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On the initial mass function and the fragmentation of molecular clouds

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Summary. We develop a simple Lagrangian model for the fragmentation process in a molecular cloud. The non-linear intermolecular potential propagates from long to short length scales the external triggering perturbation. The resulting stellar mass spectrum is sensitive to the physical quantities of the cloud and of the trigger, and is in agreement with the observational data of both field stars and open clusters.

1 Introduction

The problem of deriving from a realistic, or at least plausible, theoretic scheme the Initial Mass Function (IMF), whose knowledge is so vital for astronomy (Lynden-Bell 1977), is still open (Bodenheimer 1981).

The IMF is the result, direct or indirect (via accretion and coagulation) of the fragmentation of molecular clouds, which is believed to be the key of forming stars (Silk 1978). Previous work on this subject is characterized by some leading lines of research.

(i) Probabilistic theories (Auluck & Kothari 1954, 1965; Kushwaha & Kothari 1960; Kruszewski 1961; Kiang 1966; Larson 1973b).

(ii) Models in which a particular physical effect controls the fragmentation: density and temperature distribution (Takebe, Unno & Hatanaka 1962); changing mean molecular weight (Reddish & Wickramasinghe 1969; Reddish 1978); turbulence (Arny 1971) and heat balance (Silk 1977).

(iii) Fragment interaction models (McCrea 1961; Nakano 1966; Arny & Weissman 1973; Ferraioli & Virgopia 1979; Silk & Takahashi 1979; Bhattacharjee & Williams 1980; Bastien 1981; Zinnecker 1981).

In spite of the good ideas and the interesting remarks of the quoted papers, these attempts are not really satisfactory, either for the limits in the physical description of the situation, or the difficulties in justifying the experimental data, as discussed in Silk (1978), Scalo (1978) and Miller & Scalo (1979). Without claiming to solve the problem, in this work we present a different proposal.

We like schematizing the problem in the following way: it is a many-body problem; a very high number of particles (molecules and/or atoms, floccules, fluid elements, etc.) subject to long-range forces (self-gravitation) and to short-range potentials ('intermolecular') at some time and under some conditions prefer to aggregate in blobs, large with respect to their mutual distance, small with respect to the dimensions of the system. We adopt the point of view that molecular clouds are not self-collapsing as required by the observed star formation rate (Evans 1978; Field 1978; Mouschovias 1978). An external action will trigger the fragmentation process. We are not concerned with the analysis of the effects of a particular trigger mechanism, but with the reply of the system to the external action which we shall parametrize very simply. We refer to papers in which triggers are analysed and reviewed (Elmegreen & Lada 1977; Elmegreen & Elmegreen 1978; Woodward 1978; Evans 1978; Lada, Blitz & Elmegreen 1978; McKee & Hollenbach 1980).

Differently from the previous attempts, in our model the non-linear 'intermolecular' interactions play a conclusive role in the propagation of the external trigger perturbation from large to short scales. In particular it may happen that, as a non-equilibrium situation, critical mass densities are reached on much smaller scales than the cloud dimensions: the conditions for the gravitational collapse of a fraction of the total mass are so settled before the whole cloud could collapse. In other words we choose to treat the problem by using the tools of non-linear, non-equilibrium dynamics.*

Molecular clouds are seats of collective behaviour, fluctuations of density are correlated and, in some cases, can grow enough to enucleate a mass sufficiently large to collapse by itself. What will happen in term of the masses is that, due to the non-linear interactions, there will be a differential growth of excited modes and then a spectrum of different masses.

There is another class of phenomena: fragmentation resulting from the gravitational collapse of the cloud itself, not necessarily triggered by an external action. We will study this problem in a future work, because it requires a different starting equation: rotation of the cloud seems to be a necessary ingredient to produce an inevitable fragmentation (Larson 1973a; Larson 1978).

In Section 2 we describe our model. Sections 3 and 4 are devoted to the comparison of our results with the experimental data on the IMF. Our aim is not only to reproduce the IMF, but also to understand how the IMF depends on the physical parameters, such as magnetic fields, cloud temperature and dimensions. We comment on our model in Section 5.

2 The model

2.1 THE INITIAL RAW MATERIAL

We represent the original cloud as a dilute gas of elementary interacting constituents. For its being dilute (see Section 2.2), its essential thermodynamic properties are due to the two-body interaction consequently determined by the choice of the degree of microscopic description. We shall choose, in the following, molecules as individual elements, because intermolecular forces are well known and reproducible with confidence.

An alternative possibility would be the description in term of floccules as proposed by McCrea (1960, 1961, 1978). The floccules play the role of pseudo-particles in featuring the initial cloud of gas, and it is extremely interesting the property this description can offer to solve the problem of angular momentum. But to render quantitative McCrea's approach, two difficulties arise: (1) one had to give a quite satisfying description of compressible turbulence (and of its generation), responsible for the clumping of the gas in floccules (we try

*Dr J. Silk arrives to a similar fundamental Ansatz studying the fragmentation of a uniform collapsing spheroidal cloud (Silk 1981).

this programme in another paper); (2) one had to schematize quantitatively the interaction between floccules, perhaps holding in consideration the numerical simulations of Stone (1970) and Hausman (1981).

Another particle approach has been utilized by Larson (1978) in a three-dimensional numerical simulation of fragmentation in a collapsing cloud. Larson includes, in addition to gravity, a sort of viscosity, associated with shock fronts, representing the pressure forces acting between neighbouring pairs of particles by a repulsive interaction. Although the techniques in Larson's and our models are very different, we believe that there is a definite parallel: the repulsive interaction, with different behaviour on distance from gravity, compares with the non-linear interaction.

