Beat Detection Algorithm for Popular Music with Fast Fourier Transform

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***Abstract*—This paper presents a method for extracting the tempo of a piece of music using the Fast Fourier Transform and a noise-filtration algorithm. Upon receiving an audio signal, the program breaks the signal into seven discrete frequency ranges and uses an iterative approach to distinguish musical beats from noise. The algorithm was implemented using Python, trained on the Top 100 songs on Spotify from 2016 and 2018, and tested on 300 songs from various genres. The algorithm achieved an 86% success rate at identifying a song's tempo in beats per minute (BPM). These results were sustained across popular music from different genres and songs of various tempos.**

***Index Terms*—beat detection, peak detection, signal processing, tempo detection, computational musicology**

I. Introduction

A beat is defined as a perceivable pulse within a piece of music, usually occurring at regular intervals and providing a sense of rhythm. A slightly less rigorous but more accessible definition of the beat of a piece of music, provided by Tzanetakis, et al., is "the regular... sequence of pulses corresponding to where a human would tap his [or her or their] foot while listening to the music" [1]. In this paper, music with a clearer beat is referred to as "clappable." The underlying beat of a piece of music is closely related to its tempo, which is defined as the speed at which a piece of music is played; in particular, musical tempo is usually measured in beats per minute (BPM). The beat of a song, especially in the context of Western popular music, is one of its most recognizable and essential components. The salience of the beat within music has led to significant academic efforts into developing algorithms for beat and tempo detection within an audio signal. This paper aims to add to the discussion of beat detection algorithms by providing a novel methodology for finding the tempo of a piece of music and by focusing specifically on tailoring this methodology to modern popular music.

Beat and tempo detection have several significant applications, most notably within music curation and classification algorithms, fields that are becoming increasingly relevant in the age of digital streaming platforms such as Spotify, Apple Music, and Pandora. In addition, both music transcription and visualization software relies heavily on being able to detect beats, drum events, and tempo within a piece of music. Beat detection algorithms can also have applications outside of music, in fields such as cardiac arrhythmia detection, which relies on beat detection in order to find periodicity (or a lack of periodicity) within a signal from a heart monitor.

A. Literature Review

Computational musicology–the combination of music theory with music informatics–is a relatively new field of research with numerous unexplored applications. Recent efforts have focused largely on computationally studying musical patterns in real-time, with several new studies attempting to track audio tempo and measure BPM instantaneously. Most of these real-time techniques, however, have yet to be applied to popular music; for example, Mottaghi et al. [2] work with ballroom music, Jensen, et al. [**?**] use computer-generated audio tracks, and Cheng et al. [4] analyze music from a variety of genres and sources including classical and cinematic music. Furthermore, these studies lack a comprehensive review of their algorithmic accuracy when measuring the BPM of music. Therefore, it is uncertain if their results are transferable to the modern era of popular music.

There is also a significant body of literature discussing the applications of tempo-detection even if the detection was not completed in real-time, but these studies similarly lack clearly quantified results and do not use popular music as training or testing datasets [5]. The foundation for much of the work that algorithmically detects the beat of music is audio compression and transformation; thus, several studies explore the applications of the Discrete Fourier Transform and Huffman Coding on music analysis [4], [5], [6], [7], [**?**].

Goto and Muraoka [**?**] study popular music and develop an algorithm with upwards of 86% accuracy to track BPM in real-time, but their study is from over two decades ago and may not apply to the popular music of today. Similarly, the work of Zhu and Wang [**?**] employs an approach incorporating the Huffman Coded Domain to track the tempo of popular music with 84% accuracy.[1] While both of these works offer promising findings, popular music has changed considerably in recent years, with the integration of new audio technologies–modern popular music incorporates a different combination of sound, mastering, drums, and techniques than popular music from

[1]The algorithm correctly identified the tempo of 21 out of 25 popular songs from commercial CDs.

a decade or two decades ago; these modifications to music may have led to an increase in "noise"–sounds, instruments, or audio effects that are unhelpful when searching for the beat. Building off of the work started by Goto and Muraoka [**?**] and Zhu and Wang [**?**], we aim to create a beat-detection algorithm which can filter such noise by training it on datasets of modern popular music.

A more accurate, reliable, and updated approach to measuring the BPM of popular music can be integrated into existing research efforts such as the work of Kirovski and Attias [**?**], who propose a quantitative identifier for music using its BPM and rhythmic structure; such techniques could pave the way for new classifications and categorizations of modern music.

B. Objective

Our primary objective is to test and design a noise-reducing beat-detection algorithm which accurately measures the tempo of modern popular music using Python. Such an algorithm must be able to:

1) Break down the audio signal of a piece of music into distinct frequency ranges using the Discrete Fourier Transform
2) Develop a method to identify the peaks across these distinct frequency ranges
3) Create a process for eliminating peaks associated with noise so that only the beats within a piece of music are left
4) Measure the amount of time between detected beats to calculate the BPM

To do this, we adopt a two-pronged "training and testing" approach to iteratively build the broader algorithm, comprised of the listed components above. This requires training the algorithm and using its output as feedback to repeatedly adjust our methods related to Steps 2 and 3 until a fully optimized detection algorithm is created.

C. Hypothesis

We put forth the following hypotheses about the algorithm's performance:

1) The algorithm performs better on genres with a more "clappable" beat such as Pop, whereas the algorithm is less effective at detecting the tempo of genres characterized by more complex rhythms and greater presence of noise such as Rock.
2) The algorithm performs better on music with a faster tempo than songs with a slower tempo. If there is more time between two beats, there are more opportunities for the algorithm to identify an incorrect beat due to noise. Additionally, slower songs are generally less "clappable" than faster ones.

II. Methodology

The algorithm consists of three phases. First, upon input of an audio file (in .wav format[2]), the Fast Fourier Transform (FFT) separates the audio signal encoded in the .wav file into seven distinct frequency ranges. Second, the algorithm identifies peaks within each frequency range (which, we argue, correspond to beats in the music) and analyzes the distance between the peaks using a moving window. By distance, we refer to the amount of time (in seconds) between peaks in the audio signal. Finally, the program calculates the tempo, in beats per minute, of the piece of music by using the groups of peaks that are spaced out most evenly. We search for the peaks with the most regular spacing since these peaks are most likely to represent beats within a piece of music. The parameters of the algorithm were trained on two sets of 100 popular songs (Spotify Top 100 from 2016 and 2018) and the algorithm was then tested on two sets of 100 popular songs (Billboard Hot 100 from 2021 year-end and April 2022) and one set of 100 rock songs (Spotify '00s Rock Anthems Playlist). A full pseudocode of the algorithm is included in the Appendix and a top-level view of the algorithm's design is included in Figure 1.

