Stat 510: Exam 1

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Announcement: The exam carries 44 points but the maximum possible score is 40 points.

Problem 1: P has three jars each containing n M&M's: the first jar contains red ones, the second blue ones, and the third yellow ones. Whenever P wants an M&M, she reaches randomly into one of the jars and gets one. What is the chance that the first time she reaches into a jar and finds it empty, the other two jars contain r and s M&M's (with $0 < r, s < n$)? (12 points)

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Problem 2: (a) Consider

$$F(x) = \left[1 - \frac{1}{(1+x)^2}\right] 1(x \geq 0)\,.$$

(i) Verify that this is a proper distribution function and given a uniform random variable U, use this to generate a random variable X that has distribution function F. (4 + 4 = 8 points)
(ii) Find EX. (4 points)

(b) For a general random variable X, show that $EX = \int_0^1 F^{-1}(p)dp$. (6 points)

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Problem 3: Let $X \sim \text{Poisson}(\lambda)$. Show that for any integer $r \geq 1$, $E[X(X-1)\dots(X-r+1)]$, the r'th factorial moment of X, is λ^r. Use this formula to calculate $E(X-\lambda)^3$. (8 + 6 = 14 points)

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