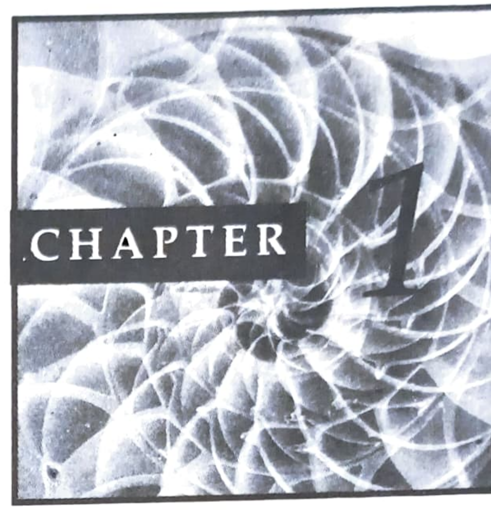


# Basic Logical Concepts

## 1.1 What Logic Is

Logic is the study of the methods and principles used to distinguish correct reasoning from incorrect reasoning. There are objective criteria with which correct reasoning may be defined. If these criteria are not known, then they cannot be used. The aim of the study of logic is to discover and make available those criteria that can be used to test arguments, and to sort good arguments from bad ones.

The logician is concerned with reasoning on every subject: science and medicine, ethics and law, politics and commerce, sports and games, and even the simple affairs of everyday life. Very different kinds of reasoning may be used, and all are of interest to the logician. In this book arguments of many varieties, on very many topics, will be analyzed. Our concern throughout will be not with the subject matter of those arguments, but with their *form* and *quality*. Our aim is to learn how to test arguments and evaluate them.

It is not the thought processes called reasoning that are the logician's concern, but the outcomes of those processes, the *arguments* that are the products of reasoning, and that can be formulated in writing, examined, and analyzed. Each argument confronted raises this question for the logician: Does the conclusion reached *follow* from the premisses used or assumed? Do the premisses *provide good reasons* for accepting the conclusion drawn? If the premisses do provide adequate grounds for accepting the conclusion—that is, if asserting the premisses to be true does warrant asserting the conclusion also to be true—then the reasoning is correct. Otherwise it is incorrect.

It would be a mistake to suppose that only the student of logic can reason well or correctly, just as it would be wrong to suppose that only the athlete who studies physiology can run well. Athletes unaware of the processes going on in their bodies often perform excellently, and some advanced students of physiology, although knowing much about the way the body functions, nevertheless perform poorly on the athletic field. Similarly, the study of logic does not give assurance that one's reasoning will be correct.

But a person who has studied logic is more likely to reason correctly than one who has never thought about the principles involved in reasoning. Partly this is because the student of logic will acquire methods for testing the correctness of reasoning, and the more easily errors are detected, the less likely they are to be allowed to stand. Among the errors detected will be those common fallacies, or "natural" mistakes in reasoning, that can be readily avoided when fully understood.

The study of logic is likely to improve the quality of one's reasoning for another reason: It gives one the opportunity to *practice* the analysis of arguments and the construction of arguments of one's own. Reasoning is something we *do* as well as understand; it therefore is an art as well as a science, with skills to be developed and techniques to be mastered. To this end this book provides an abundant supply of exercises through which those skills and techniques may be strengthened.

There are affairs in human life that cannot be fully analyzed by the methods of logic, and issues that cannot be resolved by arguments, even good ones. The appeal to emotion sometimes is more persuasive than logical argument, and in some contexts it may be more appropriate as well. But where judgments that must be relied upon are to be made, correct reasoning will in the long run prove to be their most solid foundation. With the methods and techniques of logic we can distinguish efficiently between correct and incorrect reasoning. These methods and techniques are the subject matter of this book.

## 1.2 Propositions and Sentences

We begin by examining *propositions*, the building blocks of every argument. **A proposition is something that may be asserted or denied.** Propositions in this way are different from questions, commands, and exclamations. Neither questions, which can be asked, nor commands, which can be given, nor exclamations, which can be uttered, can possibly be asserted or denied. Only propositions assert that something is (or is not) the case, and therefore only they can be true or false. Truth and falsity do not apply to questions, commands, or exclamations.

Moreover, every proposition *is* either true or false—although we may not know the truth or falsity of some given proposition. The proposition that there is life on some other planet in our galaxy is one whose truth or falsity we do not know; but either it is true that there is such extraterrestrial life, or it is not true. In short, an essential feature of propositions is that they are either true or false.

It is customary to distinguish between propositions and the *sentences* by means of which they are asserted. Two sentences that consist of different words differently arranged may in the same context have the same meaning and be used to assert the same proposition. For example,

Leslie won the election.
The election was won by Leslie.

are plainly two different sentences, for the first contains four words and the second six, and they begin differently, and so on. Yet these two declarative sentences have exactly the same meaning. **We use the term *proposition* to refer to what declarative sentences are typically used to assert.**

A sentence, moreover, is always a sentence in a particular language, the language in which it is used. But propositions are not peculiar to any language; a given proposition may be asserted in many languages. The four sentences

It is raining.
Está lloviendo.
Il pleut.
Es regnet.

are certainly different, for they are in different languages: English, Spanish, French, and German. Yet they have a single meaning, and all may be uttered to assert the same proposition.

