Stats 415 Assignment 3.

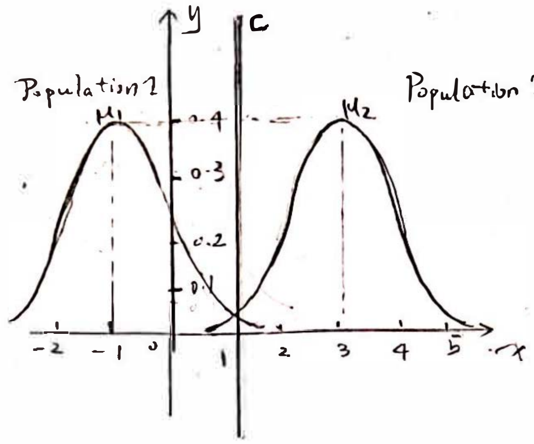
1. $\hat{\mu}_1 = -1$, $\hat{\mu}_2 = 3$. $\hat{\sigma}^2 = 1$

(a) Hence $p_1(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x+1)^2}$

$p_2(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-3)^2}$

Since we assume $\pi_1 = \pi_2 = 0.5$

$$\begin{cases} p_1(x) > p_2(x) \text{ when } x < 1. \\ p_1(x) = p_2(x) \text{ when } x = 1 \\ p_1(x) > p_2(x) \text{ when } x > 1 \end{cases}$$



(b) In practice, we do not assume $\pi_1 = \pi_2$.

Assume we still have class mean $\mu_1 = -1$, $\mu_2 = 3$

The class priors $\begin{cases} \hat{\pi}_1 = \frac{40}{100} = 0.4 \\ \hat{\pi}_2 = \frac{60}{100} = 0.6 \end{cases}$

The discriminant function $\delta_k(x) = x\dfrac{\mu_k^2}{\sigma^2} - \dfrac{\mu_k^2}{2\sigma^2} + \log \pi_k$ with the term $\log \pi_k$ will be greater for class 2 than class 1.

Hence, the new boundary value $\hat{c}$ will be less than c.

(c) $\begin{cases} \delta_1(x) = -x - \frac{1}{2} + \log 0.4 \\ \delta_2(x) = 3x - \frac{9}{2} + \log 0.6 \end{cases}$

$\Rightarrow \begin{cases} \delta_1(x) > \delta_2(x) \text{ when } x < 0.8986 \\ \delta_2(x) = \delta_2(x) \text{ when } x = 0.8986 \\ \delta_1(x) < \delta_2(x) \text{ when } x > 0.8986 \end{cases} \Rightarrow \hat{c} = 0.8986 < c$

(d) I would recommand using QDA, since Quadratic Discriminant Analysis tends to work better when the variances are very different between classes. Also we have enough observations to accurately estimate the variances.

(e) $\begin{cases} \hat{\mu}_1 = -1;\ \hat{\sigma}_1^2 = 0.25 \\ \hat{\mu}_2 = 3,\ \hat{\sigma}_2^2 = 1.5 \end{cases}$ Also, we assume $\pi_1 = \pi_2 = 0.5$.

Hence $\delta_1(x) = -\frac{1}{2}\log(0.25) - \frac{1}{2}\dfrac{(x+1)^2}{0.25} + \log 0.5$ $\Rightarrow$ $c = 0.292$

$\delta_2(x) = -\frac{1}{2}\log(1.5) - \frac{1}{2}\dfrac{(x-3)^2}{1.5} + \log 0.5$

$$\begin{cases} \delta_1(x) > \delta_2(x) \text{ when } x < 0.292 \\ \delta_1(x) = \delta_2(x) \text{ when } x = 0.292 \\ \delta_1(x) < \delta_2(x) \text{ when } x > 0.292 \end{cases}$$