第二周第二次答案

2-2.1

解:

（1）

①当$-\infty < t \leq 0$时，重合面积为零（如图 a 所示），$f_1(t) * f_2(t) = 0$。

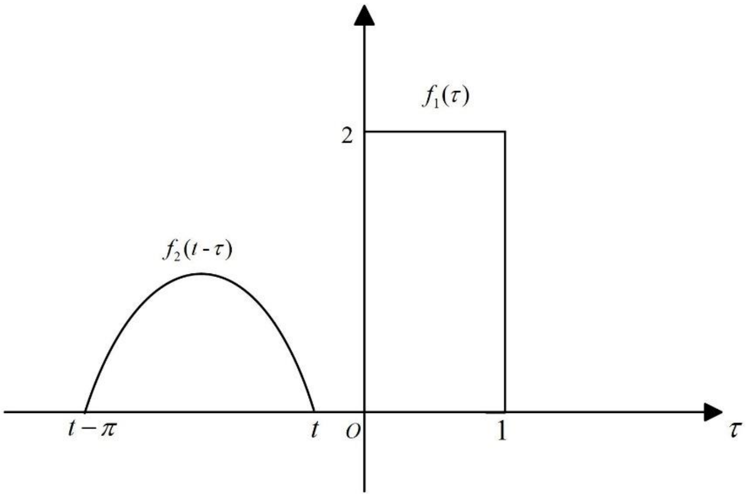

图 a

②当$0 < t \leq 1$时（图 b），$f_1(t) * f_2(t) = \int_0^t 2 \times \sin(t-\tau)\, d\tau = 2\cos(t-\tau)\,|_0^t$

$$= 2 - 2cost。$$

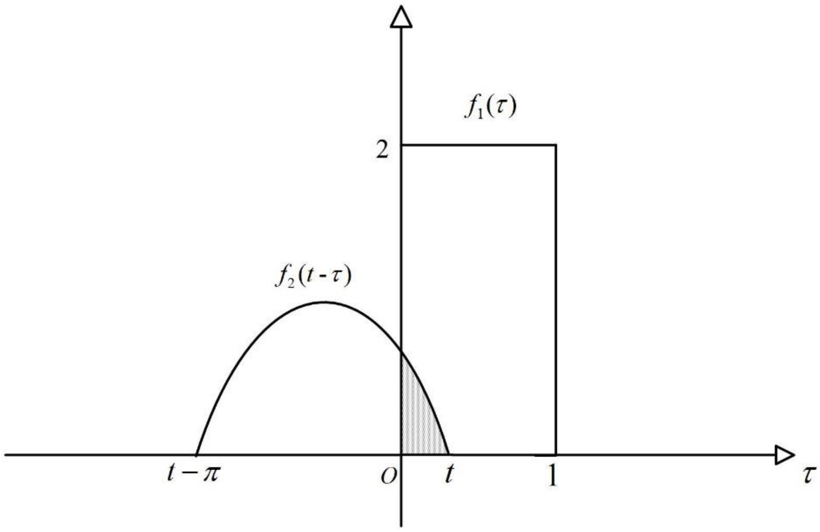

图 b

③当$t \geq 1$且$t-\pi \leq 0$，即$1 < t \leq \pi$时（图 c），$f_1(t) * f_2(t) = \int_0^1 2 \times \sin(t-\tau)\,d\tau$

$$= 2\cos(t-\tau)\,|_0^1 = 2[\cos(t-1) - cost]$$。

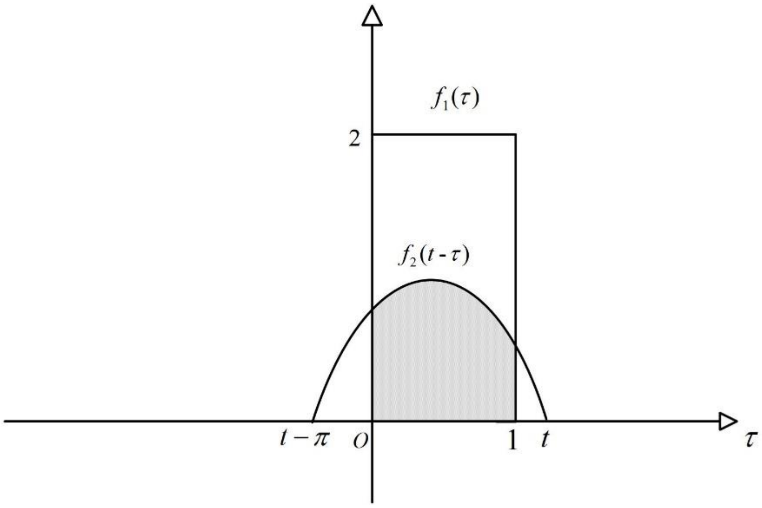

图 c

④当$0 < t-\pi \leq 1$，即$\pi < t \leq \pi+1$时（图 d），

$f_1(t) * f_2(t) = \int_{t-\pi}^1 2 \times \sin(t-\tau)\,d\tau = 2\cos(t-\tau)\,|_{t-\pi}^1 = 2[\cos(t-1) + 1]$。

