Frequency Domain filtering:

↳ $I(r,t) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ $B(r,c,u,v) = \left\{ \begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \begin{bmatrix} +1 & -1 \\ +1 & -1 \end{bmatrix} \begin{bmatrix} +1 & +1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \right\}$

$T(0,0) = 11$
$T(0,1) = 1$
$T(1,0) = 5$
$T(1,1) = 3$

Transformation of Image:

$$T(u,v) = \frac{1}{M \times N} \sum_{c=0}^{N-1} \sum_{r=0}^{M-1} f(c,r)\, \underbrace{e^{-j2\pi\left(\frac{cu + rv}{M \times N}\right)}}_{\text{Base frequency}}$$

Base Image frequency

$$T(u,v) = \begin{bmatrix} 11 & 1 \\ 5 & 3 \end{bmatrix}$$

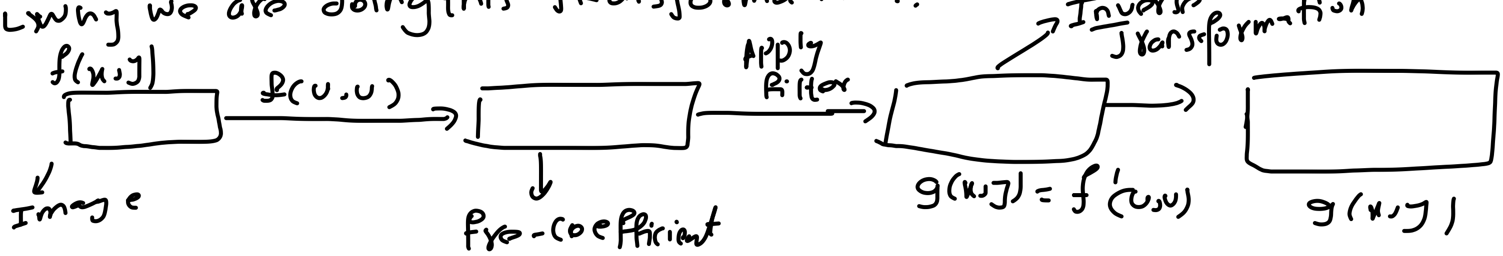
$$T(u,v) = k \sum_{r=0}^{1} \sum_{c=0}^{1} I(r,c)\, B(r,c;u,v)$$

↓ constant

→ ↳ Euler formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Question
↳ why we are doing this transformation??



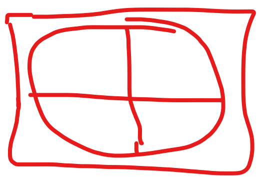
↳ $f'(u,v) = f(u,v) * w(s,t)$

① due to periodic Noise
↳ frequency domain may periodic Noise remove karna easy hota as compared to Spatial Domain

② Time complexity:
↳ $O(n)^2$ → loop wali example:

↳ Smoothing Image

Area ∝ Smoothing of Image



Inverse Transformation:

$$I(r,c) = k' \sum_{r=0}^{1} \sum_{c=0}^{1} T(u,v)\, B(r,c;u,v)$$

→ Base Image

↳ $T(u,v) = \begin{Bmatrix} 11 & 1 \\ 5 & 3 \end{Bmatrix}$

$T(0,0) = 20$ $T(1,1) = 8$
$T(0,1) = 12$
$T(1,0) = 4$

} from Mapping

$Tr(x,c) = \begin{bmatrix} 20 & 12 \\ 4 & 8 \end{bmatrix}$ But should come back this $= \begin{Bmatrix} 5 & 3 \\ 1 & 2 \end{Bmatrix}$

$I(x,c) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \xrightarrow{\text{Apply Transformation}} \boxed{\begin{bmatrix} 11 & 1 \\ 5 & 3 \end{bmatrix}}$

$I'(x,c) : \begin{bmatrix} 11 & 1 \\ 5 & 3 \end{bmatrix} \xrightarrow{\text{Apply Inverse Transformation}} \begin{Bmatrix} 20 & 1 \\ 5 & 3 \end{Bmatrix}$

Anna Tou ye Chary tha.

