CECS 456: Machine Learning (Spring 2020)

Homework #1
Due 02/17/2020

Note:
(1)You need to submit first five questions in hard-copy (print out or by-hand) before lecture.
(2) For the last question, you need to submit a report and your code to BeachBoard. Unlimited number of submissions are allowed on Beachboard and the latest one will be graded.

1. (10 points) $a = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, compute $a^T b$, $b^T a$, ab^T, and ba^T.

2. (10 points) Given $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, show that the rank of matrix xy^T is one.

3. (15 points) Given $X = [x_1, x_2, ..., x_n] \in \mathbb{R}^{m \times n}$ where $x_i \in \mathbb{R}^m$ for all i, and $Y^T = [y^1, y^2, ..., y^n] \in \mathbb{R}^{p \times n}$ where $y^i \in \mathbb{R}^p$ for all i. Show that

$$XY = \sum_{i=1}^{n} x_i (y^i)^T. \tag{1}$$

4. (5 points) Given $g(x, y) = e^x + e^{y^2} + e^{3xy}$, compute$\frac{\partial g}{\partial y}$.

5. (20 points) Consider the matrix A

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{pmatrix}$$

   (a) Compute the eigenvalues and corresponding eigenvectors of A. You are allowed to use Matlab or Python to compute the eigenvectors (but not the eigenvalues).

   (b) What is the eigen-decomposition of A?

   (c) What is the rank of A?

   (d) Is A positive definite?

   (e) Is A positive semi-definite?

   (f) Is A singular?

6. (40 points) **Perceptron for Handwritten Digits Recognition:** The handwritten digits files are in the "data" folder: train.txt and test.txt. The starting code is in the "code" folder. In the data file, each row is a data example. The first entry is the digit label ("1" or "5"), and the next 256 are grayscale values between -1 and 1. The 256 pixels correspond to a 16×16 image. You are expected to implement your solution based on the given codes. The only file you need to modify is the "solution.py" file. You can test your solution by running "main.py" file. Note that code is provided to compute a two-dimensional feature (symmetry and average intensity) from each digit image; that is, each digit image is represented by a two-dimensional vector before being augmented with a "1" to form a three-dimensional vector as discussed in class. These features along with the corresponding labels should serve as inputs to your Perceptron algorithm.

(a) (5 points)Familiarize yourself with the data by completing the *show_images()* function. Include the images you plotted into your report.

(b) (15 points)In the assignment, we already extracted two features, (symmetry and average intensity), to distinguish between 1 and 5. Familiarize yourself with the features by completing the *show_features()* function and include the 2-D scatter plot into your report. For each sample, plot the two features with a red $*$ if the label is 1 and a blue $+$ if the label is 5.

(c) (15 points)Complete the *perceptron()* function. You can test your accuracy results using the "test_accuracy()" function in "main.py".

(d) (5 points)Complete the *show_result()* function to plot the test data with the separators. Include the images you plotted into your report.

Deliverable: You should submit a report that summarizes your results and the "solution.py" file to the BeachBoard.

Note: Please read the "Readme.txt" carefully before you start this assignment. Please do NOT change anything in the "main.py" and "helper.py" files when you program.