2.2 DILUTE GAS APPROXIMATION

We represent the original cloud as a weakly non-ideal gas: this means that the scattering amplitude a among the gas particles is small compared with the mean distance $1 \sim (V/N)^{1/3}$ between themselves. Moreover, the inequality $ka \ll 1$ ($k = p/\hbar$ are the wavevectors of the particles constituting the cloud) has to be verified. At the very low temperatures of the interstellar medium, the validity of this inequality is manifest (Spitzer 1978). Under these conditions only the binary interactions between particles will be relevant. It is well known that as a consequence of the steep increase of the two particle energy of interaction $U_{12}(\mathbf{r})$ for small $\mathbf{r}$, the perturbative theory cannot be applied.

A possible way to overcome the difficulty is to replace formally the real energy $U_{12}(\mathbf{r})$ by another function which gives the same value of the diffusion amplitude a but allows the application of the perturbative theory (Bogolyubov 1947). Since the final result depends on $U_{12}(\mathbf{r})$ via a diffusion amplitude (at low temperatures) it is evident that the proposed procedure is quite equivalent to the standard one.

Therefore, we figure the system as follows: in each point $\mathbf{x}$, and for each time t, the molecular cloud is represented by a scalar field $\phi(\mathbf{x}, t)$, such that its squared modulus coincides with the density $\rho(\mathbf{x}, t)$ of the cloud in the point $\mathbf{x}$ and at the time t:

$$\rho(\mathbf{x}, t) = |\phi(\mathbf{x}, t)|^2.$$

The system is then described by the renormalized Lagrangian density:

$$L(\phi) = \frac{1}{2} \partial_\nu \phi(\mathbf{x}, t) \partial^\nu \phi(\mathbf{x}, t) - \frac{\mu^2}{2} \phi^2(\mathbf{x}, t) - V(\phi) \tag{2.1}$$

where

$$\partial_\nu = \frac{1}{v_s} \frac{\partial}{\partial t}, -\frac{\partial}{\partial x_i},$$

with v_s the velocity of elastic propagation in the matter (sound velocity). We separate the quadratic term from the potential to preserve analogy with the corresponding Lagrangian of the non-linear Klein–Gordon problem. The mass μ of the density field has the dimension of the reciprocal of a length and determines the scale of linear fluctuations (see Ma 1976). The hypothesis that the non-linear effects are relevant to the description of the model implies that the quadratic term is negligible with respect to $V(\phi)$ and so μ does not play any role in the following. $V(\phi)$ is the non-linear interaction potential which in the perturbative approach *à la* Bogolyubov gives the same result as the effective potential U_{12}. The Lagrangian model (2.1) is equivalent to picture the molecular cloud by a mean field. Furthermore the condition $ka \ll 1$ justifies the adoption of the point-like interaction represented by the local non-linear potential $V(\phi)$

$$V(\phi) = \frac{g}{4}\phi^4(\mathbf{x}, t). \tag{2.2}$$

The choice of a non-linear quartic potential is the most natural to represent dynamics dominated by binary low energy interactions and allows to relate easily the coupling constant g to the potential $U_{12}(\mathbf{r})$ (g is proportional to the corresponding diffusion amplitude). The equation of motion derived from the Lagrangian (2.1) with the potential (2.2) is:

$$\frac{1}{v_s^2}\ddot{\phi}(\mathbf{x}, t) = \nabla^2\phi(\mathbf{x}, t) - \mu^2\phi(\mathbf{x}, t) - g\phi^3(\mathbf{x}, t). \tag{2.3}$$

The initial conditions under which we study the properties of the solutions of equation (2.3) are the most simple ones consistent with the action of trigger mechanisms (ionization, driven shocks, supernovae, cloud collisions, density waves etc.). Indeed without distinguishing the nature of the acting mechanisms we can suppose, as a realistic initial condition, that the trigger interests the modes with larger wavelength.

We remark that to this Lagrangian model one can also give an hydrodynamic interpretation, briefly discussed in Appendix B.

2.3 DISCUSSION OF THE EQUATION (2.3)

To simplify the analytical treatment of equation (2.3) we suppose that $\phi(\mathbf{x}, t)$ is a one-dimensional scalar field. Fucito *et al.* (1982) and Marchesoni & Sparpaglione (1982) studied analytically and numerically the non-equilibrium properties of the one-dimensional non-linear Klein–Gordon equation (2.3), where the field $\phi(x, t)$ is defined in the interval $[-L/2, L/2]$ with periodic boundary conditions.

The Fourier transform of the field is defined as:

$$\phi(k, t) = \frac{1}{(2\pi)^{1/2}}\int_{-L/2}^{L/2} \exp(ikx)\,\phi(x, t)\,dx. \tag{2.4}$$

We shall consider as a typical initial condition:

$$\phi(k, 0) = \begin{cases} A & k \leqslant k_{\mathrm{I}} \\ 0 & k > k_{\mathrm{I}} \end{cases} \tag{2.5}$$

$$\dot{\phi}(k, 0) = 0$$

where k_{I} is the (small) wavenumber below which the energy of the system is initially confined as potential energy. The propagation of the energy from large to short wavelengths is studied in term of the time evolution of the quantity:

$$\rho(k, t) = |\phi(k, t)|^2 \tag{2.6}$$

in the region $k > k_{\mathrm{I}}$.

The main analytic predictions (confirmed by numerical simulations, obtained by discretizing equation 2.3) can be summarized as follows (for a detailed discussion see Appendix A).

