第四周第一次作业答案

4-1.1

解:

$$
由图可知f(t)=\begin{cases}\frac{2E}{\tau-\tau_1}t+\frac{E\tau}{\tau-\tau_1},t\in[-\frac{\tau}{2},-\frac{\tau_1}{2})\\E,t\in[-\frac{\tau_1}{2},\frac{\tau_1}{2})\\\frac{-2E}{\tau-\tau_1}t+\frac{E\tau}{\tau-\tau_1},t\in[\frac{\tau_1}{2},\frac{\tau}{2})\end{cases}
$$

设$f(t)$的傅里叶变换为$F(\omega)$，根据傅里叶变换的时间微分特性$\frac{d^nf(t)}{dt^n}\leftrightarrow(j\omega)^nF(\omega)$，可求得$F(\omega)$，然后根据$F(\omega)$画出$\tau=2\tau_1$情况下该脉冲的频谱图。

解法一:

易得

$$
f'(t)=\frac{2E}{\tau-\tau_1}[u(t+\frac{\tau}{2})-u(t+\frac{\tau_1}{2})]-\frac{2E}{\tau-\tau_1}[u(t-\frac{\tau_1}{2})-u(t-\frac{\tau}{2})]
$$

$$
f''(t)=\frac{2E}{\tau-\tau_1}[\delta(t+\frac{\tau}{2})-\delta(t+\frac{\tau_1}{2})]-\frac{2E}{\tau-\tau_1}[\delta(t-\frac{\tau_1}{2})-\delta(t-\frac{\tau}{2})]
$$

根据$\frac{d^nf(t)}{dt^n}\leftrightarrow(j\omega)^nF(\omega)$，得

$$
\begin{aligned}
(j\omega)^2F(\omega)&=\frac{2E}{\tau-\tau_1}(e^{j\omega\frac{\tau}{2}}-e^{j\omega\frac{\tau_1}{2}})+\frac{2E}{\tau_1-\tau}(e^{-j\omega\frac{\tau_1}{2}}-e^{-j\omega\frac{\tau}{2}})\\
&=\frac{2E}{\tau-\tau_1}(e^{j\omega\frac{\tau}{2}}+e^{-j\omega\frac{\tau}{2}})+\frac{2E}{\tau_1-I}(e^{-j\omega\frac{\tau_1}{2}}+e^{j\omega\frac{\tau_1}{2}})\\
&=\frac{4E}{\tau-\tau_1}[\cos(\frac{\omega\tau}{2})-\cos(\frac{\omega\tau_1}{2})]
\end{aligned}
$$

$$F(\omega)=\frac{4E}{(\tau_1-\tau)\omega^2}[\cos(\frac{\omega\tau}{2})-\cos(\frac{\omega\tau_1}{2})]$$

当$\tau=2\tau_1$时，带入上式可得

$$F(\omega)=\frac{-4E}{\tau_1\omega^2}[\cos(\omega\tau_1)-\cos(\frac{\omega\tau_1}{2})]=\frac{8E}{\tau_1\omega^2}\sin(\frac{3\omega\tau_1}{4})\sin(\frac{\omega\tau_1}{4})$$

$$F(\omega)=\frac{3E\tau_1}{2}Sa\left(\frac{3\tau_1}{4}\omega\right)Sa\left(\frac{\tau_1}{4}\omega\right)$$

解法二：

易得

$$f'(t)=\frac{2E}{\tau-\tau_1}[u(t+\frac{\tau}{2})-u(t+\frac{\tau_1}{2})-u(t-\frac{\tau_1}{2})+u(t-\frac{\tau}{2})]$$

$u(t+\frac{\tau}{2})-u\left(t+\frac{\tau_1}{2}\right)$和$u(t-\frac{\tau_1}{2})+u(t-\frac{\tau}{2})$分别为矩形脉冲

$g(t)=\begin{cases}1, |t|<\frac{\tau-\tau_1}{4}\\0, |t|>\frac{\tau-\tau_1}{4}\end{cases}$ 向左和向右平移$\frac{\tau+\tau_1}{4}$。

