Frequency Domain filtering:

↳ $I(r,t)=\begin{bmatrix}5 & 3\\ 1 & 2\end{bmatrix}$

$$B(r,c,u,v)=\left\{\begin{bmatrix}+1 & +1\\ +1 & +1\end{bmatrix}\ \begin{bmatrix}+1 & -1\\ +1 & -1\end{bmatrix}\ \begin{bmatrix}+1 & +1\\ -1 & -1\end{bmatrix}\ \begin{bmatrix}+1 & -1\\ -1 & +1\end{bmatrix}\right\}$$

$T(0,0)=11$
$T(0,1)=1$
$T(1,0)=5$
$T(1,1)=3$

Transformation of Image:

$$T(u,v)=\frac{1}{M\times N}\sum_{c=0}^{N-1}\sum_{r=0}^{M-1} f(c,r)\,\underbrace{e^{-j2\pi\left(\frac{cu+rv}{M\times N}\right)}}_{\text{Base frequency}}$$

Base frequency
Base Image frequency

$$T(u,v)=\begin{bmatrix}11 & 1\\ 5 & 3\end{bmatrix}$$

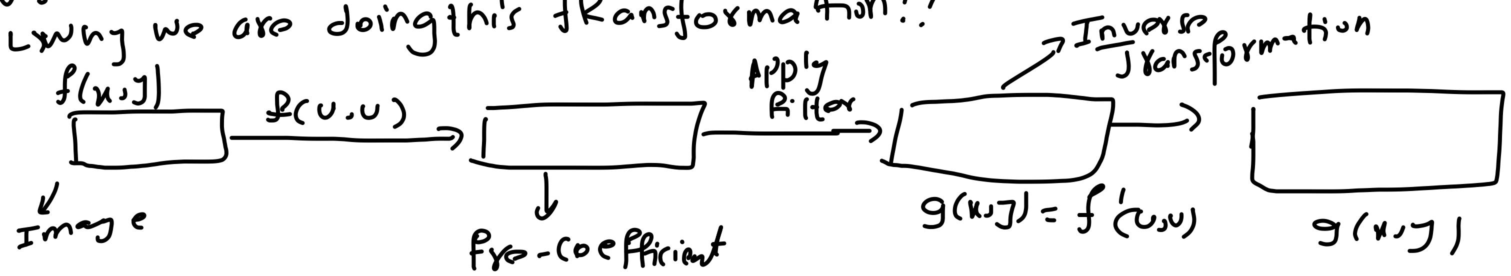
$$T(u,v)=k\sum_{r=0}^{1}\sum_{c=0}^{1} I(r,c)\,B(r,c;u,v)$$

k ↓ constant

→ ↳ Euler formula:

$$e^{j\theta}=\cos\theta+j\sin\theta$$

Question
↳ why we are doing this transformation??



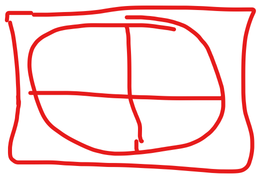
↳ $f'(u,v)=f(u,v)*w(s,t)$

1. due to periodic Noise
   ↳ frequency domain may periodic Noise remove karna easy hota as compared to Spatial Domain

2. Time complexity:
   ↳ $O(n)^2$ → loop wali example:

↳ Smoothing Image

Area ∝ Smoothing of Image



Inverse Transformation:

$$I(r,c)=k'\sum_{r=0}^{1}\sum_{c=0}^{1} T(u,v)\,B(r,c;u,v)$$

→ Base Image

↳ $T(u,v)=\begin{Bmatrix}11 & 1\\ 5 & 3\end{Bmatrix}$

$T(0,0)=20$
$T(0,1)=12$
$T(1,0)=4$
$T(1,1)=8$

} from Mapping

$Tr(x,c) = \begin{bmatrix} 20 & 12 \\ 4 & 8 \end{bmatrix}$ But should come back this $= \begin{Bmatrix} 5 & 3 \\ 1 & 2 \end{Bmatrix}$

$I(x,c) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \xrightarrow{\text{Apply Transformation}} \boxed{\begin{bmatrix} 11 & 1 \\ 5 & 3 \end{bmatrix}}$

$I'(x,c) : \begin{bmatrix} 11 & 1 \\ 5 & 3 \end{bmatrix} \xrightarrow{\text{Apply Inverse Transformation}} \begin{bmatrix} 20 & 1 \\ 5 & 3 \end{bmatrix}$

Anna Tou ye Chaay tha.

↳ Solution to get correction answer:

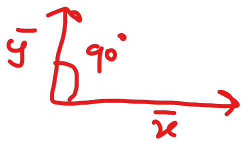
- ↳ i) $k' = 1/4$ $\quad k = 1$
- ↳ ii) $k' = 1$ $\quad k = 1/4$
- iii) $k = k' = 1/2$

In order to get correct response your base Image should have these properties:

↳ i) Orthogonal frequency:

↳ When two vector are 90° to each other

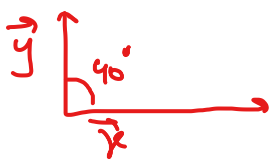
For example



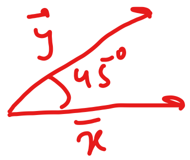
$\vec{x}$ and $\vec{y}$ are ⊥ to each other

↳ projection should be zero:

for example



→ Projection of $\vec{y}$ on $\vec{x}$ is zero



→ Projection of $\vec{y}$ on $\vec{x}$ is something

Question:

When two Vector are 90° to each other??