The same sentence can be used, in different contexts, to make very different statements. For example, the sentence

The largest state in the United States was once an independent republic.

would have been a true statement about Texas during the first half of the twentieth century, but it is now a false statement about Alaska. A change in the temporal context, plainly, may result in very different propositions, or statements, being asserted by the very same words. (The terms "proposition" and "statement" are not exact synonyms, but in the context of logical investigation they are used in much the same sense. Some writers on logic prefer "statement" to "proposition," although the latter has been more common in the history of logic. In this book, both terms are used.)

The propositions illustrated thus far have been simple: "Leslie won the election"; "It is raining"; and so on. But propositions are often *compound*, containing other propositions within themselves. Consider the following passage from an account of the last days of Hitler's Third Reich in 1945:

> The Americans and Russians were driving swiftly to a junction on the Elbe. The British were at the gates of Hamburg and Bremen and threatening to cut off Germany from occupied Denmark. In Italy Bologna had fallen and Alexander's Allied forces were plunging into the valley of the Po. The Russians, having captured Vienna on April 13, were heading up the Danube.[1]

Several propositions contained in this paragraph are compound. "The British were at the gates of Hamburg and Bremen," for example, is the *conjunction* of two propositions: "The British were at the gates of Hamburg," and "The British were at the gates of Bremen." And that conjunctive proposition is itself one component of a larger conjunction, that "the British were at the gates of Hamburg and Bremen and [the British] were threatening to cut off Germany

[1]William L. Shirer, *The Rise and Fall of the Third Reich* (New York: Simon and Schuster, 1960).

from occupied Denmark." Every proposition in this passage is asserted; that is, it is stated as true. Asserting the conjunction of two propositions is equivalent to asserting each of the component propositions themselves.

But there are other kinds of compound propositions which do not assert the truth of their components. For example, in *alternative* (or *disjunctive*) propositions, such as

> Circuit Courts are useful, or they are not useful.[2]

neither of the two components is asserted; only the compound "either-or" disjunctive proposition is asserted. If this disjunctive proposition is true, either of its components could be false. And in compound propositions that are *hypothetical* (or *conditional*), such as

> If God did not exist, it would be necessary to invent him.[3]

again neither of the components is asserted. The proposition that "God does not exist" is not asserted here; nor is the proposition that "it is necessary to invent him." Only the "if-then" proposition is asserted by the hypothetical or conditional statement, and that conditional statement might be true even though both of its components were false.

In the course of this book we shall analyze the internal structure of many kinds of propositions, both simple and compound.

## 1.3 Arguments, Premisses, and Conclusions

Propositions are the building blocks with which arguments are made. **The term *inference* refers to the process by which one proposition is arrived at and affirmed on the basis of one or more other propositions accepted as the starting point of the process.** To determine whether an inference is correct, the logician examines the propositions with which that process begins and ends, and the relations between them. This cluster of propositions constitutes an *argument*, and therefore there is an argument corresponding to every possible inference.

It is with arguments that logic is chiefly concerned. As logicians use the word **an *argument* is any group of propositions of which one is claimed to follow from the others, which are regarded as providing support or grounds for the truth of that one.** The word "argument" is often used in other senses also, of course, but in logic it has strictly the sense just explained.

In this strict sense, it is clear that an argument is not a mere collection of propositions; a passage may contain several related propositions and yet contain no *argument* at all. For an argument to be present, the cluster of propositions must have a structure. In describing this structure, the terms "premiss" and "conclusion" are commonly used. **The *conclusion* of an argument is the proposition that**

[2]Abraham Lincoln, annual message to Congress, 3 December 1861.

[3]Voltaire, *Épitre à l'Auteur du Livre des Trois Imposteurs*, 10 November 1770.

**is affirmed on the basis of the other propositions of the argument, and these other propositions, which are affirmed (or assumed) as providing support or reasons for accepting the conclusion, are the *premisses* of that argument.**

The simplest kind of argument consists of one premiss and a conclusion that is claimed to follow from it or be implied by it. The premiss and the conclusion, in that order, may each be stated in a separate sentence, as in this argument that appears on a sticker affixed to biology textbooks in the State of Alabama:

> No one was present when life first appeared on earth. Therefore any statement about life's origins should be considered as theory, not fact.

Or both the premiss and the conclusion may be stated in the same sentence, as in the following argument:

> Since it turns out that all humans are descended from a small number of African ancestors in our recent evolutionary past, believing in profound differences between the races is as ridiculous as believing in a flat earth.[4]

Even in simple arguments, the statement of the conclusion may *precede* the statement of the single premiss. When it does, the two propositions may appear in separate sentences or in the same sentence. An example of separate statements in which the conclusion is stated first is this:

> The Food and Drug Administration should stop all cigarette sales immediately. After all, cigarette smoking is the leading preventable cause of death.[5]

And an example of a combined statement in which the conclusion comes first is this:

> Every law is an evil, for every law is an infraction of liberty.[6]

Most arguments are much more complicated than these, and some arguments, containing compound propositions with several components, are exceedingly complicated, as we shall see. But every argument, whether simple or complex, consists of a group of propositions, of which one is the conclusion and the others are the premisses offered to support it.

Since an argument is made up of a group of propositions, no single proposition can, by itself, be an argument. But some compound propositions closely resemble arguments. Care must be taken not to confuse such propositions with arguments. Consider the following hypothetical proposition:

> If life evolved on Mars during an early period in its history when it had an atmosphere and climate similar to Earth's, then it is likely that life evolved on countless other planets that scientists now believe to exist in our galaxy.

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[4]David Hayden, "Thy Neighbor, Thy Self," *New York Times*, 9 May 2000.

