UNIVERSITY OF MICHIGAN
DATA MINING
STATS415

ASSIGNMENT7

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Stat415 Assignment 7.

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1. $u_1 = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$; $u_2 = \begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}$

(a) Assume the original covariance matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Hence $\begin{pmatrix} a-4 & b \\ c & d-4 \end{pmatrix}\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} = 0$ $\Rightarrow$ Hence $\begin{cases} 0.6(a-4) + 0.8b = 0 \\ -0.8(a-1) + 0.6b = 0 \end{cases} \Rightarrow a = 2.08;\ b = 1.44$

$\begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix}\begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} = 0$ $\begin{cases} 0.6c + 0.8(d-4) < 0 \\ -0.8c + 0.6(d-1) = 0 \end{cases} \Rightarrow c = 1.44;\ d = 2.92$

Hence $\begin{pmatrix} 2.08 & 1.44 \\ 1.44 & 2.92 \end{pmatrix}$ is the original coverience matrix

(b) $\frac{\lambda_1}{\lambda_1+\lambda_2} = \frac{4}{4+1} = 0.8$; Hence 80% of the variance can be explained by γ_1.

(c)

$\gamma = (1, 2)$

$\gamma_1 = 0.6 + 0.8 \times 2 = 2.2$

$\gamma_2 = -0.8 + 0.6 \times 2 = 0.4$

Hence the first and second principle components are $\begin{cases} \gamma_1 = 2.2 \\ \gamma_2 = 0.4 \end{cases}$

Q2.

```
library(ISLR)
data("College")
College$Accept.Apps<-College$Accept/College$Apps
College<-College[,c(-2,-3)]
#Split the dataset
set.seed(234)
inTrain <- createDataPartition(College$Accept.Apps, p = 0.7, list = FALSE)
training <- College[inTrain,]
testing <- College[-inTrain,]
```

(a)

```
X <- model.matrix(Accept.Apps ~ ., data = College)[, -1]
# Run PCA
collegePCA <- prcomp(x = X, center = T, scale = T)
```

