Conditional Probability Example: Tossing a Fair Coin Three Times

We will define two events based on the outcomes of tossing a fair coin three times:

- **Event E:** At least two tails appear.
- **Event F:** The first coin shows head.

To calculate the conditional probability P(E|F), which is the probability of getting at least two tails given that the first coin shows head, follow these steps:

1. **Identify the Sample Space (S):** When you toss a coin three times, the sample space SS is: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
2. **Define Event F (First coin shows head):** The outcomes that satisfy this condition are: {HHH, HHT, HTH, HTT}
3. **Define Event E (At least two tails appear):** The outcomes that satisfy this condition are: {HTT, THT, TTH, TTT}
4. **Intersection of E and F (At least two tails and first coin is head):** The outcomes that satisfy both conditions are: {HTT}
5. **Calculate Probabilities:**
    - **P(F) (Probability of first coin showing head)** = 1/2
    - **P(E∩F) (Probability of getting at least two tails and the first coin showing head)** = 1 / 8
6. **Calculate Conditional Probability P(E|F):**

6. Calculate Conditional Probability $P(E|F)$:
    - $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/8}{1/2} = \frac{1/8}{4/8} = \frac{1}{4}$

Thus, the conditional probability of getting at least two tails given that the first coin shows head is 25%

Sums based on the scenario of tossing a fair coin three times, focusing on different conditional probability questions:

1. Calculate P(E|F) where E is getting exactly one head, and F is getting a head on the first toss.
2. Calculate P(E|F) where E is getting no heads, and F is not getting a head on the third toss.
3. Calculate P(E|F) where E is getting all tails, and F is getting a tail on the first toss.
4. Calculate P(E|F) where E is getting at least one tail, and F is getting a head on the second toss.
5. Calculate P(E|F) where E is getting exactly two heads, and F is getting a head on the third toss.
6. Calculate P(E|F) where E is getting exactly two tails, and F is getting a tail on the second toss.
7. Calculate P(E|F) where E is the outcome TTH, and F is the outcome starting with a tail.
8. Calculate P(E|F) where E is getting at least one head, and F is getting a tail on the first toss.
9. Calculate P(E|F) where E is getting a sequence of HTH, and F is getting a head on the first toss.
10. Calculate P(E|F) where E is getting heads only on the first and third tosses, and F is not getting a tail on the second toss.
11. Calculate P(E|F) where E is getting tails only on the first and third tosses, and F is getting a head on the second toss.
12. Calculate P(E|F) where E is the outcome HTT, and F is the first coin showing a head.
13. Calculate P(E|F) where E is the outcome HHH, and F is getting a head on the third toss.
14. Calculate P(E|F) where E is the outcome TTT, and F is getting a tail on the third toss.
15. Calculate P(E|F) where E is getting all heads, and F is getting at least two heads.