A. Assumptions

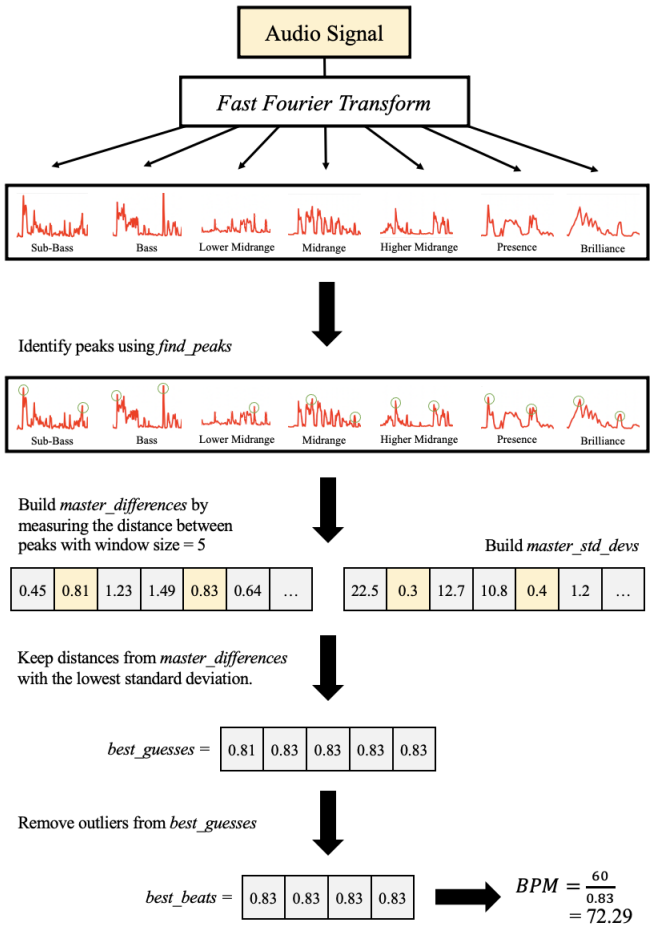
We make several assumptions about the music that the algorithm is used for. First, we assume that the analyzed music has a constant tempo and a time signature that does not change throughout the piece. Second, the logic of the algorithm is based on the premise that at least one frequency range will contain a regular half-note, quarter-note, or eighth-note pulse corresponding to the beat of the song. Though this is not a radical assumption for most music, it may cause the algorithm to fail for pieces of music with unusual beats or for slow ballads that consist solely of piano and vocals and therefore lack a clear, "clappable" beat. Similarly, the algorithm implicitly assumes that no other instruments will be playing beats at regular intervals other than the BPM of the song.[3] Third, the algorithm assumes that the distances between noisy peaks that do not correspond to the music's true tempo will always have less regularity than the peaks corresponding to beats. This assumption lays the foundation for our noise-filtration process. Finally, the algorithm is designed to analyze music consisting of two channels (left and right) with a sampling rate of 48 KHz.[4]

[2]A .wav file is a lossless audio format which leaves the original analog audio uncompressed [**?**]. The file format was created by Microsoft. The algorithm uses .wav files because of the availability of the Wave module [**?**] in Python, which allows the computer to efficiently read data from .wav audio files.

[3]While the phenomenon of a piece of music overlaying two meters over each other simultaneously ("polymeter") is unheard of in popular music, making our algorithm's assumption reasonable, polymeter is used in some non-popular music. For an example of polymeter, see "5/4" by Gorillaz, in which the pulses in the drum beat suggest a 4/4 time signature, whereas the use of accents in the guitar suggest a 5/4 time signature.

[4]Though the algorithm is currently designed for audio files matching these criteria, it can be easily modified to take in mono audio and/or accommodate a sampling rate of 44.1 KHz, another common sampling rate for audio files.

Fig. 1. Beat Detection Algorithm Design



B. Fast Fourier Transform Background

It is beyond the scope of this paper to provide an in-depth derivation of the Fast Fourier Transform, or FFT, since its use in music analysis and beat detection is well-established. However, this paper provides some of the mathematical intuition behind the FFT in order to contextualize its use in the beat detection algorithm.

The Fast Fourier Transform is a particular application of the Discrete Fourier Transform, or DFT. A brief summary of the Discrete Fourier Transform, when applied to signals, is that it transfers a signal from the time domain to the frequency domain. Specifically, the DFT takes an input vector f, with each element in the vector constituting a sample, and returns a vector of frequencies $\hat{f}$ that make up the input vector. Each element $\hat{f}_i$ of $\hat{f}$ is a complex number representing the phase and magnitude of the frequency corresponding to index i, with the real component representing the magnitude and the imaginary component representing the phase.

In particular, the DFT can be calculated using the following sum:

$$\hat{f}_k = \sum_{j=0}^{n-1} f_j \cdot e^{-i2\pi jk/n} \tag{1}$$

For simplicity, we use the following common notation:

$$\omega_n = e^{-2\pi i/n} \tag{2}$$

The sum in (1) can be trivially rewritten as the following (square) matrix-vector multiplication, assuming f is a vector with n elements:

$$\hat{f} = \begin{bmatrix} 1 & 1 & \dots & 1 & 1 \\ 1 & \omega_n & {\omega_n}^2 & \dots & {\omega_n}^{n-1} \\ 1 & {\omega_n}^2 & {\omega_n}^4 & \dots & {\omega_n}^{2(n-1)} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & {\omega_n}^{n-1} & {\omega_n}^{2(n-1)} & \dots & {\omega_n}^{(n-1)^2} \end{bmatrix} f \tag{3}$$

The innovation of the Fast Fourier Transform, or FFT, is that the matrix multiplication in (3) can be decomposed into a nested sequence of less computationally intensive matrix multiplications [**?**]. Letting F_n be the n by n matrix in (3), the matrix-vector multiplication can be written as:

$$F_{2n} = \begin{bmatrix} I_n & D_n \\ I_n & -D_n \end{bmatrix} \begin{bmatrix} F_n & 0 \\ 0 & F_n \end{bmatrix} P \tag{4}$$

The matrix P is a permutation matrix that reorders the elements of the outputted vector, as the matrix multiplication in (4) results in the correct coefficients but in the incorrect order. D_n is a diagonal matrix:

$$D_n = \begin{bmatrix} 1 & & & & \\ & \omega & & & \\ & & \omega^2 & & \\ & & & \dots & \\ & & & & \omega^{n-1} \end{bmatrix} \tag{5}$$

Multiplication by a diagonal matrix such as D_n is less costly than multiplying by the n by n matrix in (3).

The matrix decomposition in (4) is the core of the Fast Fourier Transform algorithm. The FFT requires that the number of elements in f is a power of two, namely because F_{2n} is decomposed into F_n block matrices recursively until the matrix multiplication is reduced to non-costly 2 by 2 matrix multiplications. Thus, the FFT is considered an $O(n\log(n))$ run-time operation, compared to the raw matrix multiplication of the DFT, which is of $O(n^2)$ time-complexity [**?**]. For large enough n, this discrepancy leads to significant computational benefits.

To summarize, the FFT is a computationally cheap way to decompose a signal into the frequencies that make up the signal.

C. Frequency Breakdown Using Fast Fourier Transform

The goal of the first section of the algorithm is to break down an inputted song (a .wav file) into seven different frequency ranges. The purpose of breaking down the audio signal into seven frequency ranges is the intuitive notion that the presence of beats may be more apparent in certain

frequency ranges (e.g., within certain instruments that are playing on the downbeats of a song) than in the audio signal as a whole. Using the FFT, the algorithm separates an audio file into the following seven audio ranges:

TABLE I
FREQUENCY RANGES

Name	Frequency Range
Sub-Bass	16-60 Hz
Bass	60-250 Hz
Lower Midrange	250-500 Hz
Midrange	500-2000 Hz
Higher Midrange	2000-4000 Hz
Presence	4000-6000 Hz
Brilliance	6000-20000 Hz

Source: CUI Devices. [**?**]

Our usage of the FFT for beat detection is similar to that of Cheng, et al. [4] and Scheirer, et al. [**?**], who both employ the FFT to break down audio signals into several discrete frequency ranges before identifying beats in the signal.

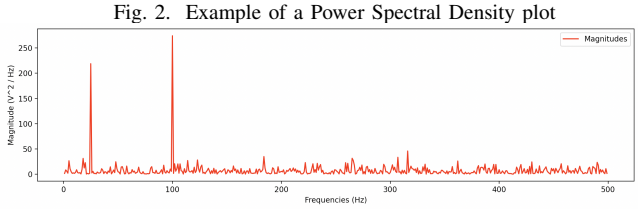
A .wav file contains a sequence of bytes in little-endian format, with the encoded integers representing an audio signal. Each element in the sequence of bytes is referred to as a "sample," which can be thought of as the value of the audio signal at an instantaneous moment in time. The music analyzed by this algorithm had a sampling rate s of 48 KHz, meaning every second of music consists of 48,000 equally spaced samples.