(1) The spectrum of the field $\rho(k, t)$ decays exponentially with k for $k > k_{\mathrm{I}}$

$$\rho(k, t) \sim A^2 \exp[-k/k_{\perp}(t)]. \tag{2.7}$$

(2) It is possible to distinguish between two time regimes:

$$\frac{1}{k_\perp(t)} \propto \begin{cases} -\dfrac{2}{k_\mathrm{I}} \ln\left(\dfrac{g^{1/2}A}{\sqrt{2}} v_\mathrm{s} t\right) & \text{'short time'} \\ (g v_\mathrm{s}^2 A^2 \ln t)^{-1/2}. & \text{'intermediate time'} \end{cases} \tag{2.8}$$

(3) The separation between the two regimes is stated by the time-scale $(gA/v_\mathrm{s}^2)^{-1/2}$ which depends on the initial conditions (energy and non-linear coupling constant). They are both non-equilibrium regimes: the system approaches to equilibrium with a logarithmic dependence on t, so that the non-equilibrium spectrum may persist for extremely long times, and may be mistaken as a stationary state if the observation time is not sufficiently long.

(4) The conclusions (2.8) continue to be valid in the limit $L \to \infty$, provided the set of initially excited modes maintains a non-zero measure. The result can be generalized to non-linear potentials with different power behaviour.

2.4 THE MASS SPECTRUM

We have to understand equation (2.3) to be three-dimensional to apply it to the realistic situation. We can expect, with a certain reliance, that in absence of effects due to particular geometries of the system, the results (2.7)–(2.8) continue to be valid: we assume that the generalization of the treatment of the previous paragraph gives the correct dependence of the spectrum on the physical quantities which determine the evolution of the process.

The Lagrangian (2.1) describes the molecular cloud regarding only the local short-range binary interactions; furthermore, the extension of the above stated analytical approach implies $g > 0$ and then a repulsive interaction between the particles at short distance, i.e. the scattering amplitude has to be positive. This agrees with the fact that, however weak the attraction is, a Bose gas can never remain dilute at low temperature.

On the other hand, the long-range forces (gravitational, magnetic, etc.) which do not appear in the model described by equation (2.1) are responsible for the overall collapse of the cloud. The decay under the long-range interactions is labelled by a mean life τ_0 – mean time of star formation (Mestel 1965).

According to Section 2.3, we can fix the initial conditions as follows:

$$|\phi(k,0)|^2 = \begin{cases} \rho_\mathrm{I} & k \leqslant k_\mathrm{I} \\ 0 & k > k_\mathrm{I} \end{cases} \tag{2.9}$$

$$\dot{\phi}(k,0) = 0.$$

The propagation of the perturbation to short wavelengths is likewise given by:

$$\rho(k,t) \sim \rho_\mathrm{I} \exp\left[-k/k_\perp(t)\right] \tag{2.10}$$

with

$$\frac{1}{k_\perp(t)} = -\frac{2\delta}{k_\mathrm{I}} \ln\left(\frac{t}{\tau}\right); \qquad \tau = \left(\frac{2}{g v_\mathrm{s}^2 \rho_\mathrm{I}}\right)^{1/2} \tag{2.11}$$

in the short time regime (δ is the proportionality factor missing in equation 2.8).

In other words, the time required to achieve a density ρ on a certain wavelength k is given by:

$$t = \tau \exp\left[-\frac{1}{2\delta}\frac{k_\mathrm{I}}{k} \ln \frac{\rho_\mathrm{I}}{\rho}\right]. \tag{2.12}$$

The above expression is valid only in the short-time regime (t less than 10τ from computer simulations): if the collapse takes place before intermediate time regime arises, equation (2.12) describes the distribution of mass density on the different scale length at that time.

Therefore, the probability that the system (2.1) with a mean life τ_0, produces a density ρ on the scale k, starting from the initial conditions (2.9) is given by:

$$P(\rho) = 1 - \exp\left\{-\frac{\tau}{\tau_0}\exp\left[-\frac{1}{2\delta}\frac{k_\mathrm{I}}{k}\ln\left(\frac{\rho_\mathrm{I}}{\rho}\right)\right]\right\}. \tag{2.13}$$

The production of the corresponding protostellar object (of mean density ρ and volume $V \sim 1/k^3$) takes place only if the conditions necessary for the condensation of the object subsist locally.

As the simplest choice, in the absence of magnetic field, we assume the validity of the Jeans criterion:

$$m \sim \frac{\rho}{k^3} = \frac{b}{\rho^{1/2}} \qquad \text{with} \qquad b = 1.86\left(\frac{RT}{G}\right)^{3/2} \tag{2.14}$$

where T is the absolute temperature, R is the perfect gas constant and G the gravitational constant.

We substitute (2.14) in (2.13) obtaining the mass spectrum $n(m)$:

$$n(m) = N_0\left\langle 1 - \exp\left\{-F_\mathrm{A}\exp\left[-F_\mathrm{B}m\ln\left(\frac{m\rho_\mathrm{I}^{1/2}}{b}\right)\right]\right\}\right\rangle \tag{2.15}$$

with

$$F_\mathrm{A} = \frac{\tau}{\tau_0} \tag{2.16a}$$

and

$$F_\mathrm{B} = \frac{1}{\delta(V_\mathrm{I}b^2)^{1/3}} = \frac{1}{\delta}\,\frac{1}{2.90\,M_\odot}\,\frac{1}{V_\mathrm{I}^{1/3}\,(\mathrm{pc})\,T\,(\mathrm{K})} \tag{2.16b}$$

where V_I is the volume interested by the initial perturbation, $M_\odot$ is the mass of the Sun. The normalization factor N_0 is fixed by the total mass M of the system and the parameters of equation (2.15)

$$\int n(m)m\,dm = M. \tag{2.17}$$

Our description of the fragmentation follows the classical fragmentation model (Hoyle 1953; Mestel & Spitzer 1956): a blob of density separates out from the medium when its mass exceeds a typical mass fixed by a criterion of instability. The probablistic treatment is in some way justified by an argument of Mestel (1977) for the plausibility of Hoyle's intuitive transition from 'can fragment' to 'will fragment' by the studies on linear and non-linear domains of collapse (Hunter 1962, 1964; Lynden-Bell 1973).