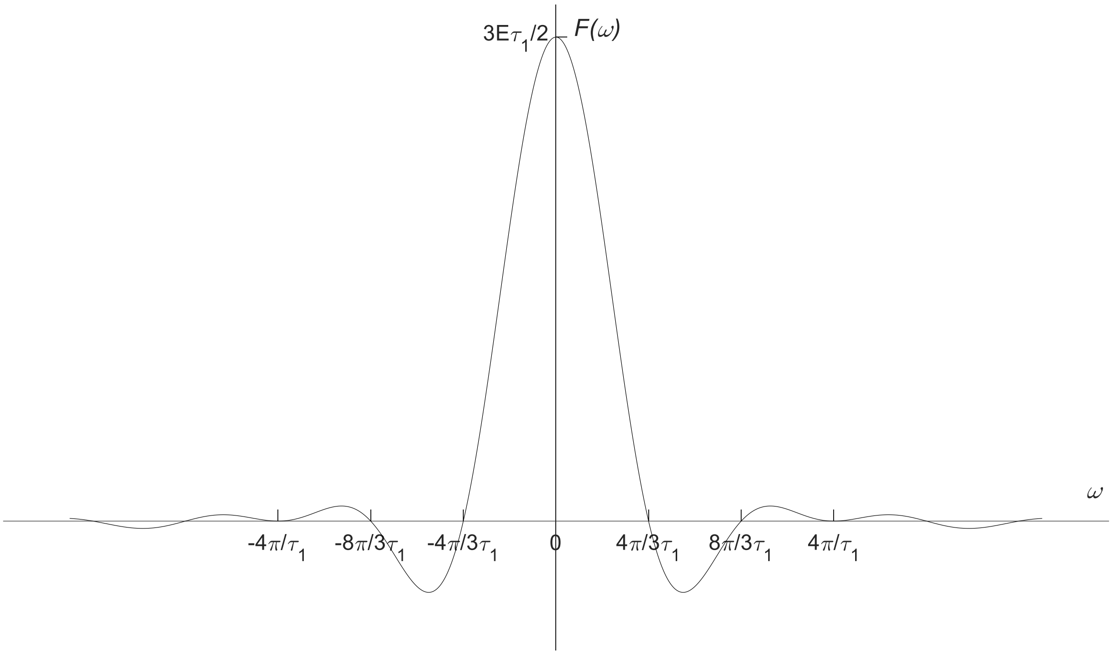
易得$g(t)$的傅里叶变换为$(\frac{\tau-\tau_1}{2})\mathrm{Sa}\left[\frac{\omega(\tau-\tau_1)}{4}\right]$，由傅里叶变换的时移特性和线性性可得$f'(t)$的傅里叶变换：

$$j\omega F(\omega)=\frac{2E}{\tau-\tau_1}\left[e^{j\frac{\omega(\tau+\tau_1)}{4}}-e^{-j\frac{\omega(\tau+\tau_1)}{4}}\right]\frac{(\tau-\tau_1)}{2}Sa\left[\frac{\omega(\tau-\tau_1)}{4}\right]$$

$$F(\omega)=\frac{(\tau+\tau_1)E}{2}\cdot Sa\left[\frac{\omega(\tau+\tau_1)}{4}\right]\cdot Sa\left[\frac{\omega(\tau-\tau_1)}{4}\right]$$

当$\tau=2\tau_1$时，$F(\omega)=\frac{3E\tau_1}{2}Sa(\frac{3\omega\tau_1}{4})Sa(\frac{\omega\tau_1}{4})$

根据上式可画出$\tau=2\tau_1$情况下的频谱图，如下图所示：



4-1.2

解:

（1）由图可知

$$f_1(t) = E_1\left[u\left(t+\frac{\tau_1}{2}\right) - u\left(t-\frac{\tau_1}{2}\right)\right]$$
$$f_2(t) = E_2\left[u\left(t+\frac{\tau_2}{2}\right) - u\left(t-\frac{\tau_2}{2}\right)\right]$$

易知

$$f_1(t) * f_2(t) = \int_{-\infty}^{t} f_1(\tau) f_2(t-\tau) d\tau \ = \frac{df(t)}{dt} * \int_{-\infty}^{t} f_2(\tau) d\tau$$

