第三周第二次作业答案

3-2.1

解:

（1）由图（a）可知$f_1(t)=\sin(\pi t)[u(t)-u(t-2)]$，其为 T=4 的周期信号，基波频率$\omega_1=\frac{\pi}{2}$。可将$f_1(t)$进行傅里叶分解，由$f_1(t)=a_0+\sum_{n=1}^{\infty}a_n\cos(n\omega_1 t)+b_n\sin(n\omega_1 t)$,其中$a_0=\frac{1}{T}\int_0^4 f_1(t)dt=0,a_n$和$b_n$可通过下式计算：

$$a_n=\frac{2}{T}\int_0^4 f_1(t)\cos(n\omega_1 t)dt \qquad\qquad b_n=\frac{2}{T}\int_0^4 f_1(t)\sin(n\omega_1 t)dt$$

将$f_1(t)$代入上式计算可得

$$\begin{aligned}
a_n&=\frac{2}{4}\int_0^2 sin(\pi t)\,cos(n\omega_1 t)\,dt=\frac{1}{2}\int_0^2 sin(\pi t)\,cos\left(\frac{n\pi t}{2}\right)dt\\
&=\frac{1}{2}\int_0^2 \frac{1}{2}\left[sin\left(\pi t+\frac{n\pi t}{2}\right)+sin\left(\pi t-\frac{n\pi t}{2}\right)\right]dt\\
&=\frac{1}{4}\int_0^2 sin\,(\pi t+\frac{n\pi t}{2})+sin\,(\pi t-\frac{n\pi t}{2})]dt\\
&=\frac{2}{4\pi(n^2-4)}\{(2-n)cos\,[(n+2)\pi]+(2+n)cos\,[(2-n)\pi]-4\}\\
&=\frac{2}{4\pi(n^2-4)}[4cos\,(n\pi)-4]=\frac{2[cos\,(n\pi)-1]}{\pi(n^2-4)}\\
&=\begin{cases}0,n=2k,k\in Z^+\\ \frac{4}{\pi(4-n^2)},n=2k+1,k\in Z^+\end{cases}
\end{aligned}$$

$$\begin{aligned}
b_n&=\frac{2}{T}\int_0^2 sin\,(\pi t)sin\,(\frac{n\pi}{2}t)dt\\
&=\frac{2}{4}\int_0^2 sin\,(\pi t)sin\,(\frac{n\pi}{2}t)dt\\
&=\frac{1}{4}\int_0^2 [cos\,(\pi-\frac{n\pi}{2})t-cos\,(\pi+\frac{n\pi}{2})t]dt\\
&=\frac{1}{4\pi}[\frac{sin\,(\pi-\frac{n\pi}{2})t}{1-\frac{n}{2}}-\frac{sin\,(\pi+\frac{n\pi}{2})t}{1+\frac{n}{2}}]|_0^2\\
&=\frac{sin\;n\pi}{\pi(n^2-4)}
\end{aligned}$$

可以发现，使用上式计算b_n，当$n \neq 2$时，$b_n = 0$;当 n=2 时，不能通过上式计算，$b_n = \frac{2}{4}\int_0^2 \sin^2(\pi t)\,dt = \frac{1}{2}$。

$$即b_n = \begin{cases} \frac{1}{2}, n = 2 \\ 0, n \neq 2 \end{cases}$$

故$f_1(t)$三角形式的FS为$f_1(t)$= $\frac{1}{2}\sin \pi t + \sum_{n=1,3,5..}^{\infty} \frac{4}{(4-n^2)\pi} \cos\frac{n\pi}{2}$t

（2）由图（b）可知$f_2(t)$为 T=4 的周期信号，基波频率$\omega_1 = \frac{\pi}{2}$，其表达式如下：

$$f_2(t) = \begin{cases} \cos(\pi t), t \in [4k - \frac{1}{2}, 4k + \frac{7}{2}], k \in Z \\ -cos(\pi t), t \in [4k + \frac{3}{2}, 4k + \frac{7}{2}], k \in Z \end{cases}$$

不难发现：

$$\begin{aligned} f_2(t) &= \cos(\pi t)[(u + \frac{1}{2}) - (u - \frac{3}{2})] - \cos(\pi t)[(u - \frac{3}{2}) - (u - \frac{7}{2})] \\ &= f_1(t + \frac{1}{2}) - f_1(t - \frac{3}{2}) \end{aligned}$$

