University of Michigan

Exam 1

Moulinath Banerjee

October 19, 2016

Announcement: The exam carries 38 points but the maximum possible score is 34 points.

Problem 1: Consider N balls labeled 1 through N and r boxes labeled 1 through r. One by one, wach of the balls is assigned,randomly to one of the r boxes. **(i)** What is the probability that the first box ends up with n_1 balls, the second with n_2, ..., the r'th with n_r balls where $\sum n_j = N$? (6 points)
(ii) Let $r > 2$. Let N_i denote the random number of balls that end up in the i'th box and N_j the number that end up in the j'th. Find $P(N_i = n_i, N_j = n_j)$. Also, find $E(N_i), E(N_j)$. $(7 + 5 = 12$ points)

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Problem 2: Consider a sequence of independent coin flips with p being the probability of the coin landing H on any single flip. Define a random variable R as the length of the run started by the first trial (so if $HHT...$ or $TTH...$ is how the sequence starts off, $R = 2$).
(i) Find the p.m.f of R and ER. (10 points)
(i) How would you find the p.m.f of the sum of the first two runs? (for the sequences $HHTTTH....$ or $TTHHHT..$, the first run has length 2 and the second has length 3) (5 points)

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Problem 3: Consider the SRSWOR set-up: A population of N voters has Np Democrats and Nq Republicans where $0 < p < 1$ and $q = 1 - p$. A sample of size n is drawn *without replacement* from the population. Let $X_j = 1$ if the j'th voter in the sample is Democrat and 0 otherwise. Find $P(X_1 = 1, X_2 = 0, X_3 = 1)$. Show that this is the same as $P(X_i = 1, X_j = 0, X_k = 1)$ for a general triplet $i < j < k$. (8 points)

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