The chunk size c, which is the amount of samples that the algorithm reads from the audio file at a time, for our algorithm was 1024 samples. Using a chunk size that is a power of two maximizes the computational benefits provided by the FFT algorithm, which is most efficient when the length of the input vector is a power of two.[5] Furthermore, 1024 is also the chunk size adopted by Mottaghi, et al. in their implementation of the Fast Fourier Transform for beat detection [5].

For each chunk f of audio data, we computed its Fast Fourier Transform, returning a vector $\hat{f}$ of complex numbers representing the frequencies that make up the audio signal. For the purposes of beat detection, it is not necessary to distinguish between the magnitude and phase components of the complex numbers in $\hat{f}$. Thus the vector $\hat{f}$ was converted to $\hat{f}_{magnitudes}$ by multiplying each element of $\hat{f}$ by its complex conjugate. This multiplication results in what is called the Power Spectral Density, or PSD, of a signal. To be precise, the PSD is measured in units V^2/Hz and indicates how much of a particular frequency is present in the inputted chunk f. The following figure is an example of the Power Spectral Density of a signal consisting of the sum of two sin functions, one with a frequency of 100 Hz and the other with a frequency of 25 Hz.

[5]Though minimizing runtime is not the primary objective of this algorithm's design, having a fast runtime is helpful for running an algorithm on hundreds of songs using different combinations of parameters, which was necessary for the training and testing of the algorithm. The program takes one to two seconds to generate a song's BPM, depending on the length of the song.

Fig. 2. Example of a Power Spectral Density plot



As shown in the figure, the peaks in the plot correspond to the frequencies with the most representation in the inputted sample. The same calculation represented in Figure 1 is performed on each chunk of 1024 samples from the inputted piece of music.

Thus, for each chunk f of audio data, we calculated a vector $\hat{f}_{magnitudes}$ of the relative power of each frequency that makes up the audio chunk. In order to understand which elements of $\hat{f}_{magnitudes}$ corresponded to which frequency ranges, we employed the following equation, where h_i is the frequency corresponding to element i of $\hat{f}_{magnitudes}$, s is the sampling rate, and c is the chunk size:

$$h_i = i \cdot \frac{s}{c} \tag{6}$$

Using equation (6), we categorized the elements of $\hat{f}_{magnitudes}$ into the seven frequency ranges listed in Table I.

In particular, for each frequency range, we take the average of all elements of $\hat{f}_{magnitudes}$ with indexes falling within that particular frequency range. Equation (7) is an example to make this step clearer and less abstract. As an example, $p_{brilliance}$, the average power level in the brilliance category, can be calculated with the following equation:

$$mean(\{\hat{f}_i \; s.t. \; 6000 \leq i \cdot \frac{s}{c} \leq 20000\}) \tag{7}$$

By performing this calculation for each chunk in the song and saving the power levels within each frequency range in separate arrays, the algorithm is able to reconstruct a graph of the power level (measured in units V^2/Hz) of each frequency range over time. To make this clearer, Figure 2 depicts the power level of the seven frequency ranges that make up Katy Perry's "Teenage Dream" plotted over time.

Once the piece of music is broken down into 7 frequency ranges, the data moves to the next stage of the algorithm.

D. Peak Detection in Frequency Ranges

The goal of this section of the algorithm is to find peaks in the envelope of the power levels of each frequency range over time. The motivation of this is the assumption that peaks in power would correspond to a beat in the song. Finding a set of peaks within each frequency range would thus be a key step to determining the tempo of a piece of music.

The algorithm first uses the *hilbert* function from the SciPy module to find the envelope of the audio signal within each frequency range. The envelope is useful as it provides a

Fig. 3. Plot of Power Level of Frequency Ranges Over Time for "Teenage Dream"

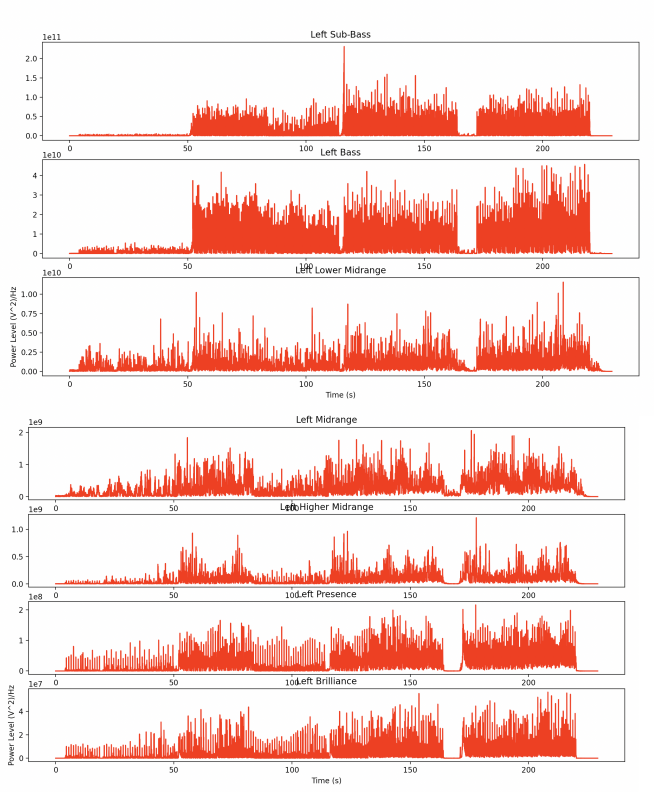
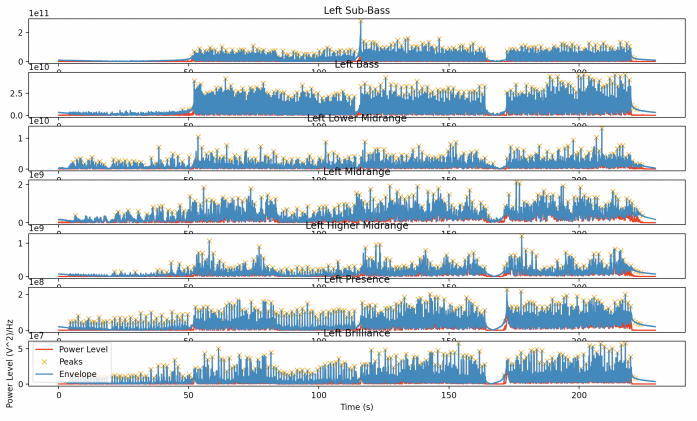


Fig. 4. Plot of Power Level of Frequency Ranges Over Time for "Teenage Dream" with Envelope and Detected Peaks



smoother estimation of the peaks of the audio data within each frequency range.

Next, the algorithm searches for peaks in the envelopes of the audio signals for each frequency range. According to the mathematical definition of a local maximum, a point x_o is a local maximum, or peak, of a function f if and only if:

$$\exists \delta \; s.t \; \forall x \; s.t \; d(x, x_o) < \delta \; we \; have \; f(x) < f(x_o) \quad (8)$$

The algorithm employs the *find_peaks* function offered by the SciPy module in order to find peaks in the power of each frequency range over time. The *find_peaks* uses a methodology similar to that of equation (8) to identify peaks in a dataset. *find_peaks* offers multiple parameters, namely the minimum distance between peaks and a minimum height for a point to be considered a peak. After tuning the minimum distance and minimum height parameters using the training data (which is elaborated on in the Training and Testing section), we determined that a minimum distance of 0.917 seconds and a minimum height of the mean power level over the entire song led to the most accurate results. Plotting the envelope and the detected peaks leads to a graph as seen in Figure 3, which shows the envelope, the signal, and the detected peaks for Katy Perry's "Teenage Dream":

E. Distinguishing Beats from Noise

We found that the *find_peaks* function often detects peaks that do not correspond to beats within a song. This is because audio signals, especially those of polyphonic music, contain large amounts of noise that may result in peaks in the song's power level that do not necessarily correspond to beats in the song. Furthermore, the design of *find_peaks* is relatively simplistic, as the algorithm considers any data point that is greater than the two points adjacent to it. According to the documentation of the function, if a minimum distance between peaks is inputted, the *find_peaks* removes the lowest peaks until all remaining peaks satisfy the minimum distance requirement. *find_peaks*, due to its simplicity, is highly susceptible to identifying noisy peaks in the data, creating the need for an approach to distinguish noisy peaks from peaks that correspond to beats in the music.