[5]"Ban Cigarettes," *Orlando Sentinel*, 27 February 1992.

[6]Jeremy Bentham, *Principles of Legislation*, 1802.

Neither the first component of this proposition—"life evolved on Mars during an early period in its history when it had an atmosphere and climate similar to Earth's"—nor the second component—"it is likely that life evolved on countless other planets that scientists now believe exist in our galaxy"—is asserted. The proposition asserts only that the former implies the latter, and both could very well be false. No inference is made in this passage, no conclusion is claimed to be true. This is a hypothetical proposition, not an argument. But now consider the following passage:

> It is likely that life evolved on countless other planets that scientists now believe exist in our galaxy, because life very probably evolved on Mars during an early period in its history when it had an atmosphere and climate similar to Earth's.[7]

In this case we *do* have an argument. The proposition that "life very probably evolved on Mars" is here asserted as a premiss, and the proposition that "life likely evolved on countless other planets" is here claimed to follow from that premiss and to be true. Thus, a hypothetical proposition may *look* very much like an argument, but it never can *be* an argument, and the two should not be confused. Recognizing arguments is a topic discussed below in section 1.5.

Finally, it should be emphasized that while every argument is a structured cluster of propositions, not every structured cluster of propositions is an argument. Consider this passage from a recent account of travel in Africa:

> Camels do not store water in their humps. They drink furiously, up to 28 gallons in a ten-minute session, then distribute the water evenly throughout their bodies. Afterward, they use the water stingily. They have viscous urine and dry feces. They breathe through their noses and keep their mouths shut. They do sweat, but only as a last resort. . . . They can survive a water loss of up to one-third of their body weight, then drink up and feel fine.[8]

There is no argument here.

## Exercises

Identify the premisses and conclusions in the following passages, each of which contains only one argument.[9]

### Example:

1. A well regulated militia being necessary to the security of a free state, the right of the people to keep and bear arms shall not be infringed.

—*The Constitution of the United States*, Amendment 2

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[7]Richard Zare, "Big News for Earthlings," *New York Times*, 8 August 1996.

[8]William Langewiesche, *Sahara Unveiled: A Journey Across the Desert* (New York: Pantheon Books, 1996).

[9]Solutions to the starred exercises may be found at the back of the book.

indication that the argument is inductive. This is so because there are some strictly deductive arguments *about* probabilities themselves.* Arguments of this kind, in which the probability of a certain combination of events is deduced from the probabilities of other events, are discussed in Chapter 14.

## 1.9 Validity and Truth

As noted earlier, a successful deductive argument is *valid*. Validity refers to a relation *between* propositions—between the set of propositions that serve as the premisses of a deductive argument, and the one proposition that serves as the conclusion of that argument. If the latter follows with logical necessity from the former, we say that the argument is valid. Since logical necessity is never achieved by inductive arguments, validity never applies to them. *Nor can validity ever apply to any single proposition by itself,* since the needed *relation* cannot possibly be found within any one proposition.

Truth and falsity, on the other hand, *are* attributes of individual propositions. A single statement that serves as a premiss in an argument may be true; the statement that serves as its conclusion may be false. That conclusion may have been validly inferred, but it makes no sense to say that any conclusion, or any single premiss, is itself valid or invalid.

Truth is the attribute of a proposition that asserts what really is the case. When I assert that Lake Superior is the largest of the five Great Lakes, I assert what really is the case, what is true. If I said that the largest of the Great Lakes is Lake Michigan, my assertion would not be in accord with the real world; therefore, it would be false. This contrast is important: **truth and falsity are attributes of individual propositions or statements; validity and invalidity are attributes of arguments.**

Just as the concept of validity does not apply to single propositions, the concept of truth does not apply to arguments. Of the several propositions in an argument, some (or all) may be true and some (or all) may be false. But the argument as a whole is neither "true" nor "false." Propositions, which are statements about the world, may be true or false; deductive arguments, which consist of inferences from one set of propositions to other propositions, may be valid or invalid.

The relations *between* true (or false) propositions and valid (or invalid) arguments lie at the heart of deductive logic. Part II of this book is largely devoted to the examination of those complex relations. However, a preliminary discussion of the relation between validity and truth is in order at this point.

We begin by emphasizing that an argument may be valid even if one or more of its premisses is not true. Every argument makes a claim about the relation between the premisses and the conclusion drawn from them; that relation may hold even if the premisses turn out to be false or the truth of the premisses is in dispute. This point was made effectively by Abraham Lincoln in

*If, for example, we learn that the probability of three successive heads in three tosses of a coin is ⅛, we may infer deductively that the probability of getting at least one tail in three tosses of coin is ⅞.