When

↳ i) $|P_1||P_2| \cos\theta = 0$ →

↳ ii) $x_1 x_2 + y_1 y_2 = 0$ (Mapping with other)

↳ we will use

example: (0,0) (0,1)

$$= \left\{ \begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \quad \begin{bmatrix} +1 & -1 \\ +1 & -1 \end{bmatrix} \quad \underset{(1,0)}{\begin{bmatrix} +1 & +1 \\ -1 & -1 \end{bmatrix}} \quad \underset{(1,1)}{\begin{bmatrix} +1 & -1 \\ -1 & -1 \end{bmatrix}} \right\}$$

$$\begin{bmatrix} +1 & +1 \\ +1 & +1 \end{bmatrix} \quad \begin{bmatrix} +1 & -1 \\ +1 & -1 \end{bmatrix}$$

$x_1 x_1 + y_1 y_2 \cdots\cdots$

$= (+1)(+1) + (+1)(-1) + (+1)(+1)\ (+1)(-1)$

$= 1 + (-1) + (+1)(-1)$

$= 1 - 1 + 1 - 1$

$= 0$

(0,0) . (0,1) = 0
(0,0) . (1,0) = 0 } should be
(0,0) . (1,1) = 0

→ If dot product to each other zero then the base Image is orthogonal Image.

-> other meaning: Mean that projection to each other is zero.
→ frequency should be choose unique and do not share anything (magnitude) thence giving unique coefficient.

② orthonormal Property:

↳ frequency should be orthonormal and plus have magnitude equal to one

for example:

$\vec{A} = [4,5]$

$|\vec{A}| = \sqrt{(4)^2 + (5)^2}$

$|\vec{A}| = \sqrt{16+25}$

$|\vec{A}| = \sqrt{41}$ #magnitude

for 2D

$M_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$|M_1| = \sqrt{(1)^2 + (1)^2 + (1)^2 + (1)^2}$

$|M_1| = \sqrt{1+1+1+1} = \sqrt{4} = 2 \neq 1$

Meth:

When we choose $k = 1/2$ and the normalize this M_i by Multiply by 1/2

$k_1 = 1/2$

Normalize

↳ Question:

Create a basic Image/filter how Magnitude is one??

$$M_1 = \left\{ \begin{bmatrix} +1/2 & +1/2 \\ +1/2 & +1/2 \end{bmatrix} \begin{bmatrix} +1/2 & -1/2 \\ +1/2 & -1/2 \end{bmatrix} \begin{bmatrix} +1/2 & +1/2 \\ -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} +1/2 & -1/2 \\ -1/2 & +1/2 \end{bmatrix} \right\}$$

$$M_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

→ Both orthogonal plus orthonormal

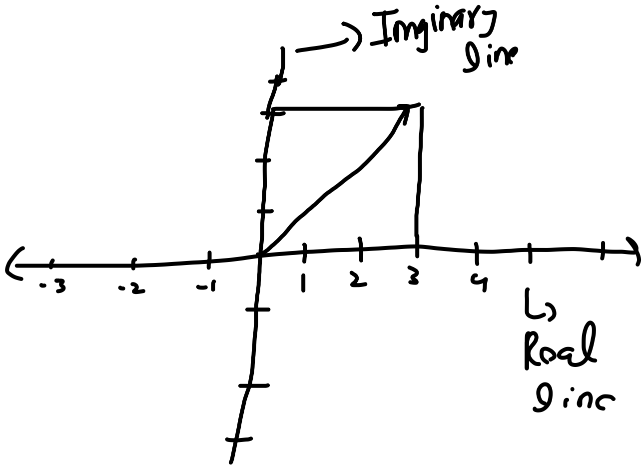
↳ use those filter you will get the same Image After the Inverse transformation.

↳ fsc concept:

→ Rectangle & polar co-ordinate:

↳ Rectangular

→



↳ Re + j Img

↳ Rectangular co-ordinate

② Polar Co-ordinate System:

↳ Magnitude → $M e^{j\theta}$

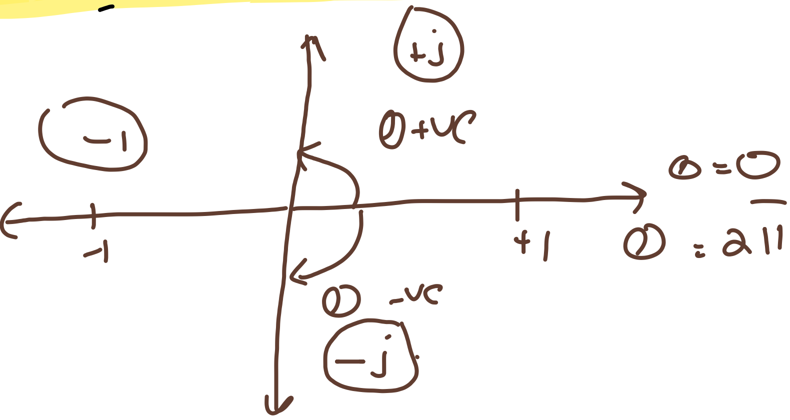
↳ Angle/direction

↳ finding Magnitude

$|M| = \sqrt{Re^2 + Img^2}$ # Magnitude

$\theta = \tan^{-1}\left|\dfrac{Img}{Re}\right|$ $Re =$ line Real (x) $Img =$ (y) # direction

Calculation of $e^{j\theta}$



for example :

$I(c) = [3, 2, 2, 5]$

$c =$

$u =$

$$f(u) = \frac{1}{N} \sum_{c=0}^{N-1} I(c)\, e^{\frac{-2\pi j c u}{N}}$$

$$f(0) = \frac{1}{4} [\ 3(\qquad\qquad] = \quad e^{-2\pi(0)}$$