We adopted the following iterative methodology to distinguish beats from noise:

1) Create a window of the first five peaks detected
2) Calculate the distance between each adjacent peak in the window
3) Compute the mean and standard deviation of the distances between the peaks in the windows and store the mean and standard deviation in separate arrays
4) Shift the window over by one peak and repeat steps (1) through (4)

We conduct this process for every frequency range, leaving us with two arrays, one array $master\_differences$ of the mean distances between the peaks from each window, and another array $master\_standard\_deviations$ of the standard deviation of the distances between the peaks from each window.

In order to separate beats from noise, the algorithm only keeps the five mean differences with the lowest standard deviations. In other words, the algorithm only keeps data from the five windows where the distance between the peaks has the highest regularity. We posit that these windows, because of the high periodicity of the peaks, are most likely to constitute "beats" rather than noise. This means that, after the filtration process, the algorithm determines the tempo of a piece of

music based on approximately 8.8 to 25 seconds of audio.[6]

Finally, left with an array, hereafter referred to as $best\_beats$, of the five average distances with the highest regularity, we remove all elements from $best\_beats$ that are outside of one standard deviation of the mean of $best\_beats$. This ensures that, if $best\_beats$ contains both double the song's tempo and the song's actual tempo, one of the two are filtered out, leaving one guess for the song's tempo.

Once outliers are removed from the $best\_beats$ array, the average of the $best\_beats$ array is taken as the algorithm's final guess for the song's tempo in beats per minute.

Table II displays the efficacy of the noise-removal process for the first 10 songs on Spotify's Top Songs of 2016 Playlist, which was part of the training dataset used for the algorithm. The "BPM Before Filter" column shows what the algorithm would have outputted for the song's tempo without filtering out windows with higher standard deviations. The "BPM After Filter" displays the algorithm's output for the song's tempo after filtering out noise by using the standard deviation of the distance between peaks. Table III showcases the efficacy of the noise-removal process across the training data by comparing the percent error before and after filtering out noise. Here, percent error refers to the average percent error between the algorithm's calculated tempos and the actual tempos of the songs in the dataset.

TABLE II
BPM OUTPUT BEFORE AND AFTER NOISE FILTRATION

Title	*Before*	*After*	*Actual*
Closer ft. Halsey	83.5	95.0	95
Love Yourself	77.3	100.4	100
One Dance	80.6	104.2	104
Starboy	88.2	93.1	93
Hello	74.0	126.1	79
Panda	145.3	144.4	143
Hurts So Good	156.4	119.9	120
Cheap Thrills ft. Sean Paul	83.1	89.9	90
Work	82.7	52.6	92
I Know What You Did Last Summer	75.8	113.7	114

*All numbers in this table are in units of BPM.

TABLE III
PERCENT ERROR BEFORE AND AFTER NOISE FILTRATION

Dataset	Before	*After*
Spotify 2016	25.4%	4.15%
Spotify 2018	26.4%	2.06%

*Peak distance held constant at 0.94 seconds.

Brief analysis of Tables II and III suggests that the noise filtration algorithm is broadly effective at distinguishing between beats and noise, or otherwise random peaks in the audio data. More detailed exploration of the effectiveness of the algorithm is included in the Results and Discussion sections.

[6]These values were calculated by estimating how many seconds of audio corresponds to five windows of five beats in music ranging from 60 to 170 beats per minute. The exact number of seconds for 5 beats depends on the BPM of the song. Though this may seem like an overly strict threshold, other algorithms such as Cheng, et al. use as little as 2.2 seconds from a piece of music to calculate the BPM.

F. Training and Testing

Upon implementing a naive Python version of the algorithm, we utilized two training datasets to build and refine our algorithm. Our training datasets were the Spotify Top 100 Songs of 2016 and the Spotify Top 100 Songs of 2018, as these playlists consisted of unique popular music from a variety of artists and genres including pop, rock, hip-hop, country, and rap. These playlists served as ideal training datasets as they contained no overlapping music and their artists were largely distinctive, meaning that although there were some artists with multiple songs in the 2016 and 2018 training sets, redundancies were majorly minimized. Between the two datasets, our algorithm was trained using feedback from 200 distinct songs and more than 150 unique artists or combinations of artists.

Our training approach included modifying the underlying logic or parameters of the following algorithms: Peak-Detection and Peak Analysis.

1) Training the Peak Detection Algorithm: As mentioned in Section D, the $find\_peaks$ function offers two parameters for manipulation: the minimum distance between peaks and the minimum height of a peak.

Through our training process, both of these parameters were altered to optimize for algorithmic accuracy.

(a) The optimal minimum distance value of the $find\_peaks$ function was 0.938 seconds. By enforcing this minimum restriction, the algorithm's calculated BPM output is capped at approximately 63.96, but in doing so, the algorithm is able to exactly detect one-fourth or one-half of the music's BPM with improved precision, in the case that the music's tempo exceeds 63.96 BPM. At the start of the algorithm's training, the minimum distance was enforced at a value equivalent to 0.320 seconds. Under these constraints, the algorithm could theoretically compute a BPM as high as 187.5. We started at this value, as the vast majority of popular music has a tempo between 60 and 180 BPM and, therefore, we enforced a minimum distance that could compute a BPM across this range. The naive algorithm was able to output BPM values within the desired range (60, 180) but the results lacked precision. Through numerous tests (as displayed in Table II), the minimum distance value was incremented, until the algorithm was fully optimized.
The intuition behind the results of Table IV and Table V is that a larger value for the minimum distance acts as a noise-blocker, especially in songs that had a slower BPM. For example, consider applying our previous algorithmic approach (using a minimum distance interval of 0.32 seconds) to a piece of music with a tempo of 60 BPM. In this scenario, the $find\_peaks$ function would *regularly* identify peaks approximately every 0.32 seconds, even if the identified peaks did not correspond to a beat. While the regularity of the peaks corresponding to the actual tempo should theoretically be higher than the regularity of noisy peaks (since these will have random marginal differences), the algorithm has difficulty differentiating

TABLE IV
$find\_peak$ MINIMUM DISTANCE VS. ALGORITHMIC ACCURACY

Minimum Distance	*Algorithmic Accuracy*
0.32 seconds	53/100
0.43 seconds	71/100
0.53 seconds	73/100
0.64 seconds	74/100
0.75 seconds	83/100
0.85 seconds	87/100
0.87 seconds	86/100
0.89 seconds	88/100
0.92 seconds	87/100
0.94 seconds	88/100
0.96 seconds	85/100
1.1 seconds	82/100

*From the Spotify 2016 Top 100 dataset.

TABLE V
$find\_peak$ MINIMUM DISTANCE VS. ALGORITHMIC ACCURACY

Minimum Distance	*Algorithmic Accuracy*
0.32 seconds	52/100
0.43 seconds	75/100
0.53 seconds	75/100
0.64 seconds	85/100
0.75 seconds	88/100
0.85 seconds	90/100
0.87 seconds	89/100
0.89 seconds	90/100
0.92 seconds	91/100
0.94 seconds	93/100
0.96 seconds	92/100
1.1 seconds	88/100

*From the Spotify 2018 Top 100 dataset.

between the two. In simple terms, the algorithm is vulnerable to being overwhelmed by noisy peaks and, as a result, it has optimal performance for larger distance intervals. This approach, however, has diminishing returns, as shown by Tables IV and V.

