Abstract Semantic Differencing for Numerical Programs

Abstract

We address the problem of computing semantic differences between a program and a patched version of the program. Our goal is to obtain a precise characterization of the difference between program versions, or establish their equivalence when no difference exists.

We focus on computing semantic differences in numerical programs where the values of variables have no a-priori bounds, and use abstract interpretation to compute an over-approximation of program differences. Computing differences and establishing equivalence under abstraction requires abstracting relationships between variables in the original program and its patched version. Towards that end, we first construct a *union program* in which these relationships can be identified, and then use a *correlating abstract domain* to compute a sound approximation of these relationships. To establish equivalence between correlated variables and precisely capture differences, our domain has to represent non-convex information. To balance precision and cost of this representation, our domain may over-approximate numerical information as long as equivalence between correlated variables is preserved.

We have implemented our approach in a tool built on the LLVM compiler infrastructure and the APRON numerical abstract domain library, and applied it to a number of challenging real-world examples, including programs from the GNU core utilities, **Mozilla Firefox and the Linux Kernel** . We evaluate over 50 patches and show that for these programs, the tool often manages to establish equivalence, reports useful approximation of semantic differences when differences exists, and reports only a few false differences.

1. Introduction

When applying a patch to a procedure, the programmer has very limited means for acquiring a description of the change the patch made to the procedure behavior.

Given a program P and a patched version of the program P' our goal is to determine the difference between ⟦P ⟧ and ⟦P' ⟧.

Our goal is to describe semantic difference between two programs (program versions) P, P' at certain program points i.e. given two programs and two locations in the programs, we want to produce all program states that are possible at the given location in one program but do not exist in the other program and vice versa.

Existing Techniques Existing techniques for proving patch equivalence will only supply the programmer with a binary answer [4] as to the (input-output) equivalence a program but no description of the difference is supplied. Further work [5] allows refining the equivalence proof by providing a set of constraints under which equivalence is desired but requires the programmer to manually deduce these. Other techniques for describing difference [**?** **?**] which rely on symbolic execution supply unsound results as they are limited by loops and essentially cover a subset of program behavior. We present a novel approach which allows for a sound description of difference for programs with loops. Our technique employs methods of abstract interpretation for over-approximating the difference in behaviors, by focusing the abstraction on the *relationships* between program behaviors i.e. between variables values (data) and conditionals (path) in the two versions.

In contrast to existing techniques, our approach allows checking for equivalence in every point of execution, and for every variable, while previous approaches focus only on input-output equivalence. This enables detection of key differences that impact the correctness of the patch: if the changed behavior includes a bug manifested by a local variable (for instance: array index out of bounds), we will detect and describe it while previous work only detected it when propagated to the output and equivalence may have been reported although a bug was introduced. This also provides a challenge as we need to carefully choose the program locations where we check for difference otherwise we will spuriously detect difference.

Correlating Program Abstracting relationships allows us to maintain focus on difference while omitting (whenever necessary for scalability) parts of the behavior that does not entail difference. In order to monitor these relationships we created a *correlating program* which captures the behavior of both the original program and its patched version. Instead of designing a correlating semantics that is capable of co-executing two programs, we chose to automatically construct the correlating program such that we can benefit from the use of standard analysis frameworks for analyzing the resulting program. Another advantage of this new construct, is that you may apply other methods for equivalence checking directly on it [8] as the correlation allows for a much more fine-grained equivalence checking (between local variables and not only output).

Correlating Abstractions Our abstraction holds data of both sets of variables, joined together and is initialized to hold equality over all matched variables. This means we can reflect relationships without necessarily knowing the actual value of a variables (we can know that $x_{old} = x_{new}$ even though actual values are unknown). We ran out analysis over the correlating program while updated the domain to reflect program behavior.

To establish equivalence between correlated variables and precisely capture differences, our domain has to maintain correlating information even when other information is abstracted away.

Since some updates may result in non-convex information (e.g. taking a condition of the form $x \neq 0$ into account), our domain has to represent non-convex information, at least temporarily. We address this by working with a powerset domain of a convex representation. To avoid exponential blowup, our join operator may over-approximate numerical information as long as equivalence between correlated variables is preserved.

