Stat415 Assignment 7.

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1. $u_1 = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$; $u_2 = \begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}$

(a). Assume the original covariance matrix is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Hence $\begin{pmatrix} a-4 & b \\ c & d-4 \end{pmatrix}\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} = 0$ $\Rightarrow$ Hence $\begin{cases} 0.6(a-4) + 0.8b = 0 \\ -0.8(a-1) + 0.6b = 0 \end{cases}$ $\Rightarrow$ $a = 2.08$; $b = 1.44$

$\begin{pmatrix} a-1 & b \\ c & d-1 \end{pmatrix}\begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} = 0$ $\qquad \begin{cases} 0.6c + 0.8(d-4) = 0 \\ -0.8c + 0.6(d-1) = 0 \end{cases}$ $\Rightarrow$ $c = 1.44$; $d = 2.92$

Hence $\begin{pmatrix} 2.0 \\ 1.4 \end{pmatrix}$ is the ori[ginal] covariance

(b). $\frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{4}{4+1} = 0.8$; Hence 80% of the variance can be explained by Y_1

(c). $X = (1, 2)$

$Y_1 = 0.6 + 0.8 \times 2 = 2.2$

$Y_2 = -0.8 + 0.6 \times 2 = 0.4$

Hence the first and second principle components are $\begin{cases} Y_1 = 2.2 \\ Y_2 = 0.4 \end{cases}$