(b) We used the mean power level of the piece of music as the minimum height value of the $find\_peaks$ function and did not test any additional values. We selected this value because a beat should intuitively have higher energy than the mean energy of a song.

2) Training the Peak Interpretation Process: After building the plot of peaks from the $find\_peaks$ function, we trained three aspects of our approach to peak analysis to achieve optimal results:

(a) *Moving Windows*. We constructed two types of moving windows: First, windows with a window size of 5 and a window jump of 5 peaks, resulting in no overlap between neighboring windows. Second, windows with a window size of 5 and a window jump of 1 peak, resulting in an intersection of 4 peaks between neighboring windows. As seen in Table VI, the overlapping window leads to a higher accuracy rate.

We hypothesize that the overlapping windows achieve higher accuracy because placing the data into non-overlapping groups of peaks may split up regions of the song with regular spacing of peaks. This could lead to the loss of data that reflects the true BPM.

TABLE VI
WINDOW TYPE VS. ALGORITHMIC ACCURACY

Dataset	Window Type	*Accuracy*
Spotify 2016	Overlapping	88/100
Spotify 2016	Non-Overlapping	85/100
Spotify 2018	Overlapping	93/100
Spotify 2018	Non-Overlapping	87/100

*Peak distance held constant at 0.94 seconds.

(b) *Noise Filtration*. Using the corresponding standard deviation value stored in $master\_std\_devs$, we devised an approach to filtering the $master\_differences$ array such that only our $best\_guesses$ were left. Initially, we attempted to filter by percentile, keeping only the distance values associated with the lowest 5%, 2%, and 1% of standard deviation values present in the master array. This approach proved to be futile, however, as even across just 1% of data, a considerable quantity of noisy peaks were still present. To enhance the strictness of the filtration process, we altered our approach and attempted to filter by rank, keeping only the top 3, 4, and 5 values (based on standard deviation). Ultimately, using this approach, a optimal value of 5 windows was achieved.

(c) *Outlier Classification*. The final step in the algorithmic process requires creating the $best\_beats$ array from the $best\_guesses$ array by removing all outliers. This necessitated selecting a threshold of standard deviations for classifying data in the $best\_beats$ array as an outlier. We first tried using a value of 2 standard deviations before reducing the threshold to 1 standard deviation for optimal results.

3) Summary of Parameter Training: Throughout the training period, the following values were tested for each parameter:

- *Minimum Distance Between Peaks*: 0.32, 0.43, 0.53, 0.64, 0.75, 0.85, 0.87, 0.89, 0.92, 0.938, 0.96, 1.1 seconds
- *Minimum Peak Height*: Average of the power level of the song
- *Window Type*: Overlapping and Non-Overlapping
- *Standard Deviation Threshold*: By percentile (Bottom 1, 2, and 5 percentile windows in terms of standard deviation) and by numerical cutoff (Bottom 3, 4, and 5 windows in terms of standard deviation)
- *Outlier Classification*: 2 standard deviations from mean, 1 standard deviation from mean

After training, we determined that the following parameters lead to optimal accuracy:

- *Minimum Distance Between Peaks*: 0.938 seconds
- *Minimum Peak Height*: Average of the power level of the song
- *Window Type*: Overlapping
- *Standard Deviation Threshold*: Bottom 5 windows in terms of standard deviation
- *Outlier Classification*: 1 standard deviation from mean

III. Results

For the purposes of this paper, the algorithm's output tempo was considered correct if it was exactly one half (½) or one fourth (¼) of the piece's accepted tempo. This is because, for music in common time (4/4), a tempo of any factor of two of the accepted tempo is equivalent. The accepted tempo was taken from TuneBat, an online music database which lists the BPM of more than 70 million songs, and compared to the BPM determined by the algorithm.

To enforce this rule when assessing the accuracy of the algorithm, we took the algorithm's calculated BPM values and multiplied them by a factor of 1, 2 or 4, depending on if the algorithm calculated the exact BPM of the inputted song, half of the BPM (½), or one fourth (¼) of the BPM, respectively. We refer to this new value as "Adjusted BPM." The BPM adjustment process is displayed in Table VII:

TABLE VII
BPM Adjusted Results

Title	*Calc. BPM*	*Adjusted BPM*	*Actual BPM*
Mood	45.47291835	90.9458367	91
Blinding Lights	42.74316109	170.9726444	171
The Good Ones	45	90	90
Every Chance I Get	37.5	150	150
Forever After All	38.00675676	152.027027	152

*From the 2021 BILLBOARD HOT 100 dataset.

For each song in our training and testing data sets, the adjusted BPM value was classified as correct if the percent error between the calculated value and the accepted value was less than 0.05. The aggregated results from our training and testing data sets, after applying the BPM adjustment process referenced above and enforcing the percent error bound of [0, 0.05), are included in Table VIII:

TABLE VIII
Algorithm Results

Dataset	*Type*	*Correct*	*Percent Error*
SPOTIFY Top 100 2016	Training	88/100	4.15%
SPOTIFY Top 100 2018	Training	93/100	2.06%
BILLBOARD Hot 100 2021	Testing	86/100	3.79%
BILLBOARD Hot 100 2022	Testing	86/100	3.82%
SPOTIFY '00s Rock Anthems	Testing	81/100	5.22%

The success of our algorithm persists across both testing data sets and is able to retain accuracy levels similar to those achieved for the training datasets.

We then tested the algorithm's success by genre. To classify the genre of songs within our testing datasets, we used the Spotify API which, when given the ID of a song produces an ID for the corresponding artist, along with a list of their common genres. From this list, we grouped songs into six distinct categories: Pop, Hip Hop, R&B, Country, Rock, and Other (which included Latin music and film music, among other miscellaneous categories). For our purposes, artists with music that was labeled as a subset of one of our categories was classified under the broader category; for example, music corresponding to dance pop was included in the Pop category. The algorithm's accuracy across the six genre categories is included in Table IX.

TABLE IX
Algorithm Testing Accuracy by Genre

Genre	*Correct*	*Incorrect*	*Success Rate*
Pop	49	12	80.3%
Hip Hop	56	4	90.3%
R&B	12	4	75.0%
Country	36	3	92.3%
Rock	86	19	81.2%
Other	14	5	73.7%
Overall	**253**	**47**	**84.3%**

*Data compiled from the testing datasets.

Next, we tested the ability of the algorithm to identify the BPM across distinct tempo ranges of music. We constructed 4 tempo ranges which spanned the range of BPM values present across our data sets. Table X tracks the accuracy of the algorithm for both the 2016 and 2018 testing sets across these distinct tempo ranges, showing how the algorithm retains accuracy irrespective of the actual tempo of the music.

TABLE X
Algorithm Testing Accuracy by Tempo Range

Tempo Range	*Algorithm Accuracy*
(0 BPM, 100 BPM]	0.831
(100 BPM, 120 BPM]	0.871
(120 BPM, 140 BPM]	0.867
(140 BPM, ∞]	0.906

The algorithm's testing results also demonstrate consistency across tempo ranges constructed relative to the value of the minimum distance parameter. As explained in Section F of the Methodology, the optimal distance of the $find\_peaks$ algorithm is equivalent to a time interval of 0.938 seconds which sets an upper bound on the maximum BPM the algorithm can compute (63.96). Table XI explores the effect this constraint has on the algorithm's ability to accurately measure the BPM, especially for pieces of music with a faster tempo. Each song was categorized into Range A or Range B based on its accepted BPM using the following bounds:

Range A: (0 BPM, 127.93 BPM]
Range B: (127.93 BPM, ∞)

The value 127.93 was calculated by multiplying the maximum BPM the algorithm can compute, 63.96, by a factor of 2. This segments our songs into two groups, Range A for which the algorithm could theoretically detect either one-half of the tempo or one-quarter of the tempo, and Range B for which the algorithm can only detect one-quarter of the tempo.