1858 in one of his debates with Stephen Douglas. Lincoln was attacking the *Dred Scott* decision, which obliged the return of slaves who had escaped into northern states to their owners in the South:

> I think it follows, [from the *Dred Scott* decision] and submit to the consideration of men capable of arguing, whether as I state it in syllogistic form the argument has any fault in it:
>
> Nothing in the Constitution or laws of any State can destroy a right distinctly and expressly affirmed in the Constitution of the United States.
>
> The right of property in a slave is distinctly and expressly affirmed in the Constitution of the United States.
>
> Therefore, nothing in the Constitution or laws of any State can destroy the right of property in a slave.
>
> I believe that no fault can be pointed out in that argument; assuming the truth of the premisses, the conclusion, so far as I have capacity at all to understand it, follows inevitably. There *is* a fault in it as I think, but the fault is not in the reasoning; but the falsehood in fact is a fault of the premisses. I believe that the right of property in a slave *is not* distinctly and expressly affirmed in the Constitution, and Judge Douglas thinks it *is*. I believe that the Supreme Court and the advocates of that decision [the *Dred Scott* decision] may search in vain for the place in the Constitution where the right of property in a slave is distinctly and expressly affirmed. I say, therefore, that I think one of the premisses is not true in fact.[48]

In the argument that he recapitulates and attacks, Lincoln finds the second premiss—that the right of property in a slave is affirmed in the U.S. Constitution—to be plainly false. The reasoning in the argument is not faulty, he points out; nevertheless, its conclusion has not been established. His logical point is correct: *An argument may be valid even when its conclusion and one or more of its premisses are false.* For the validity of an argument, we emphasize once again, depends only upon the *relation* of the premisses to the conclusion.

There are many possible combinations of true and false premisses and conclusions in both valid and invalid arguments. Consider the following illustrative arguments, each of which is prefaced by the statement of the combination it represents. With these illustrations before us, we will be in a position to formulate some important principles concerning the relations between truth and validity.

**I.** Some *valid* arguments contain *only true* propositions—true premisses and a true conclusion:

> All mammals have lungs.
> All whales are mammals.
> Therefore all whales have lungs.

**II.** Some *valid* arguments contain *only false* propositions:

> All four-legged creatures have wings.
> All spiders have four legs.
> Therefore all spiders have wings.

[48] Abraham Lincoln, in Roy R. Basler, ed., *The Collected Works of Abraham Lincoln*, vol. 3. (Rutgers University Press).

# CHAPTER 5

# CATEGORICAL PROPOSITIONS

## 5.1 THE THEORY OF DEDUCTION

Preceding chapters have dealt chiefly with language and its influence on argumentation. We turn now to argument itself, and first to the analysis of that special kind of argument called "deduction." **A deductive argument is one whose premises are claimed to provide conclusive grounds for the truth of its conclusion.** If it does provide such conclusive grounds, the deductive argument is valid. Every deductive argument is either valid or invalid: valid if it is impossible for its premises to be true without its conclusion being true also, and invalid otherwise.

The theory of deduction is intended to explain the relationship between premisses and conclusion of a valid argument, and to provide techniques for the appraisal of deductive arguments, that is, for discriminating between valid and invalid deductions. To accomplish this, two great bodies of theory have been developed. The first of these is called "classical" or "Aristotelian" logic, after the great Greek philosopher who initiated this study. The second is called "modern" or "modern symbolic" logic. Classical logic will be the topic of this and the following two chapters (chapters 5, 6, and 7); modern logic will be the topic of chapters 8, 9, and 10.

Aristotle was one of the towering intellects of the ancient world. After studying for 20 years in Plato's Academy, he became tutor to Alexander the Great; later he founded his own school, the Lyceum, where he contributed substantially to nearly every field of human knowledge. After Aristotle's death, his treatises on reasoning were gathered together and came to be called the *Organon*. The word "logic" did not acquire its modern meaning until the second century A.D., but the subject matter of logic was long determined by the content of the *Organon*.

## 5.2 Categorical Propositions and Classes

The Aristotelian study of deduction focused on arguments containing propositions of a special kind, called "categorical propositions" because they are about categories or classes. To understand the classical theory of deduction, we must begin with a very careful analysis of these propositions, which are the building blocks of that theory. Consider the argument:

> No athletes are vegetarians.
> All football players are athletes.
> Therefore no football players are vegetarians.

All three of the propositions in this argument, both premisses and the conclusion, are *categorical* propositions. **Such propositions affirm, or deny, that some class *S* is included in some other class *P*, in whole or in part.** In the example above, the three categorical propositions are about the class of all athletes, the class of all vegetarians, and the class of all football players.

(Classes were mentioned briefly in our discussion of definition in Chapter 3, where a class was explained to be the collection of all objects that have some specified characteristic in common.) There are various ways in which two classes may be related to each other.

1. If every member of one class is also a member of a second class, like the class of dogs and the class of mammals, then the first class is said to be included or contained in the second.
2. If some but perhaps not all members of one class are also members of another, like the class of females and the class of athletes, then the first class may be said to be partially contained in the second class.
3. If the two classes have no members in common, like the class of all triangles and the class of all circles, the two classes may be said to exclude one another.

These various relationships between classes are affirmed or denied by categorical propositions. The result is that there can be just four different *standard forms* of categorical propositions. They are illustrated by the four following propositions:

1. All politicians are liars.
2. No politicians are liars.
3. Some politicians are liars.
4. Some politicians are not liars.

Let us examine these four standard-form categorical propositions in greater detail.