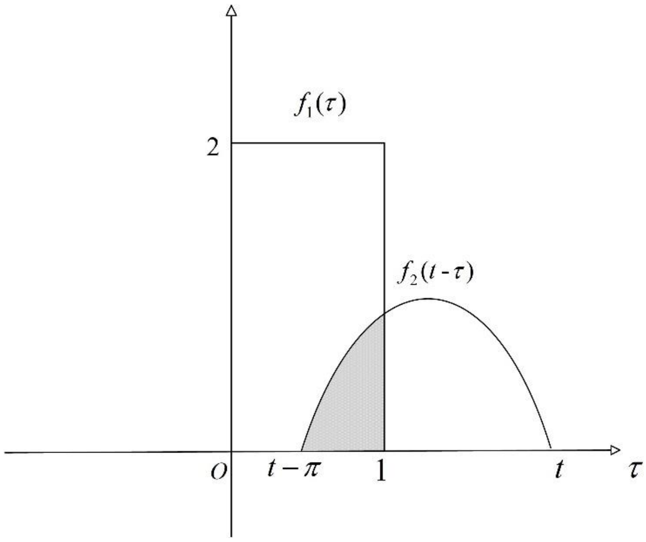

图 d

⑤当$t-\pi > 1$，即$t > \pi+1$时，重合面积为零（图 e），$f_1(t) * f_2(t) = 0$。

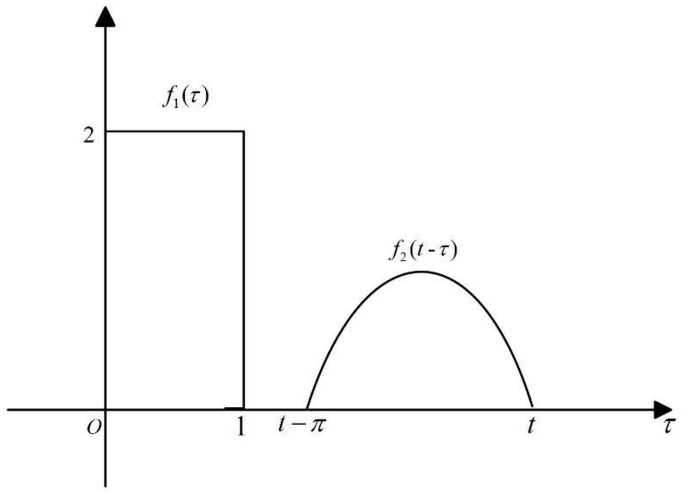

图 e

$$f_1(t)*f_2(t)=(2-2cost)[u(t)-u(t-1)]+2[\cos(t-1)-cost][u(t-1)-u(t-\pi)]+2[\cos(t-1)+1][u(t-\pi)-u(t-\pi-1)]$$

图 f 为卷积后的图像。

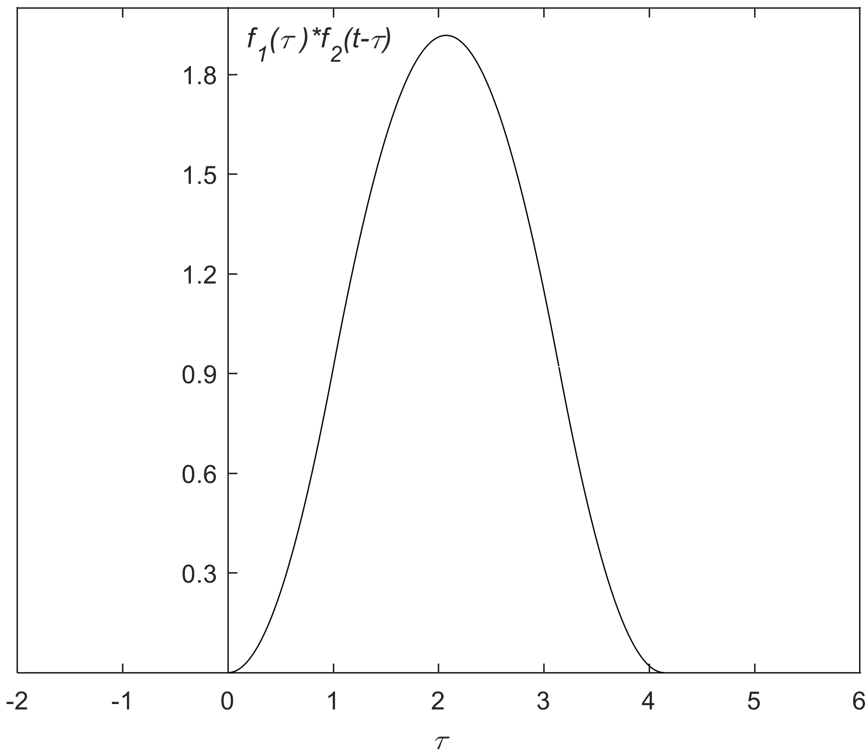

图 f

（2）$f_1(t)*f_2(t)=\int_{-\infty}^{t}f_1(\tau)d\tau*\frac{df_2(t)}{dt}=\int_{-\infty}^{t}sin\tau\, d\tau\ *\ \delta(t-1)$