TABLE XI
Algorithm Testing Accuracy by Tempo Range

Dataset	*Range A*	*Range B*
BILLBOARD Hot 100 2021	0.864407	0.878049
BILLBOARD Hot 100 2022	0.868852	0.846154

Finally, we tested our hypothesis that the algorithm would be more effective at detecting the tempo of songs with a "clappable" beat. To determine if this was the case, we tested whether the algorithm performed better with songs that received a higher ranking from TuneBat's "Danceability" score.[7] According to TuneBat, its "Danceability" score measures on a scale from 0 to 100 "how appropriate the track is for dancing based on overall regularity, beat strength, rhythm stability, and tempo." Table XII compares the average "Danceability" score (referred to as D) of the songs that the algorithm correctly calculated to the average "Danceability" of the songs that the algorithm missed.

TABLE XII
ALGORITHM TESTING ACCURACY BY DANCEABILITY

Dataset	*Avg. D. of Correct*	*Avg. D. of Incorrect*
BILLBOARD 2021	67.8	62
BILLBOARD 2022	69.0	59.4

*From the BILLBOARD HOT 100 datasets.

Table XII suggests that, on average, the algorithm performed better with songs with a higher "Danceability" score. The songs that the algorithm correctly detected were on average 12.7 percent more "Danceable", according to TuneBat's index. These results support our hypothesis that music with a clearer beat is easier to track algorithmically.

IV. DISCUSSION

A. Overview

The algorithm exhibited consistent results across training and testing data. In the three testing datasets of the Billboard Hot 100 2021, Billboard Hot 100 2022, and Spotify '00s Rock Anthems, the algorithm achieved a success rate of 86%, 86%, and 81% respectively. Furthermore, the results from Tables II and III suggest that the noise-filtration section of the algorithm is effective at differentiating between peaks in an audio signal that represent noise and peaks that represent a beat, as the algorithm was able to reduce percent error by 88% compared to a naive approach that considered all peaks in the audio signal with equal weight when calculating the tempo of a piece of music. These results are extremely encouraging and suggest that the algorithm is broadly effective at identifying beats in a piece of music.

The algorithm was also consistent across music with different genres and tempos.

Contrary to our hypothesis, the algorithm saw the highest success rates in the genres of Hip Hop (90.3%) and Country (92.3%). The high success in the Hip Hop category may be because of the presence of trap beats with clear snare sounds in Hip Hop songs. The algorithm's success for detecting the tempo of Country music may be attributable to the relative lack of noise in Country music compared to Popular music due to the use of acoustic instruments in Country music.

[7]For the purposes of this paper, "Danceability" and "clappability" are synonymous and will be referred to interchangeably.

The algorithm unexpectedly saw similar success rates for Pop (80.3%) and Rock (81.2%) music, contrary to our hypothesis that the increased levels of noise in Rock music would result in a lower success rate. This finding provides further evidence of the effectiveness of the noise-filtration portion of the algorithm.

The algorithm performed consistently across the four tempo ranges, with a slight upward trend in accuracy as the tempo increased. This matches our initial hypothesis that the algorithm will encounter greater difficulties measuring the BPM of music with a slower tempo. We suspect, however, that the algorithm's performance in the slowest tempo range (between 0 and 100 BPM) and the highest tempo range (above 140 BPM) may have been influenced by genre. Music in the highest tempo range of our dataset largely corresponds to the Hip Hop genre, which the algorithm is more accurate at measuring than Pop.[8] Furthermore, the algorithm's ability to detect the beat of music across different tempo ranges was not constrained by the parameter value chosen for the minimum distance between peaks. This indicates that the algorithm's design is equally effective at calculating one-half of the tempo and one-quarter of the tempo.

The success of the algorithm across genre, tempo, and "clappability" suggests that the algorithm's parameters and design were not overfit to the songs in the training sets, which mostly consisted of popular music. Instead, analysis of the algorithm's results suggests that the algorithm's design is broadly effective for music across a variety of cross-sections.

B. Limitations

The success of our FFT beat-detection algorithm is partially dependent on three key factors, with an ideal music track having the following characteristics:

1) the music has high audibility of the beat (often characterized by a large presence of drums)
2) the music has no tempo changes
3) the music is in a constant time signature

There were a few pieces of music within the testing datasets which were suboptimal in these categories, exemplifying key shortcomings in our algorithm. While these limitations do not apply to the majority of modern popular music, it is still a notable limitation of our approach and the design of our algorithm.

First, in some situations, our algorithm's performance suffers if the music does not have consistent drums present in the track. For example, throughout our training tests, our algorithm was regularly able to detect the BPM of music by dance pop artists such as Dua Lipa, whose music is characterized by a clear, "clappable" beat, but it outputted an incorrect tempo for Lonely by Justin Bieber, possibly due to the lack of drums and "clappability" in the track. This shortcoming, however, was not

[8]See Table IX.

universal across all tracks lacking drums, as our algorithm in many cases was still able to detect the beat.[9]

Second, the algorithm fails to account for tempo changes in music. Notably, the algorithm identified the tempo of Travis Scott's "SICKO MODE," which contains a tempo change, as 154.96 beats per minute. This result matches the tempo of the song's first and second verses (which have a BPM of 155), but does not match the song's introduction, which has a tempo of roughly 139 BPM. The presence of a clear electronic drumbeat in the first and second verses likely caused the algorithm to pick up the tempo of the verses with higher confidence than the tempo of the introduction section, leading to the output of 155 BPM.

While the algorithm correctly identified one of the tempos of Scott's "SICKO MODE," the algorithm was unable to identify either of the tempos of "Welcome to the Black Parade" by My Chemical Romance, which has a tempo of 75 BPM in its introduction section and 97 BPM in the rest of the piece. The algorithm, however, outputted 125 BPM, which does not match either tempo present in the piece.

These case studies demonstrate the variable performance of the algorithm on pieces with variable tempos.

Third, the algorithm is unable to account for changes in the time signature. For Billie Eilish's "Happier Than Ever," which includes both a tempo change and a time signature change, the algorithm's output did not match the correct tempo for the song. Notably, the algorithm calculated a BPM that differed from both of the correct BPM values, indicating that the error may have been caused by the music's variable time signature and not the change in tempo. In modern popular music, however, the vast majority of music is made in a constant time signature of four beats per measure.

These limitations reflect an opportunity for improvement in our algorithm. These improvements may be made with the use of advanced machine learning techniques which could, for example, identify a tempo change in music and accordingly calculate multiple BPM values for a given track of music with a certainty score attached to each guess for the BPM value. Adopting these changes would require further research which may build upon our algorithm to develop an even more accurate approach to tempo measurement.

Our algorithm can also be improved through greater exploration of the window size, window jump, and minimum height parameters. The window size was held constant at 5 peaks, the window jump was only tested with values of 1 and 5, and the minimum height was held constant at the mean power level of the piece of music. Further research could experiment with different values for these parameters, possibly leading to a refined version of this algorithm with higher accuracy.

[9]For an example, see "changes" by XXXTentacion, a piece of music that is relatively "unclappable" due to the its lack of drums and slow tempo. However, the algorithm correctly detected the tempo of the piece at 60 BPM. This can be explained by the fact that the piano part of "changes" falls exactly on the downbeat, with little overall noise in the track.