由公式$F_n = \frac{1}{T}\int_0^T f_2(t)e^{-jn\omega_1 t}dt$ 可得

$$F_n = \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{3}{2}} sin\left[\left(t + \frac{1}{2}\right)\pi\right] e^{-j\frac{\pi t}{2}n}dt - \frac{1}{4}\int_{\frac{3}{2}}^{\frac{7}{2}} sin\left[\left(t - \frac{3}{2}\right)\pi\right] e^{-j\frac{\pi t}{2}n}dt$$

$$= \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{3}{2}} cos(\pi t)e^{-j\frac{\pi t}{2}n}dt - \frac{1}{4}\int_{\frac{3}{2}}^{\frac{7}{2}} cos(\pi t)e^{-j\frac{\pi t}{2}n}dt$$

$$= \frac{1}{8}\int_{-\frac{1}{2}}^{\frac{3}{2}} [e^{\left(j-\frac{jn}{2}\right)\pi t} + e^{-\left(j+\frac{jn}{2}\right)\pi t}]dt - \frac{1}{8}\int_{\frac{3}{2}}^{\frac{7}{2}} [e^{\left(j-\frac{jn}{2}\right)\pi t} + e^{-\left(j+\frac{jn}{2}\right)\pi t}]dt$$

$$= \frac{1}{j(8-4n)\pi}\left[2e^{j\frac{3(2-n)}{4}\pi} - e^{j\frac{n-2}{4}\pi} - e^{j\frac{7(2-n)}{4}\pi}\right] - \frac{1}{j(8+4n)\pi}\left[2e^{-j\frac{3(2+n)}{4}\pi} - e^{j\frac{2+n}{4}\pi} - e^{-j\frac{7(2+n)}{4}\pi}\right]$$

$$= \frac{-2}{(4-n^2)\pi}e^{-\frac{j3n\pi}{4}} + \frac{1}{(4-n^2)\pi}e^{\frac{jn\pi}{4}} + \frac{1}{(4-n^2)\pi}e^{-\frac{j7n\pi}{4}}$$

$$= \frac{1}{(4-n^2)\pi}\left[-2\cos\left(\frac{3n\pi}{4}\right) + 2\cos(\pi n)\cos\left(\frac{3n\pi}{4}\right) + 2j\sin\left(\frac{3n\pi}{4}\right) - \right.$$

$$\left. 2j\cos(n\pi)\sin(\frac{3\pi n}{4})\right]$$

$$= \frac{2}{(4-n^2)\pi}[\cos(n\pi) - 1][\cos(\frac{3n\pi}{4}) - j\sin(\frac{3n\pi}{4})]$$

可以发现，当$n \neq 2$时，可使用上式计算F_n：

$$F_n=\frac{2}{(4-n^2)\pi}[\cos(n\pi)-1][\cos(\frac{3n\pi}{4})-j\sin(\frac{3n\pi}{4})]$$

当$n=2$时，不能使用上式计算F_n，易知$F_n=0$

即$F_n=\begin{cases}\frac{2}{(4-n^2)\pi}[\cos(n\pi)-1][\cos(\frac{3n\pi}{4})-j\sin(\frac{3n\pi}{4})], n\neq 2\\ 0, n=2\end{cases}$

当 n 为偶数时，$\cos n\pi-1=0$，故

$$f_2(t)=\sum_{n=\pm1,\pm3,\pm5..}^{\pm\infty}\frac{4}{(4-n^2)\pi}\sin\left(\frac{n\pi}{2}\right)\left[\sin\left(\frac{n\pi}{4}\right)+j\cos\left(\frac{n\pi}{4}\right)\right]e^{j\frac{n\pi}{2}t}$$

3-2.2

解：

由图可知$f(t)=E\cos(\frac{\pi}{\tau}t)[u(t+\frac{\tau}{2})-u(t-\frac{\tau}{2})]$，根据$F(j\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$可知：

$$F(j\omega)=\int_{-\frac{T}{2}}^{\frac{T}{2}}E\cos\left(\frac{\pi}{\tau}t\right)e^{-j\omega t}dt$$