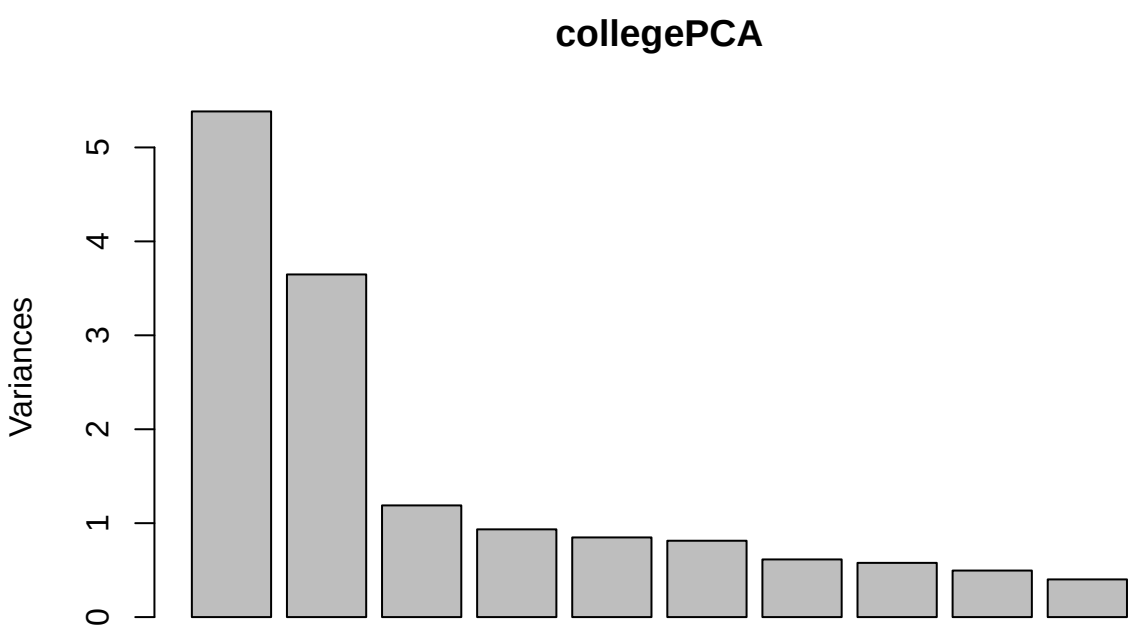
We Choose to standardize our data, Since the scales of different variables vary a lot.

```
collegePCA
```

```
## Standard deviations (1, .., p=16):
##  [1] 2.3202063 1.9099163 1.0903953 0.9665203 0.9209106 0.9014974 0.7834930
##  [8] 0.7596985 0.7038782 0.6333919 0.5883741 0.5535871 0.4293728 0.3795948
## [15] 0.2925402 0.1750902
##
## Rotation (n x k) = (16 x 16):
##                      PC1          PC2           PC3         PC4         PC5
## PrivateYes  -0.20681302  0.356455041 -0.1591399019  0.03669239 -0.03923585
## Enroll       0.03464939 -0.462454913  0.0246197785 -0.07571059 -0.06136725
## Top10perc   -0.34312918 -0.157492803  0.0002373487 -0.38473317 -0.05895401
## Top25perc   -0.31666718 -0.190913384  0.0644941854 -0.40669327  0.02206220
## F.Undergrad  0.06037906 -0.473283545  0.0155847650 -0.05457372 -0.06157156
## P.Undergrad  0.12071330 -0.332095123 -0.1454697842  0.32845546 -0.20131954
## Outstate    -0.37484355  0.073682797 -0.0709249583  0.19779008 -0.02606275
## Room.Board  -0.28237524 -0.002834249 -0.1876472860  0.51936358  0.17450305
## Books       -0.03323706 -0.098129260 -0.6600537366 -0.16214229  0.65009874
## Personal     0.13546497 -0.203608582 -0.4614837522 -0.18011244 -0.33460409
## PhD         -0.24369433 -0.300157933  0.2191702188  0.21858223  0.09364174
## Terminal    -0.24860292 -0.287163985  0.1569234220  0.27737483  0.16018974
## S.F.Ratio    0.26977520 -0.134839801  0.2949613466 -0.05055637  0.46993927
## perc.alumni -0.29132731  0.102080574  0.1587564355 -0.21746668 -0.05517069
## Expend      -0.33637777 -0.072088937 -0.2167806971  0.05856710 -0.28415926
## Grad.Rate   -0.29682521  0.025669164  0.1666259249 -0.15184996  0.20403454
##                     PC6         PC7          PC8          PC9         PC10
## PrivateYes   0.12977308 -0.16685240  0.046025151 -0.210730381  0.11463677
## Enroll       0.37968193  0.04806950  0.009264097  0.270836882  0.18228282
## Top10perc   -0.03828945  0.14658931 -0.207437186 -0.308024087  0.02285775
## Top25perc   -0.05309190  0.08900245 -0.193962229 -0.403774469 -0.03674098
## F.Undergrad  0.34030917  0.04446552  0.017143427  0.199925132  0.15102496
## P.Undergrad  0.18895873 -0.02573806  0.404669987 -0.605723298 -0.34496102
## Outstate     0.12389422 -0.08847727  0.001696581 -0.002664277  0.19633570
## Room.Board   0.25514858 -0.15582031 -0.312821881 -0.109745295  0.25526994
## Books       -0.05613622  0.16618403  0.234332295  0.080892272 -0.07551044
```

```
## Personal    -0.30706177 -0.67260614 -0.131755103  0.047793743  0.11121565
## PhD         -0.42798888 -0.07499474  0.027832793  0.073616941 -0.09598987
## Terminal    -0.42734017 -0.04113989  0.105461368  0.125249977 -0.05160145
## S.F.Ratio    0.02766480 -0.27618136 -0.111151688 -0.329572850  0.42582930
## perc.alumni  0.03380466 -0.19571483  0.741129392  0.042582398  0.36289505
## Expend      -0.00163687  0.27121084 -0.075119992  0.143535476  0.13524297
## Grad.Rate    0.37095330 -0.47981591 -0.061594994  0.210800288 -0.58735027
##                     PC11        PC12        PC13         PC14         PC15
## PrivateYes  -0.692632136  0.25873200 -0.38698443 -0.023188042  0.014246848
## Enroll      -0.249028057  0.00355242 -0.03198605  0.028758893 -0.020862258
## Top10perc    0.046221362 -0.01736028 -0.04481966  0.104186948 -0.727257149
## Top25perc   -0.088902323 -0.23347958  0.08855632 -0.145456913  0.622734534
## F.Undergrad -0.211799028 -0.02657383 -0.03857620 -0.012894604  0.021249495
## P.Undergrad  0.079513281  0.10106581  0.01339449 -0.005876036 -0.027600024
## Outstate    -0.100271062  0.26705152  0.81437712  0.045249482 -0.014967200
## Room.Board   0.214915208 -0.48079461 -0.20189361  0.045088043 -0.012466482
## Books       -0.008701055  0.03588007  0.02435974  0.068552934  0.009707638
## Personal     0.041863946 -0.02325922  0.03523255 -0.021561698  0.001521452
## PhD         -0.145412245  0.08781640 -0.11929824  0.696090060  0.118155335
## Terminal    -0.173066481 -0.01072878 -0.04630350 -0.676592742 -0.156069951
## S.F.Ratio    0.182527711  0.42380583 -0.06167845 -0.047221417  0.016532343
## perc.alumni  0.212784181 -0.22561467 -0.09581960  0.038676644  0.006200652
## Expend       0.427127703  0.54926490 -0.31140597 -0.094508901  0.205746389
## Grad.Rate    0.163783749  0.15155896 -0.07456633 -0.046905892 -0.003064506
##                     PC16
## PrivateYes   0.0207312563
## Enroll      -0.6763011086
## Top10perc    0.0193041993
## Top25perc   -0.0356159248
## F.Undergrad  0.7326758367
## P.Undergrad -0.0450587928
## Outstate     0.0253870978
## Room.Board  -0.0104115884
## Books        0.0009363273
## Personal    -0.0111575157
## PhD          0.0117603412
## Terminal    -0.0194870584
## S.F.Ratio   -0.0076363267
## perc.alumni  0.0069337649
## Expend       0.0046295077
## Grad.Rate    0.0133250183
```

```r
#Scree Plot
plot(collegePCA)
```



