

Stat415 Assignment 7.

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1.  $u_1 = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$ ;  $u_2 = \begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}$

(a) Assume the original covariance matrix is  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Hence  $\begin{pmatrix} a-4 & b \\ c & d-4 \end{pmatrix} \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} = 0$

Hence  $\begin{cases} 0.6(a-4) + 0.8b = 0 \\ -0.8(a-1) + 0.6b = 0 \end{cases}$

$\Rightarrow a = 2.08; b = 1.44$

$\begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix} \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} = 0$

$\begin{cases} 0.6c + 0.8(d-4) = 0 \\ -0.8c + 0.6(d-1) = 0 \end{cases}$

$\Rightarrow c = 1.44; d = 2.92$

Hence  $\begin{pmatrix} 2.08 & 1.44 \\ 1.44 & 2.92 \end{pmatrix}$

is the original covariance matrix

(b)  $\frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{4}{4+1} = 0.8$ ; Hence 80% of the variance can be explained by  $\gamma_1$ .

(c)  $X = (1, 2)$

$\gamma_1 = 0.6 + 0.8 \times 2 = 2.2$

$\gamma_2 = -0.8 + 0.6 \times 2 = 0.4$

Hence the first and second principle components are  $\begin{cases} \gamma_1 = 2.2 \\ \gamma_2 = 0.4 \end{cases}$

