_1 and θ_2

$\therefore$ There will be infinite values of z_1 and z_2

$\therefore$ It is an infinite group.

Q3) How many symmetric relations are possible on set A with n elements?

The cartesian product of $A \times A$ will look like this:

$$
\begin{matrix}
(a_1,a_1), & (a_1,a_2), & (a_1,a_3) & \cdots & (a_1,a_n) \\
(a_2,a_1), & (a_2,a_2), & (a_2,a_3) & \cdots & (a_2,a_n) \\
(a_3,a_1), & (a_3,a_2) & (a_3,a_3) & \cdots & (a_3,a_n) \\
\vdots & \vdots & \vdots & & \vdots \\
(a_n,a_1), & (a_n,a_2) & (a_n,a_3) & \cdots & (a_n,a_n)
\end{matrix}
$$

__/__/____

From the above representation, it is cleare that the diagonal elements would be present in the symmetric relation.

There are n elements in the diagonal.
∴ There are 2^n possible assignments.

Now, for non diagonal elements, (a_i, a_j) would be present. iff (a_j, a_i) is present.

Except the diagonal elements, there are $n^2 - n$ elements present, but since (a_i, a_j) would represent the same assignment as (a_j, a_i), we should only count them once

∴ There would be $\frac{n^2-n}{2}$ elements present and $2^{\frac{n^2-n}{2}}$ way to assign them

∴ Total possible ways to ~~assign them~~ assign them,

$$2^n \times 2^{\frac{n^2-n}{2}}$$
$$= 2^{\frac{n(n+1)}{2}}$$

$2^{\frac{n(n+1)}{2}}$ possible ways.

_/_/_

Q4) In how many ways a graph containing n edges can be decomposed into a pair of subgraphs?

We have n edges that either go in subgraph G_1 or subgraph G_2.
So, we have two Choice for each edge

$\therefore$ There are 2^n possible ~~choices~~, assignment

But, in this way of assignment, we are counting cases like $e_1 \to G_1$, $e_2 \to G_2$ and $e_2 \to G_1$, $e_1 \to G_2$, which we can see refer to the same assignment

$\therefore$ We most remove duplicates,
$$\frac{2^n}{2} = 2^{n-1} \text{ possible assignment}$$

We have considered (G_1, G_2) and (G_2, G_1) as same, for example,
$e_1, e_2 \to G_1$ and $e_3 \to G_2$ is considered same as
$e_1, e_2 \to G_2$ and $e_3 \to G_1$.