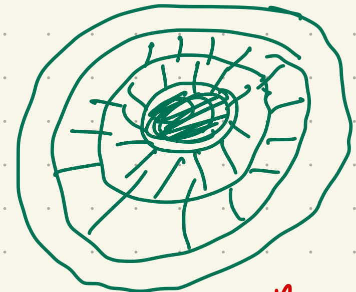
$A_1 \subseteq A_2 \subseteq A_3 \subseteq \ldots$

$$A = \bigcup_{j=1}^{\infty} A_j .$$



$$\left. P(A) = \lim_{n\to\infty} P(A_n) \right\} .$$

A_n is disjoint union of $B_1, \ldots, B_n$

Countable additivity property.

$B_1 = A_1, \quad B_2 = A_2 - A_1, \ldots,$

$B_j = A_j - A_{j-1}, \ldots$

$$\bigcup_{j=1}^{\infty} A_j = \bigcup_{j=1}^{\infty} B_j = \underline{A}$$

$$P(A) = P\left(\bigcup_{j=1}^{\infty} B_j\right) = \sum_{j=1}^{\infty} P(B_j)$$

$$= \lim_{n\to\infty}\left(P(B_1) + \ldots + P(B_n)\right)$$

$$= \lim_{n\to\infty} P(A_n)$$

Unordered subsample of size r drawn with replacement from collection $\{1, 2, \ldots, n\}$:

$$\boxed{\left\{ (k_1, k_2, \ldots, k_n) : 0 \le k_i \le r, \ \sum k_i = r \right\}}$$

Cardinality of this set is precisely the total number of unordered samples of size r.

$(0, 2, 0)$

$(1,1), (2,2), (3,3), (1,3), \boxed{(3,1)}$ -

$\boxed{(1, 0, 1)} = (k_1, k_2, k_3)$

$$(1+x+x^2+\cdots+x^r)^n = \left(\frac{1-x^{r+1}}{1-x}\right)^n$$

Find the co-efficient of x^r in this polynomial. ~~that~~ gives the number.

$$(1+x+\cdots+x^r)\times(1+x+\cdots+x^r)$$

$$\times(1+x+\cdots+x^r)$$

$$x^{i_1} \cdot x^{i_2} \cdots x^{i_n}$$

$$x^r = x^{i_1+i_2+\cdots+i_n}$$

$i_1+i_2+\cdots+i_n = r$

$A(m)$: all sequences comprising H and T of length n with exactly m H's.

Consider any $\omega \in A(m)$

$$\underbrace{P_{prod}(\{\omega\})}_{} = \underbrace{p^m q^{n-m}}_{}$$

$$P(\{\underbrace{H H \cdots H}_{m} \underbrace{T T \cdots T}_{n-m}\})$$

$$P\left(\underline{\underline{A(m)}}\right)$$

$$= |\underline{\underline{A(m)}}| \times p^m q^{n-m}$$

$$= \binom{n}{m} p^m q^{n-m}$$

Extension of NB to more than two outcomes

6 faced die

I keep on tossing die till

I see x_1 3's and

x_2 5's.

$$3 \qquad (3 \cup 5)^c \qquad 3 \qquad 5 \quad \dots$$

$$\tfrac{1}{6} \qquad \tfrac{2}{3} \qquad \tfrac{1}{6} \qquad \tfrac{1}{6}$$

r_2 5's before

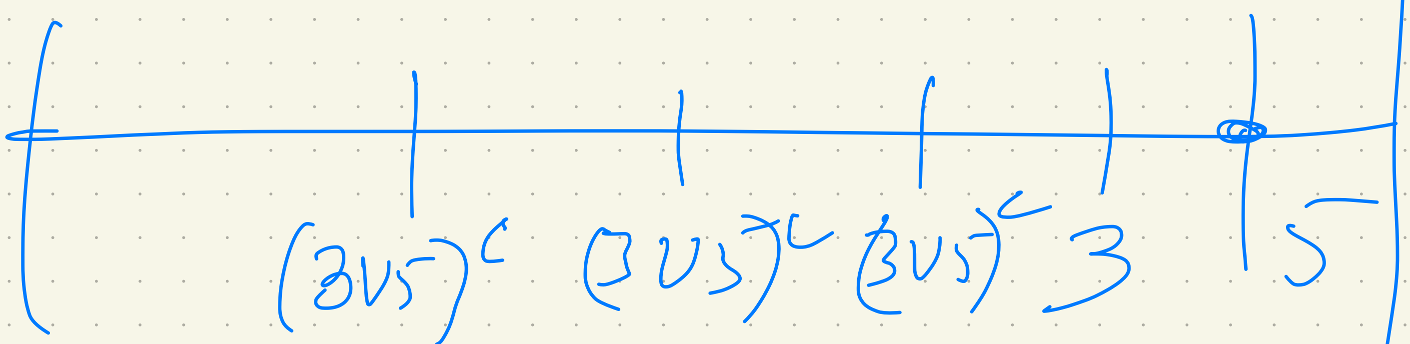
$r_1 - 1$ 3's before

$r_1 + r_2$

either a 3 or a 5

$$\left.\begin{array}{l} r_1 = 1 \\ r_2 = 1 \end{array}\right\}.$$

$3 \longrightarrow 5$



$P(\#\text{ of trials is } 5)$