In some cases, it would have been sufficient to use alternative domains that are capable of representing richer information, such as interval polyhedra [3], or other numerical domains that can represent non-convex information (e.g., [**?**]). The recent donut domain [**?**] may be of particular interest for this purpose. However, the general principle of having to preserve correlating information even when information about the values is abstracted away, holds in all of these cases.

```
int sign(int x) {
  int sgn;
  if (x < 0)
    sgn = -1
  else
    sgn = 1
 return sgn
}
```

```
int sign'(int x) {
  int sgn;
  if (x < 0)
    sgn = -1
  else if (x==0)
    sgn = 0
  else
    sgn = 1
 return sgn
}
```

Figure 1: Two simple implementations of the *sign* operation.

In this paper, we present a technique based on abstract interpretation, that is able to compute an over-approximation of the difference between numerical programs or establish their equivalence when no difference exits. The approach is based on two key ideas: (i) create a *union program* that captures the behavior of both the original program and its patched version; (ii) analyze the union program with a *correlating domain* that captures relationships between values of variables in the original program and values of variables in the patched version.

The idea of a correlating program is similar to that of self-composition [2, 10], but the way in which statements in the union program are combined is carefully designed to keep the steps of the two programs close to each other. Rather than having the patched program sequentially composed after the original program, our union program interleaves the two versions. Analysis of the union program can then recover equivalence between values of correlated variables even when equivalence is *temporarily* violated by an update in one version, as the corresponding update in the other version follows shortly thereafter.

check whether Aiken paper SAS'05 does some sort of interleaving as well

also need to say that there is the problem of choosing differencing points

1.1 Main Contributions

The main contributions of this paper are as follows:

- we phrase the problem of semantic differential analysis as an analysis of a union program — a single program that represents an original program and its patched version.
- we present an approach for analyzing differences over the union program using a correlating abstract domain. Our approach is sound — if there is a difference at a differentiation point, we cannot miss it. However, since we over-approximate differences, our approach may report false differences due to approximation.
- We have implemented our approach in a tool based on the LLVM compiler infrastructure and the APRON numerical abstract domain library, and evaluated it using over 50 patches from open-source software including GNU core utilities, Mozilla Firefox, and the Linux Kernel. Our evaluation shows that the tool often manages to establish equivalence, reports useful approximation of semantic differences when differences exists, and reports only a few false differences.

mention classical work like [? ?]

2. Overview

In this section, we informally describe our approach with a few simple example programs.

```
int sign(int x) {
  int x' = x;
  Guard G1 = (x < 0);
  Guard G1' = (x' < 0);
  int sgn;
  int sgn';
  if (G1) sgn = -1;
  if (G1') sgn' = -1;
  Guard G2' = (x' == 0);
  if (!G1') if (G2') sgn' = 0;
  if (!G1') if (!G2') sgn' = 1;
  if (!G1) sgn = 1;
}
```

Figure 2: Correlating program $sign \bowtie sign'$.

2.1 Motivating Example

Consider the simple example program of Fig. 1, inspired by an example from [9]. For this example, we would like to establish that $sign$ and $sign'$ only differ in the case where $x = 0$.

As a first naive attempt one could try to analyze each version of the program separately and compare the (abstract) results. However, this is clearly unsound, as equivalence under abstraction does not entail concrete equivalence. For example, using a interval analysis [**?**] would yield that in both programs the value of `sgn` ranges in the same interval $[-1, 1]$, missing the fact that $sign$ never returns the value 0.

Establishing equivalence under abstraction To establish equivalence under abstraction, we need to abstract relationships between the values of variables in $sign$ and $sign'$. Specifically, we need to track the relationship between the values of `sgn` in both versions and see whether we can establish their equivalence.

We reduce the problem of analysis across two programs to the problem of analyzing a single program by constructing a *correlating program*. The correlating program represents the behaviors of both programs, and allows us to reason about relationships between variables in both.

Fig. 2 shows the correlating program for the programs of Fig. 1. Using the correlating program, we can directly track the relationship between `sgn` in $sign$ and its corresponding variable `sgn'` in $sign'$. Unfortunately, any domain with convex constraints will still fail to capture the precise relationship between `sgn` and `sgn'`. For example, using the polyhedra abstract domain [**?**], the relationship between `sgn` and `sgn'` in the correlating program would be $\langle \mathbf{constraint - here} \rangle$.