The first example—All politicians are liars—is a *universal affirmative* proposition. It is about two classes, the class of all politicians and the class of all liars, saying that the first class is included or contained in the second. *A universal affirmative proposition says that every member of the first class is also a member of the second class.* In the present example, the subject term "politicians" designates the class of all politicians, and the predicate term "liars" designates the class of all liars. Any universal affirmative proposition may be written schematically as

universal affirmative

All *S* is *P*.

where the letters *S* and *P* represent the subject and predicate terms, respectively. The name "universal affirmative" is appropriate, because the proposition *affirms* that the relationship of class inclusion holds between the two classes and says that the inclusion is *complete* or universal: All members of *S* are said to be members of *P* also.

universal negative

The second example—No politicians are liars—is a *universal negative* proposition. It denies of politicians universally that they are liars. Concerned with two classes, *a universal negative proposition says that the first class is wholly excluded from the second,* which is to say that there is no member of the first class that is also a member of the second. Any universal negative proposition may be written schematically as

No *S* is *P*.

where, again, the letters *S* and *P* represent the subject and predicate terms. The name "universal negative" is appropriate because the proposition *denies* that the relation of class inclusion holds between the two classes—and denies it *universally:* No members at all of *S* are members of *P*.

particular affirmative

The third example—Some politicians are liars—is a *particular affirmative* proposition. Clearly, what the present example affirms is that some members of the class of all politicians are (also) members of the class of all liars. But it does not affirm this of politicians universally: Not all politicians universally, but, rather, some particular politician or politicians, are said to be liars. This proposition neither affirms nor denies that *all* politicians are liars; it makes no pronouncement on the matter. It does not literally say that some politicians are *not* liars, although in some contexts it might be taken to suggest it. The literal, minimal interpretation of the present proposition is that the class of politicians and the class of liars have some member or members in common. For definiteness, we adopt that minimal interpretation here.

The word "some" is indefinite. Does it mean "at least one" or "at least two" or "at least a hundred"? Or how many? For the sake of definiteness, although this position may depart from ordinary usage in some cases, it is customary to regard the word "some" as meaning "at least one." Thus a particular affirmative proposition, written schematically as

Some *S* is *P*.

*says that at least one member of the class designated by the subject term* S *is also a member of the class designated by the predicate term* P. The name "particular affirmative" is appropriate because the proposition *affirms* that the relationship of class inclusion holds, but does not affirm it of the first class universally but only *partially, i.e.,* of some particular member or members of the first class.

The fourth example—Some politicians are not liars—is a *particular negative* proposition. This example, like the one preceding it, does not refer to politicians universally but only to some member or members of that class; it is particular. But unlike the third example it does not affirm that the particular members of the first class referred to are included in the second class; this is precisely what is denied. A particular negative proposition, schematically written as

Some $S$ is not $P$.

Particular negative

*says that at least one member of the class designated by the subject term* S *is excluded from the whole of the class designated by the predicate term* P.

Not all standard-form categorical propositions are as simple and straightforward as the four examples just considered. The subject and predicate terms of a standard-form categorical proposition always designate classes, but those terms may be complicated expressions rather than single words. For example, the proposition "All candidates for the position are persons of honor and integrity" has the phrase "candidates for the position" as its subject term and the phrase "persons of honor and integrity" as its predicate term.

It was traditionally held that all deductive arguments were analyzable in terms of classes, categories, and their relations. Thus the four standard-form categorical propositions just explained:

| | |
|---|---|
| universal affirmative propositions | (called **A** propositions) |
| universal negative propositions | (called **E** propositions) |
| particular affirmative propositions | (called **I** propositions) |
| particular negative propositions | (called **O** propositions) |

were thought to be the building blocks of all deductive arguments. A great deal of logical theory—as we shall see—has been built up concerning these four kinds of propositions.

## Exercises

Identify the subject and predicate terms in, and name the form of, each of the following propositions.

***1.** Some historians are extremely gifted writers whose works read like first-rate novels.

**2.** No athletes who have ever accepted pay for participating in sports are amateurs.

**3.** No dogs that are without pedigrees are candidates for blue ribbons in official dog shows sponsored by the American Kennel Club.

**4.** All satellites that are currently in orbits less than ten thousand miles high are very delicate devices that cost many thousands of dollars to manufacture.

***5.** Some members of families that are rich and famous are not persons of either wealth or distinction.

6. Some paintings produced by artists who are universally recognized as masters are not works of genuine merit that either are or deserve to be preserved in museums and made available to the public.
7. All drivers of automobiles that are not safe are desperadoes who threaten the lives of their fellows.
8. Some politicians who could not be elected to the most minor positions are appointed officials in our government today.
9. Some drugs that are very effective when properly administered are not safe remedies that all medicine cabinets should contain.

*10. No people who have not themselves done creative work in the arts are responsible critics on whose judgment we can rely.

## 5.3 QUALITY, QUANTITY, AND DISTRIBUTION

### A. Quality

Every standard-form categorical proposition is said to have a **quality**, either affirmative or negative. **If the proposition *affirms* some class inclusion, whether complete or partial, its quality is affirmative.** Thus both universal affirmative propositions and particular affirmative propositions are affirmative in quality, and their letter names, **A** and **I** respectively, are thought to come from the Latin word, "AffIrmo," meaning "I affirm." **If the proposition *denies* class inclusion, whether complete or partial, its quality is negative.** Thus both universal negative propositions and particular negative propositions are negative in quality, and their letter names, **E** and **O**, respectively, are thought to come from the Latin word "nEgO," meaning "I deny."

### B. Quantity

Every standard-form categorical proposition is said to have a **quantity** also, universal or particular. **If the proposition refers to *all* members of the class designated by its subject term, its quantity is universal.** Thus the **A** and **E** propositions are universal in quantity. **If the proposition refers only to *some* members of the class designated by its subject term, its quantity is particular.** Thus the **I** and **O** propositions are particular in quantity.

Every standard-form categorical proposition begins with one of the words "all," "no," or "some." These words show the quantity of the proposition. "All" and "no" indicate that the proposition is universal; "some" indicates that the quantity is particular. The quantifier "no" serves additionally to indicate the negative quality of the **E** proposition.