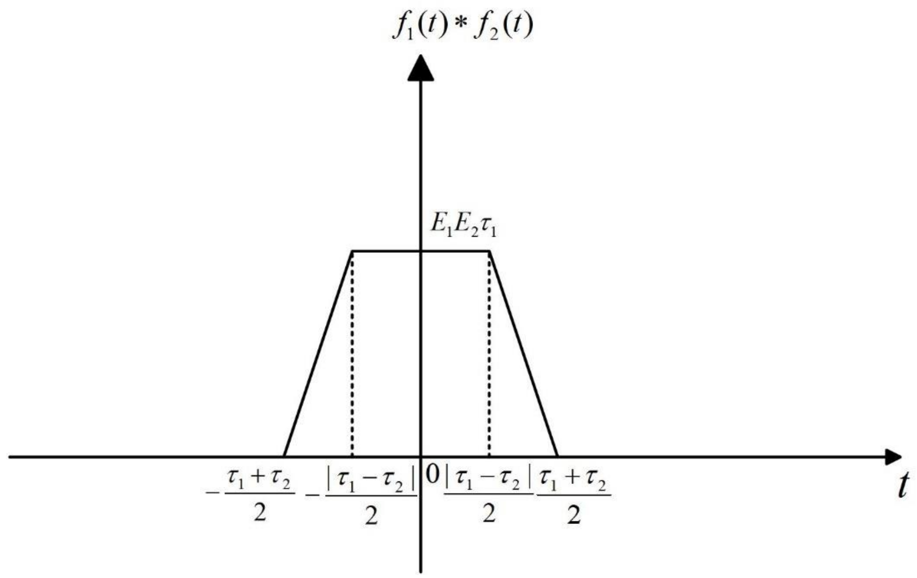
$$= E_1[\delta(t+\frac{\tau_1}{2})-\delta(t-\frac{\tau_1}{2})] * \int_{-\frac{\tau_1}{2}}^{t} E_2[u(\tau+\frac{\tau_2}{2})-u(\tau-\frac{\tau_2}{2})]d\tau$$

$$= E_1\left[\delta(t+\frac{\tau_1}{2})-\delta(t-\frac{\tau_1}{2})\right] * E_2\left[(t+\frac{\tau_2}{2})u(t+\frac{\tau_2}{2})-(t-\frac{\tau_2}{2})u(t-\frac{\tau_2}{2})\right]$$

$$= E_1E_2[(t+\frac{\tau_1+\tau_2}{2})u(t+\frac{\tau_1+\tau_2}{2})-(t+\frac{\tau_1-\tau_2}{2})u(t+\frac{\tau_1-\tau_2}{2})$$

$$-(t+\frac{\tau_2-\tau_1}{2})u(t+\frac{\tau_2-\tau_1}{2})+(t-\frac{\tau_2+\tau_1}{2})u(t-\frac{\tau_2+\tau_1}{2})]$$

根据上式可画出$f_1(t) * f_2(t)$的图形，如下图所示：



（2）已知

$$FT[f_1(t)] = E_1\tau_1 Sa\left(\frac{\omega\tau_1}{2}\right)$$

$$FT[f_2(t)] = E_2\tau_2 Sa(\frac{\omega\tau_2}{2})$$

由频域卷积定理$f_1(t) * f_2(t) \leftrightarrow F_1(\omega)\cdot F_2(\omega)$得

$$F_1(\omega)\cdot F_2(\omega) = E_1E_2\tau_1\tau_2 Sa(\frac{\omega\tau_1}{2})Sa(\frac{\omega\tau_2}{2})$$

故$f_1(t) * f_2(t)$的频谱为

$$E_1E_2\tau_1\tau_2 Sa(\frac{\omega\tau_1}{2})Sa(\frac{\omega\tau_2}{2})$$

本题与 4-1.1 题均是求梯形脉冲信号的频谱。4-1.1 题利用了傅里叶变换的时间微分特性$\frac{d^n f(t)}{dt^n} \leftrightarrow (j\omega)^n F(\omega)$，而本题利用了傅里叶变换的卷积性质，即$f_1(t) * f_2(t) \leftrightarrow F_1(\omega)\cdot F_2(\omega)$。