2.5 PARAMETERS IN THE MASS SPECTRUM AS FUNCTIONS OF THE CLOUD PROPERTIES

Equation (2.15), which gives the mass spectrum, presents two parameters F_A and F_B besides the normalization factor. We can display their dependence on the main physical quantities.

We now summarize how these parameters depend on both macroscopic (mean density ρ of the blob, temperature T, magnetic field $\mathbf{B}$, total mass M, etc.) and microscopic quantities (essentially the interaction potential between the particles of the cloud).

F_A is the ratio of the non-equilibrium time-scale τ, strongly characterized by the potential of equation (2.2), to the time of gravitational collapse τ_0. This one is given by (Hunter 1962):

$$\tau_0 = \left(\frac{3\pi}{32\,G\rho\alpha}\right)^{1/2} \tag{2.18a}$$

with

$$\alpha = \left(\frac{3}{4\pi}\right)^{2/3} \frac{1}{G\rho^{2/3}m^{1/2}} \left|\frac{d^2r}{dt^2}\right| \tag{2.18b}$$

where r is the linear dimension of the spherical distribution of mass m. When $\alpha = 1$ we are in the free-fall regime, when $\alpha = 0$ there is equilibrium. We can write a motion equation for the collapse in which we introduce the pressure of the gas and the magnetic field (very roughly as a pressure term); so that we get for α:

$$\alpha = 1 - 3\left(\frac{3}{4\pi}\right)^{1/3} \frac{1}{G\rho^{4/3}m^{2/3}} \left[RT\rho + x\frac{B^2}{4\pi}\right] \tag{2.18c}$$

where x is the exponent in the expression of the magnetic field as function of the gas density (Mouschovias 1976):

$$B \propto \rho^x \qquad \text{with} \qquad 1/3 \lesssim x \lesssim 1/2. \tag{2.19}$$

F_B depends strongly on the expression we adopt for the minimum typical mass which can collapse by itself. In Section 2.4 we presented the simplest case in which the pressure effect dominates. Now we refer to the situation in which the effects of the magnetic field $\mathbf{B}$ prevail (Mouschovias & Spitzer 1976):

$$m = \zeta \frac{B^3}{\rho^2} \tag{2.20}$$

with

$$\zeta = \frac{3.5 \times 10^{-3}}{G^{3/2}}.$$

Consequently the expression for the mass spectrum changes as follows:

$$n(m) = N_0 \left\langle 1 - \exp\left\{-F_A \exp\left[-F'_B m^{1/2} \ln\left(\frac{m}{\zeta}\frac{\rho_I^2}{B^3}\right)\right]\right\}\right\rangle \tag{2.21}$$

where

$$F'_B = \frac{1}{4\delta\zeta^{1/6}} \frac{1}{V_I^{1/3}B^{1/2}} = \frac{1}{6.70\,\delta\,M_\odot^{1/2}} \frac{1}{V_I^{1/3}(\mathrm{pc})\,B^{1/2}(\mu\mathrm{G})}. \tag{2.22}$$

3 Comparison with the observational data

The expression (2.15) of the stellar mass spectrum we are going to compare with the experimental data, was obtained under the following simplifying hypotheses.

(i) We made an assumption on the geometry of the system. Indeed we applied equation (2.8) to the molecular cloud understanding that the effective spatial dimensions are $d = 3$. It means, for example, that we excluded particular configurations of the original cloud (filaments, disc-like features, etc.) but that we also neglected effects due to non-linear coupling of spatial dimensions and to angular momentum.

(ii) We separated the short-range repulsive interaction between particles, substituted in the approximation of weakly non-ideal gas by the non-linear local potential (2.2), from the long-range forces (gravitational, magnetic, etc.) which are responsible for the overall collapse of the system. The effects on the cloud of both these groups of forces were put together in a naïve probabilistic scheme.

(iii) We neglected the mutual interactions which take place between the protostellar objects and the remaining gas of the cloud (gas ionization near the collapsing object, coagulation, accretion, etc.).

(iv) We used in our evaluation of $n(m)$, as the only relevant ingredient, the spectrum of $\rho(x)$; this corresponds to the hypothesis of mean field. Presumably effects of intermittency are not negligible in the mechanism of star formation; they can only justify the non-uniform spatial distribution of stars (Mandelbrot 1977).

Nevertheless we believe that the comparison of the experimental data with the predictions for the initial stellar mass spectrum (2.15) and (2.21), allows us to stress the non-linear distinctive features – that we regard as controlling the fragmentation process – and to show (at least qualitatively) the dependence on the properties of the original cloud.

First we consider the situation in which the effects due to the presence of the magnetic field are negligible: $n(m)$ is given by (2.15). To be uniform with other works, in the figures we plot the IMF $\xi(m)$ instead of the mass spectrum $n(m)$:

$$\xi(m) = m\frac{n(m)}{0.434}.$$

We fit our expression for the IMF with the universal or mean mass function for the open clusters of Taff (1974) and van den Bergh (1961) with arbitrary normalization and we get a good agreement with $F_A = 0.1$ and $F_B = 0.34\,M_\odot^{-1}$ (Fig. 1).