We observe that the names "universal affirmative," "universal negative," "particular affirmative," and "particular negative" uniquely describe each of the four standard forms by mentioning first its quantity and then its quality.

## C. General Schema of Standard-Form Categorical Propositions

Between the subject and predicate terms of every standard-form categorical proposition occurs some form of the verb "to be" (accompanied by the word "not" in the case of the **O** proposition). This verb serves to connect the subject and predicate terms and is called the "copula." In the schematic formulations given in the preceding section, only "is" and "is not" appear, but depending on how the proposition is worded otherwise, some other form of the verb "to be" may be more appropriate. For example, in the following three propositions,

> Some Roman emperors were monsters.
> All squares are rectangles.
> Some soldiers will not be heroes.

"were," "are," and "will not be" serve as copulas. The general skeleton or schema of a standard-form categorical proposition consists of four parts: first the quantifier, then the subject term, next the copula, and finally the predicate term. This schema may be written as

> Quantifier (subject term) copula (predicate term).

## D. Distribution

On the class interpretation, the subject and predicate terms of a standard-form categorical proposition designate classes of objects, and the proposition is regarded as being about these classes. Propositions may refer to classes in different ways, of course. A proposition may refer to *all* members of a class, or it may refer to only *some* members of that class. Thus the proposition

> All senators are citizens.

refers to or is about *all* senators but does not refer to all citizens. It asserts that each and every member of the class of senators is a citizen, but it makes no assertion about all citizens. It does not affirm that each and every citizen is a senator, but it does not deny it either. Any **A** proposition, of this form,

All $S$ is $P$.

is thus seen to refer to *all* members of the class designated by its subject term, $S$, but does not refer to all members of the class designated by its predicate term, $P$.

The technical term "distribution" is introduced to characterize the ways in which terms can occur in categorical propositions. **A proposition *distributes* a term if it refers to all members of the class designated by the term.** Let us examine each of the standard-form categorical propositions, to see which terms are distributed or undistributed in them.

First, consider the **A** Proposition. As we noted above, using the example "All senators are citizens," the *subject* term of an **A** proposition *is distributed* in (or by) that proposition. But the *predicate* term of an *A* proposition is **undistributed** in (or by) it.

Next consider the **E** proposition. An **E** proposition such as

No athletes are vegetarians.

asserts of each and every athlete that he or she is not a vegetarian. The whole of the class of athletes is said to be excluded from the class of vegetarians. All members of the class designated by its subject term are referred to by an **E** proposition, which is therefore said to distribute its subject term. At the same time, in asserting that the whole class of athletes is excluded from the class of vegetarians, it is also asserted that the whole class of vegetarians is excluded from the class of athletes. The given proposition clearly asserts of each and every vegetarian that he or she is not an athlete. An **E** proposition, therefore, refers to all members of the class designated by its predicate term and is said to distribute its predicate term also. **E** *propositions distribute both their subject and their predicate terms.*

The situation is different with respect to **I** propositions. Thus,

Some soldiers are cowards.

makes no assertion about all soldiers and makes no assertion about all cowards either. It says nothing about each and every soldier, nor about each and every coward. Neither class is said to be either wholly included or wholly excluded from the other. *Both subject and predicate terms are undistributed in any particular affirmative proposition.*

The particular negative or **O** proposition is similar in that it, too, does not distribute its subject term. Thus the proposition

Some horses are not thoroughbreds.

says nothing about *all* horses but refers to *some* members of the class designated by the subject term. It says of this part of the class of all horses that it is excluded from the class of all thoroughbreds, that is, from the *whole* of the latter class. Given the particular horses referred to, it says that each and every member of the class of thoroughbreds is *not* one of those particular horses. When something is said to be excluded from a class, the whole of the class is referred to, just as, when a person is excluded from a country, all parts of that country are forbidden to that person. *The particular negative proposition does distribute its predicate term, but not its subject term.*

We may summarize these remarks on distribution as follows: Universal propositions, both affirmative and negative, distribute their subject terms, whereas particular propositions, whether affirmative or negative, do not distribute their subject terms. Thus the *quantity* of any standard-form categorical proposition determines whether its *subject* term is distributed or undistributed. Affirmative propositions, whether universal or particular, do not distribute their predicate terms, whereas negative propositions, both universal and particular, do distribute their predicate terms. Thus the *quality* of any standard-form categorical proposition determines whether its predicate term is distributed or undistributed.

The following diagram summarizes this information and may be useful in helping one to remember which propositions distribute which of their terms.

| | predicate term undistributed | predicate term distributed |
|---|---|---|
| subject term distributed | *A*: All *S* is *P*. | *E*: No *S* is *P*. |
| subject term undistributed | *I*: Some *S* is *P*. | *O*: Some *S* is not *P*. |

## Exercises

Name the quality and quantity of each of the following propositions, and state whether their subject and predicate terms are distributed or undistributed.

*1. Some presidential candidates will be sadly disappointed people.

2. All those who died in Nazi concentration camps were victims of a cruel and irrational tyranny.

3. Some recently identified unstable elements were not entirely accidental discoveries.

4. Some members of the military-industrial complex are mild-mannered people to whom violence is abhorrent.

*5. No leader of the feminist movement is a major business executive.

6. All hard-line advocates of law and order at any cost are people who will be remembered, if at all, only for having failed to understand the major social pressures of the late twentieth century.