$$=[(1-cost)u(t)]*\delta(t-1)$$

$$=[1-\cos(t-1)]u(t-1)$$ （卷积时移特性）

图 g 为卷积后的图像。

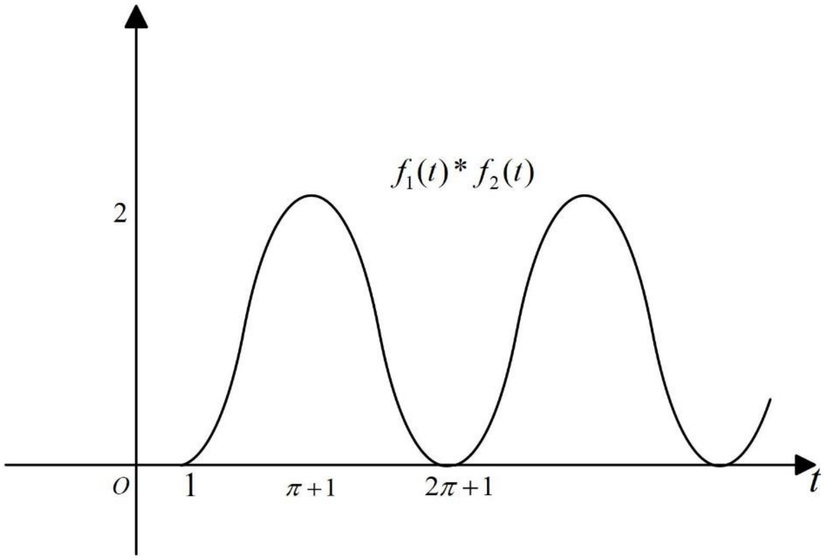

图 g

2-2.2

解:

（1）$r(t)=h(t)*e(t)=e^{-2t}u(t)*\{e^{-t}[u(t)-u(t-2)]+\beta\delta(t-2)\}$

$=\int_{-\infty}^{\infty}e^{-2(t-\tau)}u(t-\tau)\{e^{-\tau}[u(\tau)-u(\tau-2)]+\beta\delta(\tau-2)\}\,d\tau$

$=\int_{-\infty}^{\infty}e^{-2t+\tau}u(t-\tau)[u(\tau)-u(\tau-2)]\,d\tau+\int_{-\infty}^{\infty}\beta\delta(\tau-2)e^{-2(t-\tau)}u(t-\tau)\,d\tau$

$=\int_{0}^{2}e^{-2t+\tau}u(t-\tau)\,d\tau+\beta e^{-2(t-2)}u(t-2)$

$=\int_{0}^{\min(2,t)}e^{-2t+\tau}\,d\tau+\beta e^{-2(t-2)}u(t-2)$

$=e^{-2t+\tau}[u(t)-u(t-2)]|_0^t+e^{-2t+\tau}u(t-2)|_0^2+\beta e^{-2(t-2)}u(t-2)$

$=(e^{-t}-e^{-2t})[u(t)-u(t-2)]+(e^{-2t+2}-e^{-2t}+\beta e^{-2(t-2)})u(t-2)$

$=(e^{-t}-e^{-2t})[u(t)-u(t-2)]+e^{-2t}(e^{2}-1+\beta e^{4})u(t-2)$

（2）$t>2$时，

$r(t)=h(t)*e(t)=[e^{-2t}u(t)]*\{x(t)[u(t)-u(t-2)]+\beta\delta(t-2)\}$

$=e^{-2t}*\{x(t)[u(t)-u(t-2)]+\beta\delta(t-2)\}$

$=\int_{-\infty}^{\infty}e^{-2(t-\tau)}\{x(\tau)[u(\tau)-u(\tau-2)]+\beta\delta(\tau-2)\}\,d\tau$

$=\int_{-\infty}^{\infty}e^{-2(t-\tau)}x(\tau)[u(\tau)-u(\tau-2)]\,d\tau+\int_{-\infty}^{\infty}\beta\delta(\tau-2)e^{-2(t-\tau)}\,d\tau$

$$= \int_0^2 e^{-2(t-\tau)} x(\tau)\, d\tau + \beta e^{-2(t-2)}$$

$$= e^{-2t}(\int_0^2 e^{2\tau} x(\tau)\, d\tau + \beta e^4)$$

要使$t > 2$时，$r(t) = 0$，那么 $e^{-2t}\left(\int_0^2 e^{2\tau} x(\tau)\, d\tau + \beta e^4\right) = 0$，易得

$$\beta = -e^{-4} \int_0^2 e^{2\tau} x(\tau)\, d\tau$$