↳ Solution to get correction answer:

↳ i) $k' = 1/4$ $\quad k = 1$

↳ ii) $k' = 1$ $\quad k = 1/4$

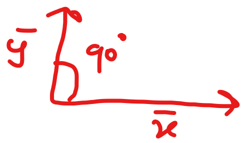
iii) $k = k' = 1/2$

In order to get correct response your base Image should have these properties:

↳ i) Orthogonal frequency:

↳ When two vector are 90° to each other

For example

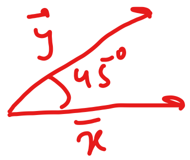


$\vec{x}$ and $\vec{y}$ are $\perp$ to each other

↳ projection should be zero:

for example

→ Projection of $\vec{y}$ on $\vec{x}$ is zero



→ Projection of $\vec{y}$ on $\vec{x}$ is something

Question:

When two vector are 90° to each other??

When

↳ i) $|f_1||f_2| \cos\theta = 0$ →

↳ ii) $x_1 x_2 + y_1 y_2 = 0$ (Mapping with other)

↳ we will use

example: (0,0) (0,1)

$$= \left\{ \begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \quad \begin{bmatrix} +1 & -1 \\ +1 & -1 \end{bmatrix} \quad \begin{bmatrix} +1 & +1 \\ -1 & -1 \end{bmatrix}_{(1,0)} \quad \begin{bmatrix} +1 & -1 \\ -1 & -1 \end{bmatrix}_{(1,1)} \right\}$$

$$\begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \quad \begin{bmatrix} +1 & -1 \\ +1 & -1 \end{bmatrix}$$

$x_1 y_1 + x_2 y_2 \cdots$

$= (+1)(+1) + (+1)(-1) + (+1)(+1) + (+1)(-1)$

$= 1 + (-1) + (+1)(-1)$

$= 1 - 1 + 1 - 1$

$= 0$

$$\left.\begin{array}{l} (0,0) \cdot (0,1) = 0 \\ (0,0) \cdot (1,0) = 0 \\ (0,0) \cdot (1,1) = 0 \end{array}\right\} \text{should be}$$

→ If dot product to each other zero then the base Image is orthogonal Image.

-> other meaning: Mean that projection to each other is zero.

→ frequency should be choose unique and do not share anything (magnitude) thence giving unique coefficient.

② orthonormal Property:

↳ frequency should be orthonormal and plus have magnitude equal to one

for example:

$\vec{A} = [4, 5]$

$|\vec{A}| = \sqrt{(4)^2 + (5)^2}$

$|\vec{A}| = \sqrt{16 + 25}$

$|\vec{A}| = \sqrt{41}$ #magnitude

for 2D

$M_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$|M_1| = \sqrt{(1)^2 + (1)^2 + (1)^2 + (1)^2}$

$|M_1| = \sqrt{1+1+1+1} = \sqrt{4} = 2 \neq 1$

Meth:

When we choose $k = 1/2$ and the normalize this M_i by Multiply by $1/2$

$k_1 = 1/2$

Normalize

↳ Question:
Create a basic Image/filter how Magnitude is one??

$$M_1 = \left\{ \begin{bmatrix} +1/2 & +1/2 \\ +1/2 & +1/2 \end{bmatrix} \quad \begin{bmatrix} +1/2 & -1/2 \\ +1/2 & -1/2 \end{bmatrix} \quad \begin{bmatrix} +1/2 & +1/2 \\ -1/2 & -1/2 \end{bmatrix} \quad \begin{bmatrix} +1/2 & -1/2 \\ -1/2 & +1/2 \end{bmatrix} \right\}$$

$$M_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

→ Both orthogonal plus orthonormal

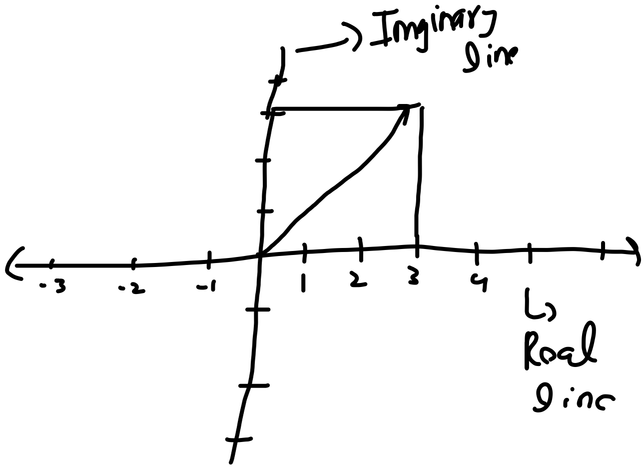
↳ use those filter you will get the same Image After the Inverse transformation.