According to the data for the single open clusters (see references quoted in Scalo 1978) the proposed spectrum flattens and changes sign of the slope for masses smaller than $1\,M_\odot$. On the contrary it does not show the flattening of Taff (1974) plots for masses larger than $10\,M_\odot$ (where the uncertainties are large because of the scarcity of high mass stars).

For the range of masses in which our IMF reproduces the more reliable data (between 1 and $10\,M_\odot$), the mean value of the spectral index $\gamma(m)$, defined as the logarithmic derivative of $n(m)$ with respect to m, is $\langle\gamma\rangle \lesssim -3.0$. If we apply the definition of $\gamma(m)$ to the mass spectrum (2.15), we get γ proportional to F_B. With (2.16b) we conclude in agreement with Burki (1977), that the spectral index is generally smaller for large clusters.

In Fig. 1 we adopt a similar procedure for the field stars (observed IMF from Scalo 1978; Miller & Scalo 1979; Lequeux 1979). In this case we fit the low mass end of the spectrum quite well while in the high mass end the obtained plot is below the observations.

A leading role in the interpretation of such disagreement is played by the above simplifying hypothesis (iii): Bastien (1981) shows that for the field stars (which presumably have not the same formation time) the effects of the fragment collisions, as well as the accretion from the remaining gas, contribute in a conclusive way to increase the production of protostellar objects of large mass.

Using the expressions (2.16a)–(2.16b) for F_A and F_B, it is possible to study the dependence of $n(m)$ on the temperature. It is necessary then to fix a normalization

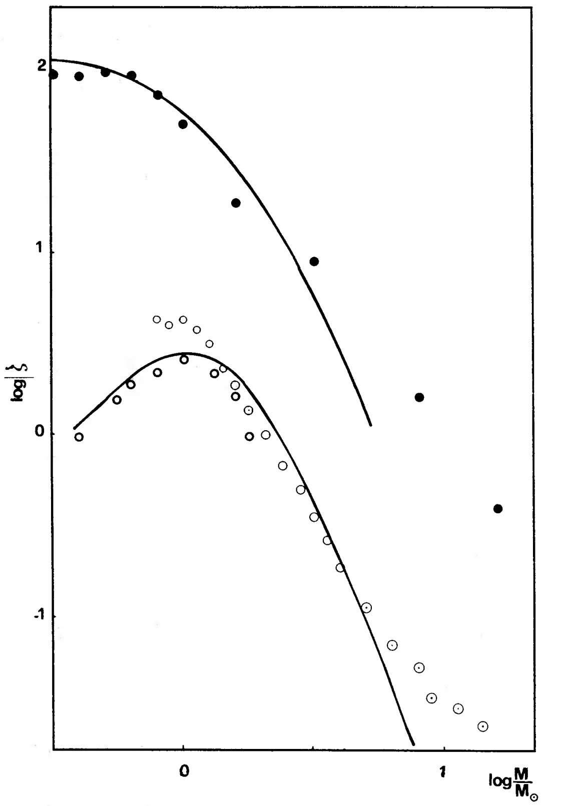

Figure 1. Fits of the initial mass function of field stars from Miller & Scalo (1978) (•) and of the mean mass functions in the open clusters of van den Bergh (1961) (○) and Taff (1974) (⊙). Normalization is arbitrary. The upper curve is obtained putting $F_A = 0.1, F_B = 0.70\,M_\odot^{-1}$ in equation (2.15), in the second one $F_A = 0.1, F_B = 0.34\,M_\odot^{-1}$.

condition to get consistency between the plots at different temperatures. We choose as reference the plot in Fig. 1 for the open clusters: the normalization condition is given by equation (2.17); the integral is computed between the minimum mass $10^{-2}\,M_\odot$ and the maximum mass $\simeq 100\,M_\odot$.

Assuming that the plot in Fig. 1 corresponds to open clusters whose progenitor molecular cloud had temperature T_0, Fig. 2 represents the altered spectrum if the temperature of the cloud was $T_0/2$ or $2\,T_0$, the other parameters being unchanged.

In the hypotheses of dominance of the magnetic field, we got for $n(m)$ the expression (2.21). Equation (2.21) fits in an even more satisfactory way both the open clusters and field stars data. We plot in Fig. 3 the best fits of (2.21) in both cases with arbitrary normalization. the corresponding values for F_A and F_B' are given in the figure caption. The increased good quality of fits in comparison with Fig. 1 seems to give evidence to the assertion of Mouschovias (1978) 'the star formation process may be understood as a purely magnetic phenomenon'.

The consequent values of the spectral index for the field stars are: $\langle\gamma\rangle = +0.5$ for $m \lesssim 1\,M_\odot$, $\langle\gamma\rangle = -2.5$ for $1\,M_\odot < m < 10\,M_\odot$, $\langle\gamma\rangle = -4.0$ for $m > 10\,M_\odot$ to be compared with the values determined from the analysis of observations by Miller & Scalo (1979) which are correspondingly $-1.4, -2.5, -3.3$ and by Lequeux (1979), $\langle\gamma\rangle = -3.0$ for $m > 2\,M_\odot$.

From equation (2.22) it is possible to show the dependence on the magnetic field of $n(m)$ in our model. According to the procedure adopted for the temperature, we call B_0 the value of the magnetic field which corresponds to the open clusters diagrams in Fig. 3, and plot in Fig. 4 the mass function with $B_0/2$ and $2B_0$.