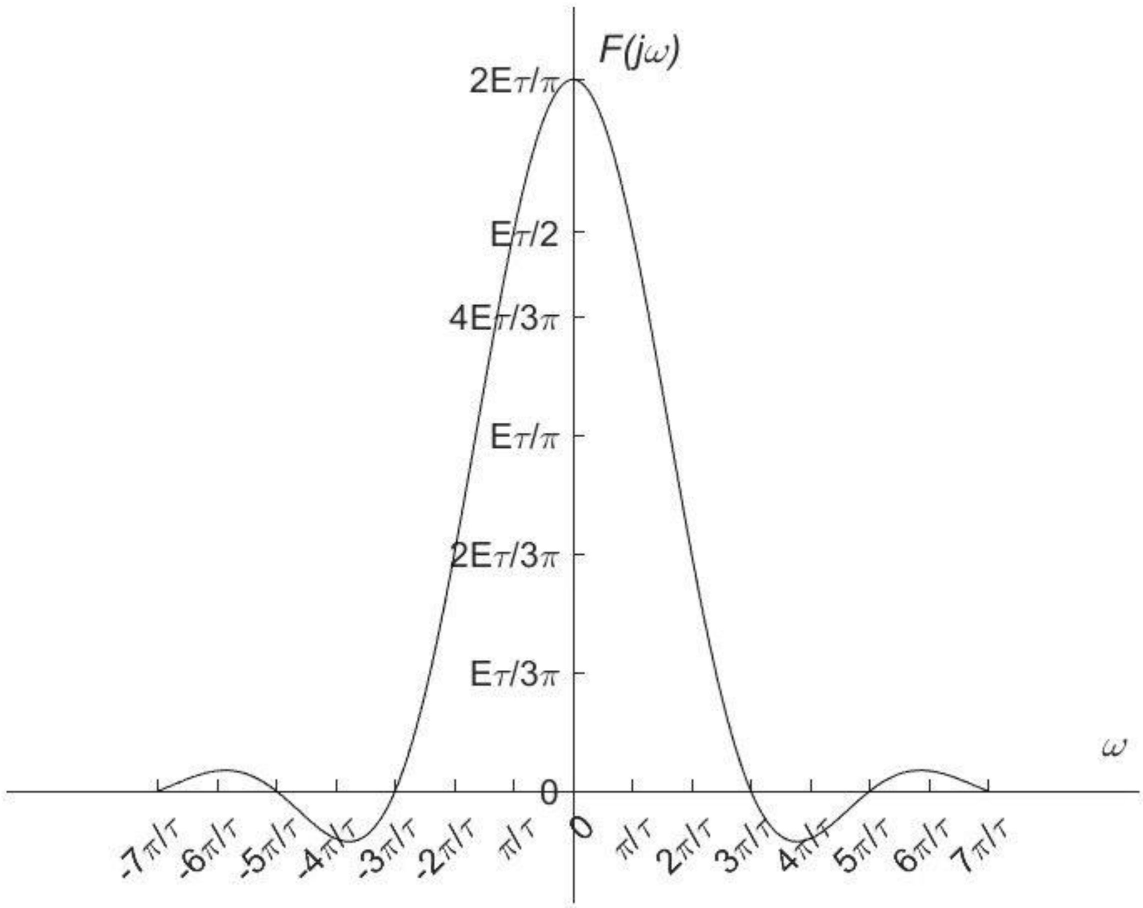
$$=\frac{E}{2}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left(e^{\frac{\pi}{\tau}tj}+e^{-\frac{\pi}{\tau}tj}\right)e^{-j\omega t}dt$$

$$=\frac{E}{2}\left[\frac{e^{-\frac{\tau}{2}(\frac{\pi}{\tau}-\omega)}}{j\left(\frac{\pi}{\tau}-\omega\right)}-\frac{e^{\frac{\tau}{2}(\frac{\pi}{\tau}+\omega)}}{j\left(\frac{\pi}{\tau}+\omega\right)}-\frac{e^{\frac{\tau}{2}(\frac{\pi}{\tau}-\omega)}}{j\left(\frac{\pi}{\tau}-\omega\right)}+\frac{e^{-\frac{j\tau}{2}(\frac{\pi}{\tau}+\omega)}}{j\left(\frac{\pi}{\tau}+\omega\right)}\right]$$

$$=\frac{E}{2j}\left[\frac{e^{-\frac{nj}{2}}\cdot e^{\frac{j\omega\pi}{2}}-e^{\frac{j\pi}{2}}\cdot e^{-\frac{j\omega\pi}{2}}}{\frac{\pi}{\tau}-\omega}+\frac{e^{-\frac{j\pi}{2}}\cdot e^{\frac{j\omega\pi}{2}}-e^{\frac{j\pi}{2}}\cdot e^{\frac{j\omega\pi}{2}}}{\frac{\pi}{\tau}+\omega}\right]$$

$$=\frac{2E\pi\tau\cos(\frac{\omega\tau}{2})}{\pi^2-\omega^2\tau^2}=\frac{2E\tau\cos(\frac{\omega\tau}{2})}{\pi[1-(\frac{\omega\tau}{\pi})^2]}$$

根据$F(j\omega)$的表达式可画出频谱图，如下图所示：



F(jω)
2Eτ/π
Eτ/2
4Eτ/3π
Eτ/π
2Eτ/3π
Eτ/3π
0
ω
-7π/τ
-6π/τ
-5π/τ
-4π/τ
-3π/τ
-2π/τ
π/τ
0
π/τ
2π/τ
3π/τ
4π/τ
5π/τ
6π/τ
7π/τ