Notes on Quadrotor

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1 Mathematic

Assume a star with Equatorial coordinate (δ, α), the latitude is ϕ_0, the camera is pointing at (a, A) in Horizontal coordinate.

The $x - y - z$ coordinate in Equatorial frame is

$$\boldsymbol{p}_e = (\cos\delta\cos(\alpha + \omega t),\ \cos\delta\sin(\alpha + \omega t),\ \sin\delta) \tag{1}$$

and in Horizontal frame is

$$\boldsymbol{p}_h = \boldsymbol{R}_x\left(\phi_0 - \frac{\pi}{2}\right)\boldsymbol{p}_e \tag{2}$$

and in camera frame is

$$\boldsymbol{p}_c = \boldsymbol{R}_x\left(\frac{\pi}{2} - a\right)\boldsymbol{R}_z(A)\boldsymbol{p}_h \tag{3}$$

Then the image coordinate of the star could be expressed as

$$\begin{aligned}
x &= \cos A\cos\delta\cos(\alpha + \omega t) - \sin A(\cos\delta\sin\phi_0\sin(\alpha + \omega t) + \sin\delta\cos\phi_0) \\
y &= \sin\delta\cos(A\cos\phi_0\sin a - \cos a\sin\phi_0) + \cos\delta(\cos(\alpha + \omega t)\sin a\sin A + \\
&\quad \cos a\cos\phi_0 + \cos A\sin a\sin\phi_0)\sin(\alpha + \omega t)) \\
z &= \cos\delta\sin(\alpha + \omega t)(\cos a\cos A\sin\phi_0 - \sin a\cos\phi_0) + \cos a\sin A\cos\delta\cos(\alpha + \omega t) + \\
&\quad \cos a\cos A\sin\delta\cos\phi_0 + \sin a\sin\delta\sin\phi_0
\end{aligned} \tag{4}$$