↳ fsc Concept:

→ Rectangle & polar co-ordinate:

↳ Rectangular

→



↳ $\underbrace{Re + j\,Img}$

↳ Rectangular co-ordinate

② Polar Co-ordinate System:

↳ Magnitude → $M e^{j\theta}$

↳ Angle/direction

↳ finding Magnitude

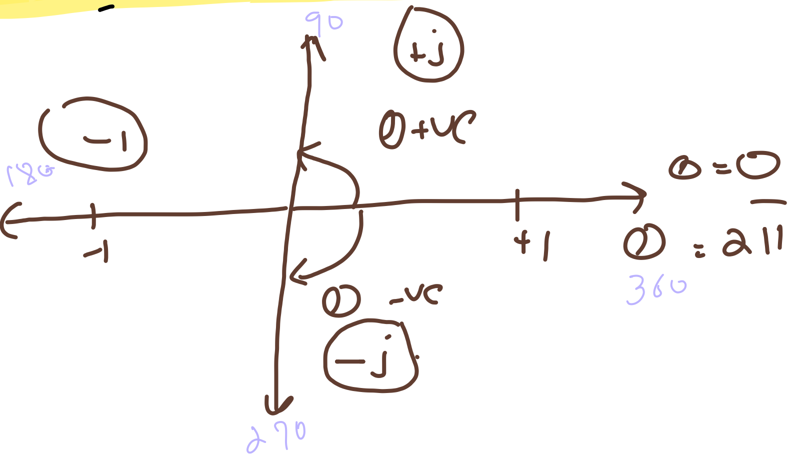
$|M| = \sqrt{Re^2 + Img^2}$ # Magnitude

$\theta = \tan^{-1}\left|\frac{Img}{Re}\right|$ # direction

Re = line Real (x)

Img = (y)

Calculation of $e^{j\theta}$



for example : Compute DFT(discreate fourier transformation) for this 1 D image

$I(c) = [3, 2, 2, 5]$ $c =$ $u =$

$$f(u) = \frac{1}{N} \sum_{c=0}^{N-1} I(c)\, e^{\frac{-2\pi j c u}{N}}$$

→ her u is zero

$$f(0) = \frac{1}{4}\left[3(1) + 2(1) + 2(1) + 5(1)\right] = \quad e^{-2\pi(0)}$$

→ $2(\ )$ ↳ $-2\pi\,(j=0)(c=?)$ ↓ J is not a value

$$f(0) = \frac{3+2+2+5}{4} = \frac{12}{4} = 3$$

$$f(1) = \frac{1}{4}\left[3(j) + 2(+1) + 2(-1) + 5(+1)\right]$$

$$= \frac{1}{4}\left[3(j) - 2j - 2 + 5\right]$$

$$= \frac{1}{4}\left[\right.$$

→ $3(1)$ ↳ $3\, e^{\frac{-2\pi j\,(u=1)\,(c=3)}{N=4}}$ ↳ $3\, e^{\frac{-2\pi j \times 3}{4}}$ ↳ $3\, e^{\frac{-6\pi(j}{4}}$ ↳ $3(-j)$ ↳ $3(-j)$

→ $2(e^{\frac{-2\pi j (1)(1)}{4}}$ ↳ $2(e^{\frac{-4\pi j}{4}}$ ↳ $2(e^{-\pi j})$

→ $2(-j)$ ↳ $2(e^{\frac{-2\pi(1)(2)}{4}})$ ↳ $2(e^{-\pi j})$ ↳ $2(+1)$

example :
↳ compute DFT of the sequence $f(u) = \{1, 0, 0, 1\}$

$$f[k] = \sum_{x=0}^{N-1} f(u)\, e^{-j2\pi k u/N} \qquad k = \{0, 1, 2 \dots N-1\}$$

