Stats 413 Assignment 4.

5.6. $v_{ji} = \begin{cases} 1 & i=j \\ -1 & i=d \\ 0 & \text{otherwise} \end{cases}$ $E(Y|V_1, \dots V_{d-1}) = \eta_0 + \eta_1 V_1 + \dots + \eta_{d-1} V_{d-1}$

Pf. $E(Y|V_1, \dots, V_{d-1}) = \eta_0 + \eta_1 V_1 + \dots + \eta_{d-1} V_{d-1}$

$= \eta_0 + \eta_1 (1) + \dots + \eta_{d-1} V_0$

$= \eta_0 + \eta_1 + \dots + \eta_{d-1} V_0$

Since we need to prove that $\alpha_j = \begin{cases} \eta_j & j \neq d \\ -(\eta_1 + \eta_2 + \dots + \eta_{d-1}) & j = d \end{cases}$, we need to prove that level mean $j-1$.

We know that $\sum \alpha_j = 0$. Hence we can simple assume

$\alpha_j = \begin{cases} \eta_j & j = d \\ \eta_1 + \eta_2 + \dots + \eta_{d-1} & j \neq d \end{cases}$

Hence the overall mean is given by

$E(Y|V_1 \dots V_{d-1}) = \eta_1 + \eta_2 + \dots + \eta_{d-1} = 1 + 1 + \dots + n-1 = 2 + \dots + 0 = 2$

Hence the level mean $n_1 = j \Rightarrow j-1$ ; Q.E.D

5.7

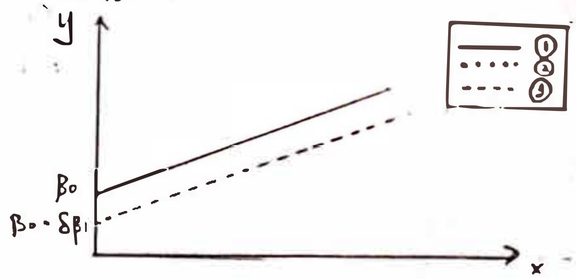
① $E(Y|X=x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$. (Slope $\beta_1$ ; intercept $\beta_0 + \beta_2 x_2 + \beta_3 x_3$)

② $E(Y|X=x) = \beta_0 + \beta_1 x_1 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3$ (Slope $(\beta_1 + \beta_{12} x_2 + \beta_{13} x_3)$ intercept $\beta_0$)

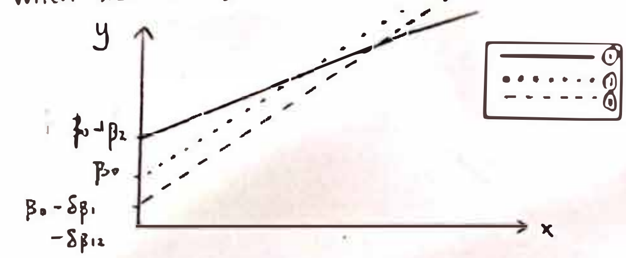
③ $E(Y|X=x) = \beta_0 + \beta_1 (x_1 - \delta) + \beta_{12} (x_1 - \delta) x_2 + \beta_{13} (x_1 - \delta) x_3$ (Slope $(\beta_1 + \beta_{12} x_2 + \beta_{13} x_3)$ intercept $\beta_0 - \delta(\beta_1 + \beta_{12} x_2 + \beta_{13} x_3$

Then the plot of $Y$ against $X$ can be drawn by; we always assume $\beta_0, \beta_1, \beta_2, \beta_3, \beta_{12}, \beta_{13}, \delta > 0$

When $x_2, x_3 = 0$.




When $x_2 = 1$, $x_3 = 0$




When $x_2 = 0$, $x_3 = 1$