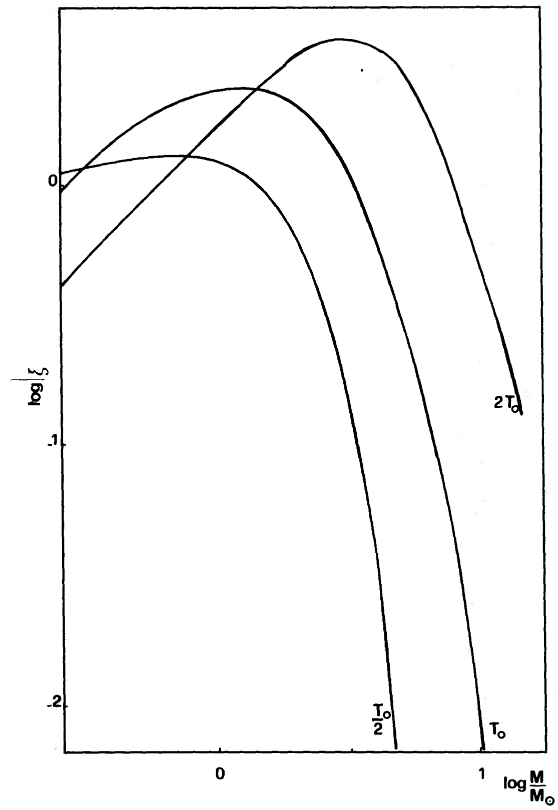

Figure 2. $\xi(m)$ dependence on the temperature T as predicted from equation (2.15). The T_0 labelled curve is the best fit for the open clusters data given in Fig. 1 and T_0 is the corresponding absolute temperature.

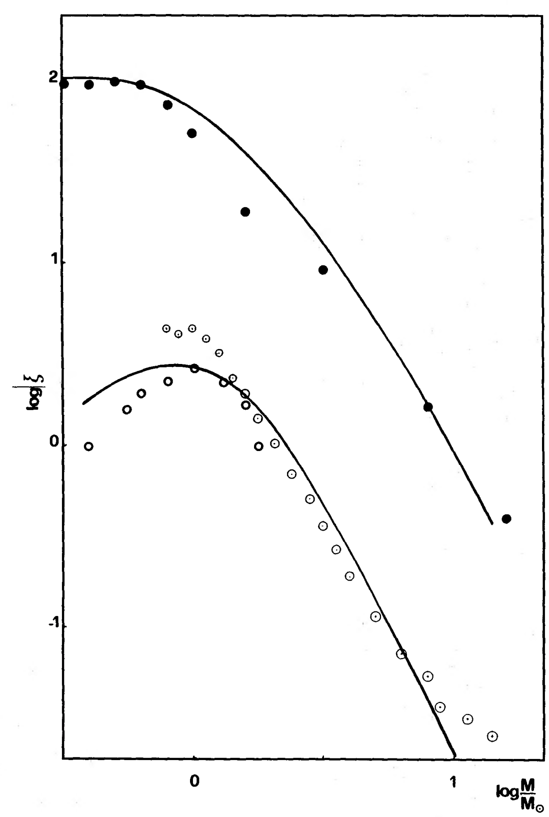

Figure 3. As in Fig. 1 for the case of magnetic field dominance. The field stars data are fitted putting $F_A = 0.05, F_B = 3.0 M_\odot^{-1}$ in equation (2.21), the open clusters ones with $F_A = 0.1, F_B = 1.5 M_\odot^{-1}$.

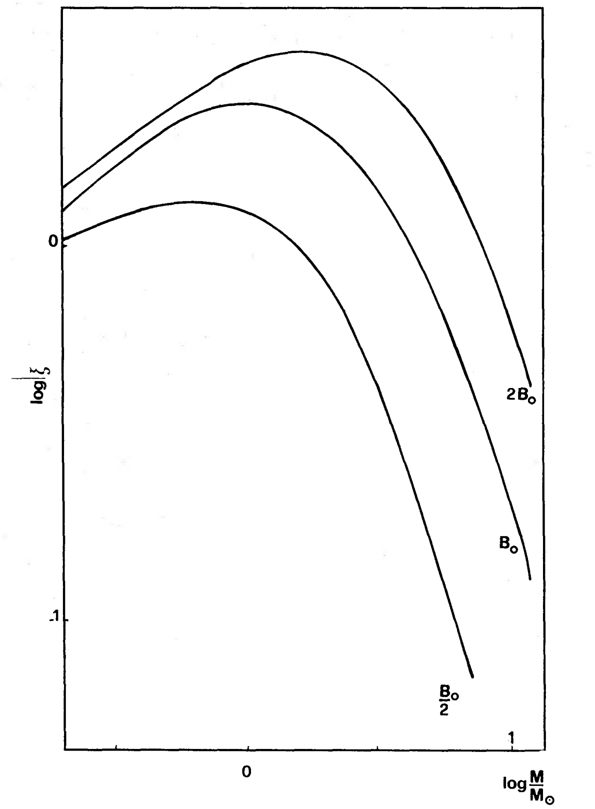

Figure 4. $\xi(m)$ dependence on the magnetic field B as predicted from equation (2.21). The B_0 labelled curve is the best fit for the open clusters data given in Fig. 3 and B_0 is the corresponding magnetic field.

Finally we notice that while the phenomenon of intermittency is very relevant in the understanding of cluster structure, probably it is not so important in the determination of the stellar mass spectrum. A very similar situation is found in the theory of turbulence (Monin & Yaglom 1975).

