TUTORIAL 3 (week 4)

The Simplex Method

Input: A linear program in standard form

Output: An optimal solution

1: Construct canonical form and obtain a basic feasible solution
2: **while** There are negative reduced costs **do**
3: Select entering variable with most positive/negative reduced cost (Alysson's notation / slides notation).
4: Select leaving variable using the ratio test.
5: Change basis.
6: **end while**

1. Solve the following LP problem by the Simplex Method.

$$\begin{aligned} &\max z = x_1 + x_2 + x_3 \\ &2x_2 + x_3 \leq 1 \\ &2x_1 + x_2 + 2x_3 \leq 1 \\ &x_1 + 2x_2 \leq 1 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

2. Solve the following LP problem, or identify that the problem is unbounded or infeasible, by using the Simplex Method.

$$\begin{aligned} &\max z = 2x_1 + 3x_2 \\ &x_1 - 2x_2 \leq 4 \\ &2x_1 + x_2 \leq 18 \\ &x_2 \leq 10 \\ &x_1, x_2 \geq 0 \end{aligned}$$

3. Solve the following LP problem, or identify that the problem is unbounded or infeasible, by using the Simplex Method. Provide a graphical interpretation.

$$\max z = 5x_1 + 3x_2$$
$$-x_1 + x_2 \leq 4$$
$$x_1 - 2x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

4. The tableau method consists in writing the problem in a table and doing simplex iterations by doing pivot operations in this table. The basic variables can be identified in the tableau by finding the columns of an identity matrix. The reduced costs can be found in the row z for the columns of the non-basic variables.

	x_1	x_2	x_3	x_4	RHS
x_1	1	1/2	1/2	0	4
x_4	0	1/2	−1/2	1	1
z	0	0	1	0	8

   (a) Identify the basic variables in the solution corresponding to the tableau above. What are their values ?

   (b) Identify the non-basic variables and their reduced costs. Is this solution optimal for a maximisation problem ?

   (c) Who will enter the basis and who will leave in the next iteration ?