7. Some recent rulings of the Supreme Court were politically motivated decisions that flouted the entire history of American legal practice.

8. No harmful pesticides or chemical defoliants were genuine contributions to the long-range agricultural goals of the nation.

9. Some advocates of major political, social, and economic reforms are not responsible people who have a stake in maintaining the status quo.

*10. All new labor-saving devices are major threats to the trade union movement.

## 5.4 The Traditional Square of Opposition

Standard-form categorical propositions having the same subject and predicate terms may differ from each other in quality or in quantity or in both. This kind of differing was given the technical name **opposition** by older logicians,

## Exercises

What can be inferred about the truth or falsehood of the remaining propositions in each of the following sets (1) if we assume the first to be true, and (2) if we assume the first to be false?

*1. a. All successful executives are intelligent people.
b. No successful executives are intelligent people.
c. Some successful executives are intelligent people.
d. Some successful executives are not intelligent people.

2. a. No animals with horns are carnivores.
b. Some animals with horns are carnivores.
c. Some animals with horns are not carnivores.
d. All animals with horns are carnivores.

3. a. Some uranium isotopes are highly unstable substances.
b. Some uranium isotopes are not highly unstable substances.
c. All uranium isotopes are highly unstable substances.
d. No uranium isotopes are highly unstable substances.

4. a. Some college professors are not entertaining lecturers.
b. All college professors are entertaining lecturers.
c. No college professors are entertaining lecturers.
d. Some college professors are entertaining lecturers.

# 5.5 Further Immediate Inferences

There are other kinds of immediate inference, in addition to those associated with the traditional Square of Opposition. In this section we shall present three of these other types.

## A. Conversion

The first kind of immediate inference, called **conversion,** proceeds by simply *interchanging the subject and predicate terms* of the proposition. Conversion is perfectly valid in the case of E and I propositions. Clearly, "No men are angels" can be uttered to make the same assertion as "No angels are men," and either can be validly inferred from the other by the immediate inference called conversion. Just as clearly, "Some writers are women" and "Some women are writers" are logically equivalent, so by conversion either can be validly inferred from the other. **One standard-form categorical proposition is said to be the *converse* of another when it is formed by simply interchanging the subject and predicate terms of that other proposition.** Thus "No idealists are politicians" is the converse of "No politicians are idealists," and each can validly be inferred from the other by conversion. The term **convertend** is used to refer to the premiss of an immediate inference by conversion, and the conclusion of that inference is called the **converse.**

Note that the converse of an **A** proposition does not in general follow validly from that **A** proposition. Thus, if our original proposition is "All dogs are animals," its converse, "All animals are dogs," does not follow from the original proposition at all, the convertend being true while its converse is false. Traditional logic recognized this fact, of course, but asserted that something *like* conversion was valid for **A** propositions. From an **A** proposition (*All S is P*), its subaltern **I** proposition (*Some S is P*) can be validly inferred on the traditional Square of Opposition, as explained in Section 5.3. The **A** proposition says something about *all* members of S, but the **I** proposition makes a more limited claim, about only *some* members of **S**. We have just seen that conversion of an **I** proposition is perfectly valid.

So, given the **A** proposition (*All S is P*), its subaltern (*Some S is P*) can validly be inferred by subalternation, and from that subaltern the proposition (*Some P is S*) can validly be inferred by conversion. Hence by a combination of subalternation and conversion, *Some P is S* can validly be inferred from *All S is P*. This pattern of inference, called **conversion by limitation (or "conversion *per accidens*"), proceeds by interchanging subject and predicate terms *and* changing the quantity of the proposition from universal to particular.** Thus it was traditionally claimed that from the premiss "All dogs are animals" the conclusion "Some animals are dogs" could validly be inferred, the inference being called "conversion by limitation." This type of conversion will be considered further in the next section.

Observe that the converse obtained as the outcome of conversion by limitation is *not equivalent* to the **A** proposition from which it is derived. The reason is that conversion by limitation requires a change in quantity, from universal to particular. The proposition that results from this conversion by limitation is therefore not an **A** but an **I** proposition; it cannot have the same meaning as its convertend, and hence cannot be logically equivalent to it. But the converse of an **E** proposition is an **E** proposition, and the converse of an **I** proposition is an **I** proposition; in these cases, the convertend and the converse do have the same quantity and are logically equivalent.

Finally, note that the conversion of an **O** proposition is not, in general, valid. The **O** proposition "Some animals are not dogs" is plainly true; its converse is the proposition "Some dogs are not animals," which is plainly false. An **O** proposition and its converse are not, in general, equivalent.

The converse of a given proposition always contains exactly the same terms as the given proposition (their order being reversed) and always has the same quality. The following table was traditionally held to give a complete picture of this immediate inference:

**TABLE OF VALID CONVERSIONS**

| CONVERTEND | CONVERSE |
|---|---|
| **A**: All *S* is *P*. | **I**: Some *P* is *S* (by limitation) |
| **E**: No *S* is *P*. | **E**: No *P* is *S*. |
| **I**: Some *S* is *P*. | **I**: Some *P* is *S*. |
| **O**: Some *S* is not *P*. | (conversion not valid) |

## B. Obversion

The next type of immediate inference to be discussed is called **obversion.** Before explaining it, we shall return briefly to the notion of a "class" and introduce some new ideas that will enable us to discuss obversion more easily. **A *class* is the collection of all objects having a certain common attribute that we refer to as the "class-defining characteristic."** Thus the class of all humans is the collection of all things that have the characteristic of being human, and its class-defining characteristic is the attribute of being human. The class-defining characteristic need not be a "simple" attribute in any sense, for *any* attribute determines a class. Thus the complex attribute of being left-handed and red-headed and a student determines a class—the class of all left-handed, red-headed students.