4 The physical content of the parameters F_A and F_B

4.1 F_A

We use the results of previous paragraphs to determine the dependence of F_A on the physical quantities connected with the properties of the cloud. We put together equations (2.16) and (2.18) and get easily, but for a numerical factor 0(1) (with the simplest choice of $\alpha = 1$):

$$F_A = \frac{\tau}{\tau_0} \sim \left(\frac{G\rho_0}{g v_s^2 \rho_I}\right)^{1/2} \tag{4.1}$$

where ρ_0 is the mean density of the unperturbed cloud.

To proceed further we need to connect the non-linear coupling constant g of the mean field picture (2.2) with the interaction potential between the particles in the low energy approximation of Section 2.2. It is a standard procedure. Once chosen, the shape of the intermolecular potential among the various conventional accepted ones (for instance the Morse potential), we can fix its parameters using the experimental data (for clouds of H_2, H and He, see Harrison 1962; Moore, Datz & Van der Valk 1966). In the low energy regime, the procedure is further simplified by the possibility of applying the Born approximation.

It is easy to verify that the potential $V(\phi)$ in the non-relativistic approximation, is equivalent to a point-like intermolecular potential. The request that the respective scattering amplitudes be equal (Bogolyubov method) yields the following estimate for g: $g \sim 10^{-2} \div 10^{-3}\, G/v_s^2$. It follows that

$$F_A \simeq c\left(\frac{\rho_0}{\rho_I}\right)^{1/2} \qquad \text{with} \qquad c \simeq 10 \div 30. \tag{4.2}$$

The values of F_A giving the best fits of our model are $F_A \simeq 1.0 \div 0.5$, so we can conclude that $\rho_I \simeq 10^3 \div 10^5\, \rho_0$. This result is in good agreement with the detailed studies on the action of particular triggers on the cloud. As an example, this conclusion on the enhancement of density due to the external perturbation is consistent with the theoretical calculations of Elmegreen & Lada (1977) and the numerical simulations of Woodward (1978).

Finally we note that the values of F_A resulting from the comparison with the observational data and the direct evaluations starting from reasonable values of the physical quantities, confirm the hypothesis done in Section 2 that the mean time of star formation τ_0 and the characteristic time of the short time regime, as defined in Section 2.3, are of the same order.

4.2 F_B

Let us introduce in equation (2.16b) $\delta = 1/3$ (as estimated numerically) and the typical value of the temperature in a molecular cloud $T = 10$ K (Spitzer 1978). We obtain F_B as function only of the dimension of the region initially interested by the trigger. If we refer to the value of F_B which gives the best fits (e.g. $F_B = 0.34\, M_\odot^{-1}$) we have $V_I^{1/3} \simeq 0.3$ pc.

The parameter F_B is determined by the typical masses in the system; indeed the expression (2.16b) can be transformed easily in:

$$F_B = \frac{3}{\mathcal{M}^{1/3} m_{JI}^{2/3}} \tag{4.3}$$

where

$$\mathcal{M} = \rho_I V_I$$

and

$$m_{JI} = \frac{b}{\rho_I^{1/2}}.$$

We can essentially say: there was an external perturbation which compressed in a volume V_I at a density ρ_I, a mass $\mathcal{M}$. The smallest Jeans mass in this region is m_{JI}. If for instance we refer to the sequence of formation of OB associations, in the model developed by Elmegreen & Lada (1977), we can exploit their evaluation of the mass involved by the trigger: $\mathcal{M} \simeq 5 \times 10^3 M_\odot$. The number density of the molecular clouds is typically $n \simeq 10^2 \div 10^3\,\text{cm}^{-3}$ (Spitzer 1978), or using the conclusions of the previous section about ρ_I and the estimate of V_I, we obtain the same mass $\mathcal{M}$.

To fix the order of magnitude of m_{JI}, we must look to dynamical and thermic processes which determine the minimum mass that can fragment out. Various theoretical calculations (Mestel & Spitzer 1956; Low & Lynden-Bell 1976; Silk 1977, 1978) give values for the minimum mass between 0.007 and $0.2\, M_\odot$, at temperature $T = 10$ K and density

$n = 10^{10}\,\mathrm{cm}^{-3}$. In our situation the density is not so high; from the conditions on F_A, it comes out that m_{JI} has to be a little larger than the former values. In particular, if $\mathcal{M} = 5 \times 10^3 M_\odot$ and $F_B = 0.34 M_\odot^{-1}$, we get $m_{JI} \simeq 0.3 M_\odot$. The expressions (2.15) and (2.21) for the mass spectrum are very sensitive to the relation between the mass that can fragment and the density of the gas from which the mass enucleates.

We saw in Figs 1 and 3 that different choices of the Jeans criterion and of the magnetic field dominated fragmentation give different plots of $\xi(m)$. Therefore we relaxed the assumption of the Jeans criterion and we concluded that by increasing the negative exponent $m \propto \rho^{-\epsilon}$, we gain in the fitting results. This means that particular care has to be taken in the hypothesis about the 'true' dimensionality of the cloud, as we already stressed in Section 2.4.

All the remarks we have presented cannot have a predictive meaning because our model is extremely simplified; the previous discussion has only the purpose to provide further elements of plausibility to the model itself. We have shown how the fundamental physical quantities come out with the right order of magnitude.

5 Conclusions

The principal aim of this paper has been to put in evidence the non-linear, non-equilibrium characteristics ruling the process of fragmentation.