$N = 4$
↳ lenght of Input

$$F[k] = \sum_{u=0}^{3} f(u)\, e^{-2\pi j k u/4}$$

$$= f(0)\, e^{0} + f(1)\, e^{-2\pi j k(1)/4} + f(2)\, e^{-2\pi j k(2)/4} + f(3)\, e^{-j2\pi k(3)/4}$$

$$= (1)(1) + 0(0) + 0 + (1)\, e^{\frac{-j3\pi k}{2}}$$

$$= 1 + 0 + 0 + e^{\frac{-j3\pi k}{2}} = 1 + e^{-j\frac{3\pi k}{2}}$$

When $K=0$

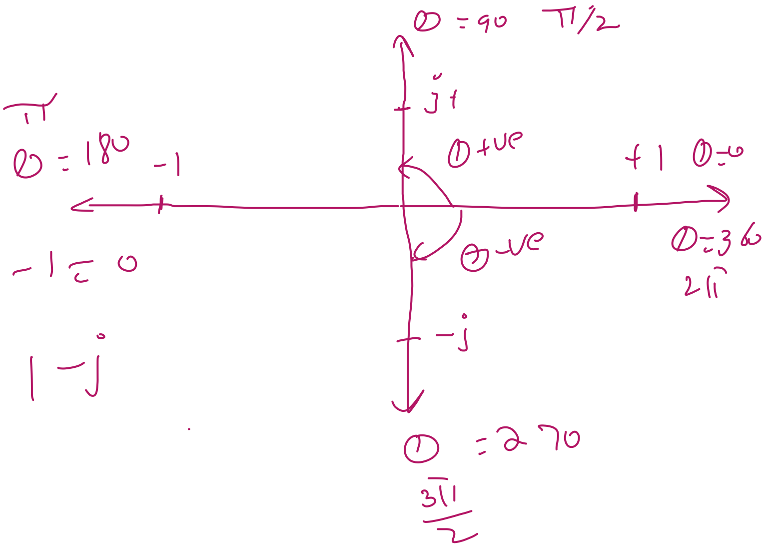
$f[0] = 1 + e^{0} = 1+1 = 2$

$f[1] = 1 + e^{-3\pi(1)/2} = 1+j$

$f[2] = 1 + e^{-3\pi(2)/2} = 1 + e^{-3\pi j} = 1-1 = 0$

$f[3] = 1 + e^{-3\pi(3)/2} = 1 + e^{-9\pi j/2} = 1-j$

$F(k) = \{2, 1+j, 0, 1-j\}$



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↳

$I(c) = [3, 2, 2, 5]$

$f(u) = [3+0j, \frac{1}{4}+\frac{3}{4}j, -\frac{1}{2}+0j, \frac{1}{4}-\frac{3}{4}j]$ → Rectangular cordinate

↳ In polar cordinate

↳ $Me^{j\theta}$

↳ $M = \sqrt{Re^2 + Img^2}$ #Magnitude

Phase $= \theta = Tan^{-1}\left|\frac{Img}{Re}\right|$ #Angle

$F(u) = \frac{1}{N}\sum_{c=0}^{N-1} I(c)\, e^{-2\pi juc/N}$ ↳ for transformation

$F^{-1}(u) = \sum I(c)\, e^{+2\pi juc/N}$ ↳ for Inverse Transformation

F(u) will transformation of Image and $F^{-1}(u)$ will Inverse the transformation for I P Image

Output in Polar co-ordinate

$F[0] = 3+0j$

↳ $M = \sqrt{(3)^2 + (0j)^2}$

$\boxed{M = 3}$

↳ Angle $= \theta = Tan^{-1}\left(\frac{0j}{3}\right) = Tan^{-1}(0/3) = 0$

$F[0] = 3 = 3e^{0}$ → (In polar co-ordinate form)

$F[1] = \frac{1}{4} + \frac{3}{4}j$

↳ $M = \sqrt{(1/4)^2 + (3/4)^2} = \sqrt{\frac{1}{16} + \frac{9}{16}} = \sqrt{\frac{1+9}{16}} = \sqrt{\frac{10}{16}} = \frac{\sqrt{10}}{16} = \frac{\sqrt{10}}{4}$