We need 14 eigenvalues for us to explain 95% of the variances in the data.

```
#loadings fo the first and second PCs
collegePCA$rotation[,1]
```

```
##  PrivateYes      Enroll   Top10perc   Top25perc F.Undergrad P.Undergrad
## -0.20681302  0.03464939 -0.34312918 -0.31666718  0.06037906  0.12071330
##    Outstate  Room.Board       Books    Personal         PhD    Terminal
## -0.37484355 -0.28237524 -0.03323706  0.13546497 -0.24369433 -0.24860292
##   S.F.Ratio perc.alumni      Expend   Grad.Rate
##  0.26977520 -0.29132731 -0.33637777 -0.29682521
```

```
collegePCA$rotation[,2]
```

```
##   PrivateYes       Enroll    Top10perc    Top25perc  F.Undergrad  P.Undergrad
##  0.356455041 -0.462454913 -0.157492803 -0.190913384 -0.473283545 -0.332095123
##     Outstate   Room.Board        Books     Personal          PhD     Terminal
##  0.073682797 -0.002834249 -0.098129260 -0.203608582 -0.300157933 -0.287163985
##    S.F.Ratio  perc.alumni       Expend    Grad.Rate
## -0.134839801  0.102080574 -0.072088937  0.025669164
```

The first component is proportional to each variable, and the second component measures the difference between the first pair of variables and the second pair of variables.