To this purpose we described the molecular cloud by a mean field with the only dynamics of intermolecular interactions, neglecting the presence of more involved mechanisms of interactions among the components of the evolving cloud. We showed that the consequent transfer of trigger perturbation from long to short length scales is nevertheless reliable. In particular this yields the prediction of a strong dependence of the stellar mass spectrum both in the initial physical (micro- and macroscopic) characteristics of the original molecular cloud, and on the interactions of the cloud itself with external actions. The results make us guess a rich taxonomy of situations of star formation, contrary to the hypothesis of universality advanced in the seventies.

In the future we shall address our efforts to improve the proposed model to make it more realistic; in particular we will discuss with deeper insight the contents of points (i)–(iv) of Section (3.1). The extension of the peculiar ideas which inspired this model to situations where external triggers are absent, will equally be studied.

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Appendix A

We present the solution of equation (2.3) for the spectrum $\rho(k,t) = |\phi(k,t)|^2$. It is known that since $\phi(x,0)$ is analytic as a function of x, the solution $\phi(x,t)$ will remain analytic at any finite time (Brezis 1981). This means that as time goes on, singularities (simple poles) of $\phi(x,t)$ appear in the complex x-plane and creep towards the real axis, accumulating on it at infinitely long times. We may relate these singularities to the large k behaviour of ρ by means of the theorem of residues. Let us consider the integral defining $\phi(k,t)$ (equation 2.4 in the complex x-plane). At positive k we close the contour in the lower complex half plane. We obtain the following expression for $\phi(k,t)$:

$$\phi(k,t) = (2\pi)^{1/2} i \sum_{\mathrm{j}} R_j \exp(ikx_j - |k \cdot y_j|) \tag{A1}$$

where the sum runs over all poles located in the relevant half plane, R_{j} being their residue and $x_{\mathrm{j}} + iy_{\mathrm{j}}$ their location. We therefore obtain the following asymptotic behaviour of $\rho(k,t)$ at large k:

$$\rho(k,t) \propto \exp[-2|ky_{\mathrm{s}}(t)|] \tag{A2}$$

where $y_{\mathrm{s}}(t)$ is the imaginary part of the location of the pole which lies nearest to the real axis. To evaluate $y_{\mathrm{s}}(t)$, extending the approach of Frisch & Morf (1981), we consider the solution $\phi(x+iy=z,t)$ of the complex partial differential equation:

$$\frac{1}{v_{\mathrm{s}}^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial z^2} - \mu^2\phi - g\phi^3. \tag{A3}$$

We first consider the case of the ordinary differential equation obtained from equation (A3) by neglecting the 'Laplacian' term $\partial^2\phi/\partial z^2$, with initial complex condition ϕ_0. We may estimate the location of the poles by computing the time needed to reach infinity starting from a given point on the imaginary axis. Since the motion is one-dimensional (all along the imaginary axis), we may introduce a potential:

$$U(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{g}{4}\phi^4. \tag{A4}$$

The time needed to reach infinity starting from the point $iIm\,\phi_0$ may be easily computed from the conservation of 'energy' in the one-dimensional motion:

$$v_{\mathrm{s}}t = \int_{Im\phi_0}^{\infty} \frac{d\phi}{\sqrt{2U(Im\,\phi_0) - 2U(\phi)}}. \tag{A5}$$

At large $Im\,\phi_0$ the time t behaves as:

$$v_s t = \left(\frac{2}{g}\right)^{1/2} \frac{1}{Im\,\phi_0}. \tag{A6}$$

From equation (A6) we obtain the location of the pole at large $Im\,\phi_0$, that is at short t:

$$Im\,\phi_0 \sim \left(\frac{2}{g}\right)^{1/2} \frac{1}{v_s t}. \tag{A7}$$

A typical initial condition:

$$\phi(x, 0) = A \cos k_0 x \tag{A8}$$

yields in the complex plane:

$$\phi(x + iy) = A\left[\cos(k_0 x)\cosh(k_0 y) - i \sin(k_0 x)\sinh(k_0 y)\right]. \tag{A9}$$

We have at large $|y|$:

$$Im\,\phi_0 \sim A \exp(k_0 |y|). \tag{A10}$$

According to our hypothesis, the nearest singularity of $\phi_0(z, t)$ to the real axis will be located where $Im\,\phi_0$ as estimated from equation (A10), will be of the order of that given by equation (A7), hence:

$$|y_s(t)| \propto -\frac{1}{k_0} \ln(v_s t A g^{1/2}). \tag{A11}$$

We now comment on the effect of the previously neglected Laplacian term. We claim that the analysis we just made retains its validity at very short times. The Laplacian term will not in fact be able to prevent $\phi(z, t)$ from escaping to infinity if its initial imaginary part is large enough.

The Laplacian term will on the contrary be essential in introducing singularities in the part of the complex x-plane which corresponds to small initial values of $Im\,\phi$. We do not present the solution of equation (2.3) for the intermediate time regime, because it is of no interest for the results of this paper.

Appendix B

If we change variable in equation (2.3) from $\phi(x, t)$ to $\rho(x, t)$, in the one-dimensional case, for simplicity, we get:

$$\frac{1}{v_s^2}\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho}{\partial x^2} - 2\mu^2 \rho - 2g\rho^3 + \frac{1}{2\rho}\left[\frac{1}{v_s^2}\left(\frac{\partial \rho}{\partial t}\right)^2 - \left(\frac{\partial \rho}{\partial x}\right)^2\right]. \tag{B1}$$

The last term goes to zero if the equation of continuity is satisfied, and this is true in our model, because the Lagrangian density (2.1) does not have sources and sinks (it is conservative). So the above equation reduces to the equation describing the sound propagation in a non-linear medium.