V. Conclusion

Computational musicology is a field of growing importance with new applications in streaming, music curation, and transcription. In this paper, we contribute to the discussion by developing an algorithm using the Fast Fourier Transform and a noise-filtration approach to find the tempo of a piece of music. Our findings indicate that even a simple approach (relative to other published beat-detection algorithms) can achieve high accuracy rates when accompanied by a rigorous noise reduction algorithm.

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Appendix

A. Pseudocode

B. Full Algorithm Results

TABLE XIII
BILLBOARD HOT 100 2021 RESULTS

Title	*Calc. BPM*	*Adj. BPM*	*Actual BPM*
Levitating Feat Dababy	102.6	102.6	103.0
Save Your Tears Remix	117.7	117.7	118.0
Blinding Lights	85.5	171.0	171.0
Mood	90.9	90.9	91.0
Good 4 U	84.2	168.4	167.0
Kiss Me More	74.0	74.0	111.0
Leave The Door Open	74.0	148.0	148.0
Drivers License	95.7	95.7	144.0
Montero	89.6	89.6	89.0
Peaches	89.9	89.9	90.0
Butter	110.3	110.3	110.0
Stay	85.0	169.9	170.0
Deja Vu	117.9	117.9	91.0
Positions	72.3	144.6	144.0
Bad Habits	125.6	125.6	126.0
Heat Waves	80.6	80.6	81.0
Without You	92.2	92.2	93.0
Forever After All	76.0	152.0	152.0
Go Crazy	93.8	93.8	94.0
Astronaut In The Ocean	75.0	150	150.0
34+35	110.3	110.3	110.0
What You Know Bout Love	84.0	84.0	84.0
My Ex 's Best Friend	62.5	125.0	125.0
Industry Baby	74.9	149.7	150.0
Therefore I Am	93.8	93.8	94.0
Up	83.0	165.9	166.0
Fancy Like	79.9	79.9	80.0
Dakiti	110.0	110.0	110.0
Best Friend	93.8	93.8	145.0
Rapstar	108.2	108.2	81.0
Heartbreak Anniversary	65.0	65.0	89.0
For The Night	63.1	126.1	126.0
Calling My Phone	52.6	105.1	105.0
Beautiful Mistakes	99.0	99.0	99.0
Holy	86.8	86.8	87.0
On Me	78.1	78.1	78.0
You Broke Me First	124.4	124.4	124.0
Traitor	100.1	100.1	101.0
Back In Blood	73.4	146.9	147.0
I Hope	75.0	75	76.0
Dynamite	76.0	76.0	114.0
Wockesha	85.0	85.0	82.0
You Right	64.5	129.0	129.0
Beat Box	80.1	160.3	160.0
Laugh Now Cry Later	67.0	133.9	134.0
Need To Know	65.0	129.9	130.0
Wants And Needs	67.9	135.9	136.0
Way 2 Sexy	67.9	135.9	136.0
Telepatia	84.0	84.0	84.0
Whoopty	119.3	119.3	140.0
Lemonade	70.0	139.9	140.0
Good Days	121.8	121.8	121.0
Starting Over	89.0	89.0	89.0
Body	93.8	93.8	94.0
Willow	84.0	84.0	81.0
Bang !	70.1	140.3	140.0
Better Together	108.6	108.6	138.0
You 're Mines Still	89.0	89.0	87.0
Every Chance I Get	75.0	150	150.0
Essence	106.3	106.3	104.0
Chasing After You	66.0	132.0	132.0
The Good Ones	90.0	90	90.0
Leave Before You Love Me	119.7	119.7	120.0
Glad You Exist	104.2	104.2	104.0
Lonely	120.1	60.0	79.0
Beggin	67.0	133.9	134.0
Streets	45.0	90.0	90.0
Whats Next	65.0	130.0	130.0
Famous Friends	102.0	102.0	102.0
Lil Bit	119.7	119.7	120.0
Thot Shit	65.0	129.9	130.0
Late At Night	98.7	98.7	99.0
Kings & Queens	114.2	114.2	130.0
Anyone	116.2	116.2	116.0
Track Star	113.3	113.3	130.0
Time Today	68.6	137.2	137.0
Cry Baby	65.0	129.9	130.0
All I Want For Christmas Is You	75.0	150	150.0

TABLE XIV
BILLBOARD HOT 100 2021 RESULTS

Title	*Calc. BPM*	*Adj. BPM*	*Actual BPM*
Heat Waves	80.6	80.6	81.0
Stay	85.0	169.9	170.0
Super Gremlin	73.1	73.1	73.0
Abcdefu	122.3	122.3	122.0
Ghost	77.1	154.1	154.0
We Do N't Talk About Bruno	102.6	102.6	103.0
Enemy	77.1	77.1	77.0
Thats What I Want	118.0	118.0	88.0
Woman	108.2	108.2	108.0
Easy On Me	114.6	114.6	142.0
Big Energy	106.1	106.1	106.0
Bad Habits	125.6	125.6	126.0
Shivers	70.5	141.0	141.0
Cold Heart Pnau Remix	115.7	115.7	116.0
Need To Know	65.0	129.9	130.0
Levitating Feat Dababy	102.6	102.6	103.0
Save Your Tears Remix	117.7	117.7	118.0
Til You Cant	80.1	160.3	160.0
Pushin P	77.5	77.5	78.0
One Right Now	97.0	97.0	97.0
Industry Baby	74.9	149.7	150.0
What Happened To Virgil	70.1	140.3	140.0
I Hate U	80.1	80.1	107.0
Hrs And Hrs	70.1	140.3	140.0
Sweetest Pie	124.4	124.4	124.0
Mamiii	93.7	93.7	94.0
Good 4 U	84.2	168.4	167.0
Ahhh Ha	78.1	156.2	156.0
Light Switch	92.1	92.1	92.0
Doin This	114.8	114.8	115.0
You Right	64.5	129.0	129.0
Fingers Crossed	110.3	110.3	109.0
Buy Dirt	89.0	89.0	89.0
Bam Bam	94.9	94.9	95.0
Surface Pressure	90.0	90	90.0
Fancy Like	79.9	79.9	80.0
Blick Blick	70.0	139.9	140.0
Sand In My Boots	70.1	70.1	70.0
Love Nwantiti	93.1	93.1	93.0
Never Say Never	70.1	140.3	140.0
Aa	104.2	104.2	104.0
Boyfriend	119.7	119.7	90.0
Thinkin With My Dick	81.2	81.2	81.0
Drunk	119.7	119.7	120.0
Beers On Me	73.1	146.1	146.0
Knife Talk	73.1	146.1	146.0
Broadway Girls	75.0	75	75.0
Shes All I Wan Na Be	80.0	160.0	160.0
Numb Little Bug	85.1	85.1	85.0
Nobody Like U	104.6	104.6	105.0
The Motto	117.7	117.7	118.0
Slow Down Summer	78.1	156.2	156.0
23	98.0	98.0	98.0
Peru	30.9	123.6	108.0
The Family Madrigal	70.0	140.0	141.0
Handsomer	81.8	81.8	82.0
Sometimes	122.1	122.1	122.0
To Be Loved By You	73.1	146.1	146.0
Heart On Fire	119.7	119.7	120.0
Circles Around This Town	75.0	150	150.0
Computer Murderers	86.5	173.1	173.0
Petty Too	77.1	154.1	154.0
Do We Have A Problem	119.7	119.7	120.0
To The Moon	106.5	106.5	144.0
Me Or Sum	80.4	160.7	161.0
Nail Tech	75.0	150	150.0
Flower Shops	85.2	85.2	128.0
No Interviews	79.0	158.0	158.0
Never Wanted To Be That Girl	74.0	74.0	74.0
Banking On Me	67.6	135.2	135.0
Barbarian	109.2	109.2	98.0
Beautiful Lies	85.2	85.2	84.0
Bones	114.3	114.3	114.0
By Your Side	79.0	158.0	158.0
City Of Gods	75.0	150	147.0
Closer	74.0	74.0	95.0
Comeback As A Country Boy	76.0	76.0	76.0
Dos Orugitas	127.1	127.1	94.0