(b)

```
library(pls)
```

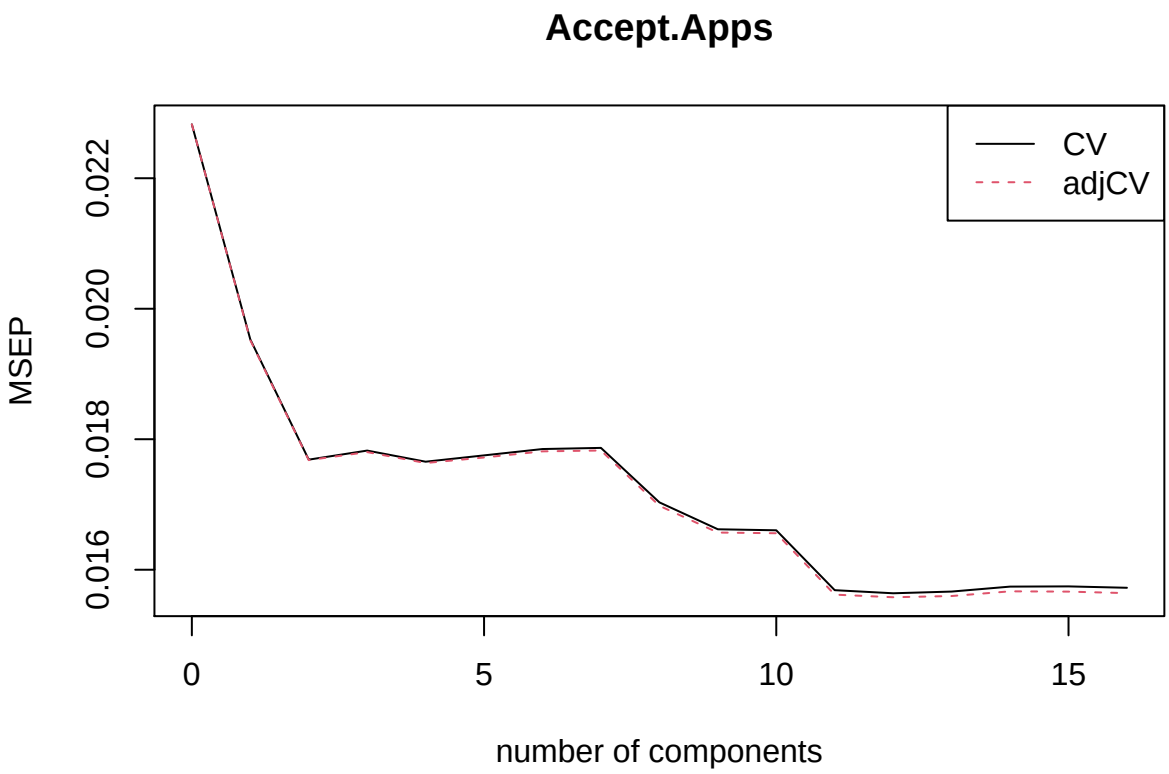
```
## Warning: package 'pls' was built under R version 4.0.3
```

```
##
## Attaching package: 'pls'

## The following object is masked from 'package:caret':
##
##     R2

## The following object is masked from 'package:corrplot':
##
##     corrplot

## The following object is masked from 'package:stats':
##
##     loadings
```

```
collegePCR <- pcr(Accept.Apps ~ ., data = training, scale = TRUE, validation = "CV")
summary(collegePCR)
```

```
## Data:    X dimension: 545 16
##  Y dimension: 545 1
## Fit method: svdpc
## Number of components considered: 16
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV          0.1511   0.1398    0.133   0.1335   0.1329   0.1332   0.1336
## adjCV       0.1511   0.1397    0.133   0.1334   0.1328   0.1331   0.1335
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV      0.1337   0.1305   0.1289    0.1289    0.1252    0.1251    0.1252
## adjCV   0.1335   0.1303   0.1287    0.1287    0.1250    0.1248    0.1249
##        14 comps  15 comps  16 comps
## CV       0.1255    0.1255    0.1254
## adjCV    0.1252    0.1252    0.1251
##
## TRAINING: % variance explained
##              1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X              33.66    56.40    63.62    69.64    75.27    80.14    83.90
## Accept.Apps    14.84    22.91    23.75    24.30    25.05    25.22    25.98
##              8 comps  9 comps  10 comps  11 comps  12 comps  13 comps  14 comps
## X              87.37    90.47     93.07     95.33     97.30     98.46     99.29
## Accept.Apps    29.73    31.19     31.26     35.38     36.12     36.65     36.66
##              15 comps  16 comps
## X               99.81    100.00
## Accept.Apps     37.21     37.46
```

```
validationplot(collegePCR, val.type = "MSEP", legendpos = "topright")
```



Hence the value m is chosen with 14

```
collegePCR.pred <- predict(collegePCR, College[-inTrain, names(College) != 'Salary'], ncomp = 14)
PCRTestMSE <- mean((collegePCR.pred - College[-inTrain, "Accept.Apps"])^2)
PCRTestMSE #test error obtained
```

```
## [1] 0.01333012
```

(C)

```
PCRTestMSE
```

```
## [1] 0.01333012
```

```
#                  TestMSE     TrainMSE
#Best reduced OLS   0.01441521  0.01315906
#Ridge Regression   0.01432566  0.01334943
#Lasso              0.01554418    0.01405796
```

The PC regression performs the lowest testMSE for this dataset. Although it requires a lot of calculation, the differences of the testMSEs are significant. So PCR should be chosen as the approach to analyze this dataset.