**Every class has associated with it a complementary class, or *complement,* which is the collection of all things that do not belong to the original class.** Thus the complement of the class of all people is the class of all things that are *not* people. The class-defining characteristic of the complementary class is the (negative) attribute of *not being a person.* The complement of the class of all people contains no people but contains everything else: shoes and ships and sealing wax, and cabbages—but no kings, since kings are people. It is sometimes convenient to speak of the complement of the class of all persons as "the class of all nonpersons." The complement of the class designated by the term *S* is then designated by the term *non-S*; and we may speak of the term *non-S* as being the complement of the term *S*.[3]

We are using the word "complement" in two senses: one being the sense of class complement, the other the sense of the complement of a term. The two senses, although different, are very closely connected. One term is the (term) complement of another just in case the first term designates the (class) complement of the class designated by the second term. It should be noted that just as a class is the (class) complement of its own complement, a term is the (term) complement of its own complement. A sort of "double negative" rule is involved here, so that we need not have strings of "non's" prefixed to a term. Thus we should write the complement of the term "voter" as "nonvoter," but we should write the complement of "nonvoter" simply as "voter" rather than "nonnonvoter." One must be careful not to mistake contrary terms for complementary terms, as in identifying "cowards" and "nonheroes." The terms "coward" and "hero" are contraries, in that no person can be both a coward and a hero, but not everyone—and certainly not everything—need be either one or the other. Thus the complement of the term "winner" is not "loser" but "nonwinner," for although not everything—or even everyone—is either a winner or a loser, absolutely everything is either a winner or a nonwinner.

[3]Sometimes we reason using what is called the *relative complement* of a class, its complement within some other class. For example: within the class of "children of mine" there is a subclass, "daughters of mine," whose relative complement is another subclass, "children of mine who are not daughters" or "sons of mine." But obversion and other immediate inferences normally rely upon the absolute complement of classes, as defined above.

Now that we understand what is meant by the complement of a term, the process of obversion is easy to describe. In **obversion,** the subject term remains unchanged, and so does the quantity of the proposition being obverted. **To obvert a proposition, we change its quality and replace the predicate term by its complement.** Thus the **A** proposition

All residents are voters.

has as its obverse the **E** proposition

No residents are nonvoters.

These two propositions, it is clear, are logically equivalent, and therefore either one can validly be inferred from the other. Obversion is a valid immediate inference when applied to *any* standard-form categorical proposition. Thus the **E** proposition

No umpires are partisans.

has as its obverse the logically equivalent **A** proposition

All umpires are nonpartisans.

Similarly, the obverse of the **I** proposition

Some metals are conductors.

is the **O** proposition

Some metals are not nonconductors.

And finally the **O** proposition

Some nations were not belligerents.

has as its obverse the **I** proposition

Some nations were nonbelligerents.

The term **obvertend** is used to refer to the premiss of an immediate inference by obversion, and the conclusion is called the **obverse.** Every standard-form categorical proposition is logically equivalent to its obverse, so obversion is a valid form of immediate inference for any standard-form categorical proposition. To obtain the obverse of a proposition, we leave the quantity and the subject term unchanged, change the quality of the proposition, and replace the predicate term by its complement. The following table gives a complete picture of all valid obversions:

**TABLE OF OBVERSIONS**

| OBVERTEND | OBVERSE |
|---|---|
| A: All *S* is *P*. | E: No *S* is non-*P*. |
| E: No *S* is *P*. | A: All *S* is non-*P*. |
| I: Some *S* is *P*. | O: Some *S* is not non-*P*. |
| O: Some *S* is not *P*. | I: Some *S* is non-*P*. |

## C. Contraposition

The third variety of immediate inference to be discussed introduces no new principles, for it can be reduced, in a sense, to the first two. **To form the *contrapositive* of a given proposition, we replace its subject term by the complement of its predicate term and replace its predicate term by the complement of its subject term.** Thus the contrapositive of the **A** proposition

All members are voters.

is the **A** proposition

All nonvoters are nonmembers.

That these two are logically equivalent will be evident upon a moment's reflection, and from this it is clear that contraposition is a valid form of immediate inference when applied to **A** propositions. Contraposition introduces nothing new, for we can get from any **A** proposition to its contrapositive by first obverting it, next applying conversion, and then applying obversion again. Thus, beginning with "All *S* is *P*," we obvert it to obtain "No *S* is non-*P*," which converts validly to "No non-*P* is *S*," whose obverse is "All non-*P* is non-*S*." Thus the contrapositive of any **A** proposition is the obverse of the converse of the obverse of that proposition.

Contraposition is most useful in working with **A** propositions, but it is a valid form of immediate inference when applied to **O** propositions also. Thus the contrapositive of the **O** proposition

Some students are not idealists.

is the somewhat cumbersome **O** proposition

Some nonidealists are not nonstudents.

which is logically equivalent to the first. Their logical equivalence can be shown by deriving the contrapositive a step at a time through obverting, converting, and then obverting again, as in the following schematic derivation: "Some *S* is not *P*" obverts to "Some *S* is non-*P*," which converts