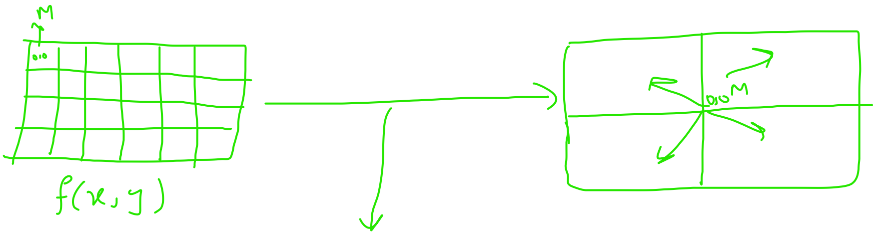
↳ θ = phase $= Tan^{-1}\left(\frac{3/4}{1/4}\right) = 71.56^{\circ}$

$= \frac{\sqrt{10}}{4} e^{71.56}$ → In polar co-ordinate form (same for other f[2], f[3])

Discreate fourier transformation In 2D Image:

↳ What actually happening In transformation??

↳ for example: If we Image and have some object in the Image



→ Amplitude is the strenght of Signal.

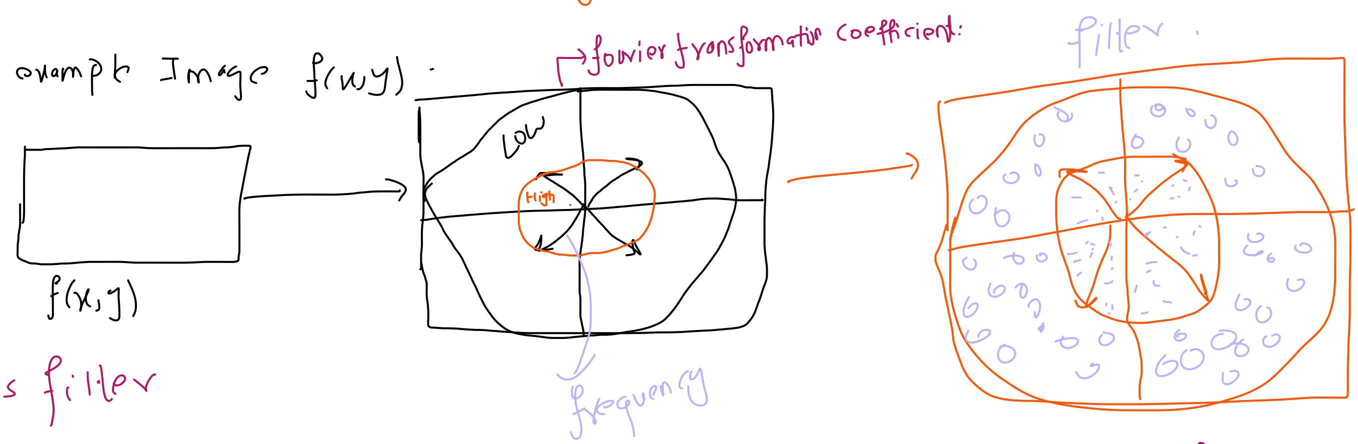
M(0,0) co-ordinate is shifted from M(0,0) from F(x,y) to other location
After shifting the location we process the Image and then Apply the Inverse transformation.

→ Two Types of filter:

↳ Low pass filter → Low pass filter will allow the Low frequency shades pixel and block other (used for smoothing the Image)

↳ High pass filter → High pass filter will allow the High frequency pixel/shade and block other from the Images. (used for sharpning the filter)

for example Image f(x,y).



↳ Lowpass filter

$$H(U,V) = \begin{cases} 0 & ; \; D(U,V) \geq D_0 \\ 1 & ; \; D(U,V) < D_0 \end{cases}$$

D_0 is the threshold

↳ $0 \leq D_0 \leq N/2$ ← for 1D

↳ $0 \leq D_0 \leq \frac{\min(N,M)}{2}$ ← for 2D Image

$$D(U,V) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$$

How to select D_0??

↳ Same as you selected in spaital Domain like

↳ $K = 2m - 1$ → for spaital Domain