TABLE XV
BILLBOARD HOT 100 2021 RESULTS

Title	*Calc. BPM*	*Adj. BPM*	*Actual BPM*
Ocean Avenue	115.3	115.3	174
Hysteria	93.4	93.4	93
Boulevard Of Broken Dreams	83.7	167.4	167
Cant Stop	90.7	90.7	91
What Ive Done	119.7	119.7	120
Chop Suey	125.2	125.2	125
Seven Nation Army	124.4	124.4	124
Mr Brightside	98.7	98.7	148
Kryptonite	98.7	98.7	99
Sugar Were Going Down	81.1	162.1	162
Misery Business	86.5	173.1	173
Dani California	95.3	95.3	96
Holiday	73.2	146.5	147
Last Resort	90.5	90.5	91
In The End	104.9	104.9	105
I Miss You	110.3	110.3	110
Californication	97.0	97.0	96
Youre Gon Na Go Far Kid	126.1	126.1	126
Toxicity	75.8	75.8	77
Dance Dance	114.3	114.3	114
The Middle	108.2	108.2	162
The Kill	122.5	122.5	91
I Write Sins Not Tragedies	117.7	117.7	176
Take A Look Around	100.8	100.8	102
Bring Me To Life	95.1	95.1	95
Savior	112.5	112.5	112
Numb	109.9	109.9	110
Best Of You	111.3	111.3	130
The Diary Of Jane	83.7	167.4	167
Take Me Out	103.8	103.8	105
Welcome To The Black Parade	125.7	125.7	97
I Hate Everything About You	89.3	89.3	89
Sex On Fire	76.4	152.7	153
Im Not Okay	89.9	89.9	90
Like A Stone	108.2	108.2	108
The Pretender	86.5	173.1	173
When You Were Young	113.9	113.9	130
Face Down	92.8	92.8	93
Its Been Awhile	117.7	117.7	117
Island In The Sun	114.8	114.8	115
She Hates Me	110.3	110.3	110
Uprising	127.1	127.1	128
Yellow	86.5	173.1	173
Smooth Criminal	127.3	127.3	127
American Idiot	93.1	93.1	93
The Anthem	88.7	88.7	89
Thnks Fr Th Mmrs	103.0	103.0	155
Paralyzer	106.1	106.1	106
All My Life	84.0	167.9	168
The Reason	82.7	82.7	83
No One Knows	125.6	125.6	126
In Too Deep	115.3	115.3	116
Supermassive Black Hole	120.2	120.2	120
First Date	95.8	95.8	96
Youth Of The Nation	98.0	98.0	98
Rollin	95.0	95.0	96
Complicated	78.1	78.1	78
Gives You Hell	100.1	100.1	100
Fake It	66.2	132.4	132
Joker And The Thief	88.4	88.4	78
Miss Murder	119.0	119.0	144
Higher	78.0	155.9	156
Im Just A Kid	109.9	109.9	110
Beverly Hills	87.9	87.9	88
Dear Maria Count Me In	90.5	90.5	91
Use Somebody	114.0	114.0	137
Through Glass	105.7	105.7	106
Wish You Were Here	84.8	169.6	170
Last Nite	104.2	104.2	104
Steady As She Goes	124.9	124.9	124
Are You Gon Na Be My Girl	104.9	104.9	105
Want You Bad	105.7	105.7	106
Welcome Home	77.1	154.1	154
Reptilia	79.0	158.0	158
Hate To Say I Told You So	67.8	135.5	136
Jerk It Out	67.0	133.9	134
I Bet You Look Good On The Dancefloor	104.7	104.7	103
Float On	100.8	100.8	101

Algorithm 1 Construction of Frequency Ranges

Input: Array of audio data $\{audio\_input\}$ in bytes and array of indices $\{[sub\_bass\_indices], [bass\_indices], etc.\}$ corresponding to each frequency range in Table I calculated using Equation (6)

Output: Array of power levels of frequency ranges over time

$chunk \leftarrow 1024$

$data \leftarrow audio\_input.readframes(chunk)$ {$readframes$ is a command from the Wave module that reads $chunk$ bytes from an audio file at a time. The while loop ends once the algorithm has read through the full audio file.}

while $len(data) > 0$ **do**

$chunk\_fft \leftarrow fft(data)$ {$chunk\_fft$ is an array of complex numbers calculated for each chunk of data}

$fft\_magnitudes \leftarrow chunk\_fft \cdot conj(chunk\_fft)$

$sub\_bass \leftarrow mean(fft\_magnitudes[sub\_bass\_indices])$

$bass \leftarrow mean(fft\_magnitudes[bass\_indices])$

$low\_mid \leftarrow mean(fft\_magnitudes[low\_mid\_indices])$

$mid \leftarrow mean(fft\_magnitudes[mid\_indices])$

$high\_mid \leftarrow mean(fft\_magnitudes[high\_mid\_indices])$

$pres \leftarrow mean(fft\_magnitudes[pres\_indices])$

$bril \leftarrow mean(fft\_magnitudes[bril\_indices])$

$data \leftarrow input.readframes(chunk)$

$sub\_bass\_array.append(sub\_bass)$

$bass\_array.append(bass)$

... {Value for each frequency range is appended onto a separate array.}

$brilliance\_array.append(bril)$

end while

return sub_bass_array, $bass\_array$, low_mid_array, mid_array, $high\_mid\_array$, $pres\_array$, $brilliance\_array$

Algorithm 2 Detection of Peaks with Moving Window

Input: sub_bass_array, $bass\_array$, low_mid_array, mid_array, $high\_mid\_array$, $pres\_array$, $brilliance\_array$

Output: Master_differences (an array of the average distances between peaks) and Master_std_devs (an array of the std. deviation of the distance between peaks within each window)

$master\_differences \leftarrow []$

$master\_std\_devs \leftarrow []$

{All of the analysis below this comment is performed on every frequency range, but only one frequency range is included in the pseudocode for brevity.}

$sub\_bass\_envelope \leftarrow hilbert(sub\_bass\_array)$

$sub\_bass\_peaks \leftarrow find\_peaks(sub\_bass\_envelope)$

{Both $hilbert$ (a function for finding the envelope of a signal) and $find\_peaks$ are available in the SciPy module, as stated before. $find\_peaks$ returns the indexes of the inputted array that correspond to peaks.}

for $i = 0$, $i{+}{+}$, while $i < length(sub\_bass\_peaks)$ **do**

$window \leftarrow sub\_bass\_peaks[i : i + 6]$

$diff\_array \leftarrow [window[4] - window[3], window[3] - window[2], window[2] - window[1], window[1] - window[0]]$

$avg\_diff \leftarrow mean(diff\_array)$

$std\_dev \leftarrow std(diff\_array)$

$master\_differences.append(avg\_diff)$

$master\_std\_devs.append(std\_dev)$

end for

{Notice that $master\_differences$ and $master\_std\_devs$ are indexed such that element i of $master\_differences$ is the average distance between the peaks of window i, and element i of $master\_std\_devs$ is the standard deviation of the distance between the peaks of window i}

return $master\_differences$, $master\_std\_devs$

Algorithm 3 Selection of Best Guesses for Peaks

Input: $master\_differences$, $master\_std\_devs$

Output: Tempo of the $audio\_input$ in beats per minute

$best\_indexes \leftarrow$ [i such that $master\_std\_devs[i]$ is in the 5 lowest standard deviations]

$best\_guesses \leftarrow master\_differences[best\_indexes]$

$best\_guesses \leftarrow remove\_outliers(best\_guesses)$

return mean($best\_guesses$)