An obvious, but prohibitively expensive, solution to the problem is to use disjunctive completion [**?**], moving to a powerset domain in which every abstract state is a set of convex objects (e.g., set of polyhedra). However, using such domain would significantly limit the applicability of the approach.

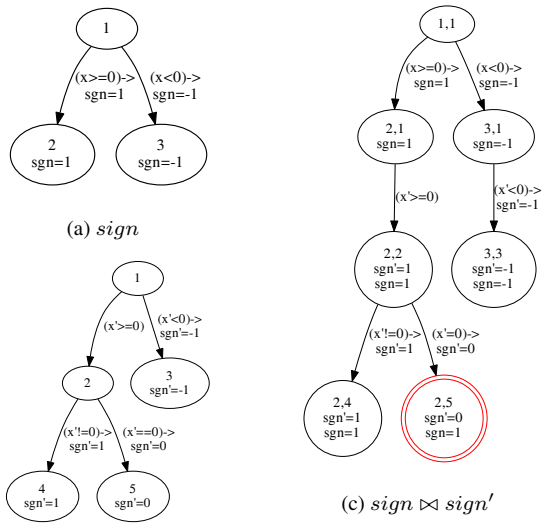
The desirable solution is a partially disjunctive domain, in which only certain disjunctions are kept separate during the analysis, while others are merged. The challenge in our setting is in keeping the partition fine enough such that equivalence could be preserved, without reaching exponential blowup.

How to partition We base our choice of explicit disjunctions to be kept based on their effect on variable equivalence.

Widening

3. Preliminaries

We use the following standard concrete semantics definitions for a program:



(a) $sign$

(b) $sign'$

(c) $sign \bowtie sign'$

Figure 3: Transition systems for the programs of Fig. 1 and Fig. 2.

Program A program P,...We are able to convert any program to this format...We exclude recursion for now.

Program Location A program location $loc \in Loc$,

Program Label A program label $lab \in Lab$, is a unique identifier for a certain location in a program. Every location has a label (usually the program counter). We also define two special labels for the start and exit locations of the program as $begin$ and fin respectively.

Concrete State A concrete program state is a tuple $\sigma \equiv \langle loc, values \rangle \in \Sigma$ mapping the set of (integer) program variables to their concrete (integer) value at a certain program location loc i.e. $values : Var \rightarrow Val$. The set of all possible states of a program P is denoted $[\![P]\!]$.

Concrete Trace A program trace $\pi \in \Sigma^*$, is a sequence of states $\sigma_0, \sigma_1, ...$ describing a single execution of the program. Each of the states corresponds to a certain location in the program where the trace originated from. Every program can be described by the set of all possible traces for its run $\Pi \subseteq \Sigma^*$. We refer to these semantics as concrete state semantics. We also define the following standard operations on traces:

- $label : [\![P]\!] \rightarrow Lab$ maps a state to the program label at which it appears.
- $last : [\![P]\!]^* \rightarrow [\![P]\!]$ returns the last state in a trace.
- $pre : \Pi \rightarrow 2^{[\![P]\!]^*}$ for a trace π is the set of all prefixes of π.

4. Concrete Semantics Delta

In this section we will define the notion of difference between concrete semantics of two programs based on a standard concrete semantics for a program.

4.1 Concrete State Differencing

Comparing two different programs P and P' under our concrete semantics means comparing *traces*. A trace is composed of concrete states thus we first need a way to compare states. Since a state σ is a mapping $Var \rightarrow Val$ we first need a correspondence between variables in P (i.e. Var) and those in P' (i.e. Var').

Variable Correspondence A variable correspondence $VC \in Var \times Var'$, is a partial mapping between 2 sets of program variables. Any variable in Var may be matched with any in Var' and vice versa. Naturally, the comparison between states will occur between values mapped to corresponding variables, as described by VC.

In our analysis, the correspondence can be determined by the user but our experience suggests that in most cases the set of variables stays the same ($Var = Var'$) over subsequent versions and in cases where Var does change, it's by the addition of new variables or removal of an old one. Therefore we concluded that for our purposes, matching variables by name is sufficient for finding a precise difference. Thus we define our standard correlation to be: $VC_{EQ} \triangleq \{(v, v') | v \in Var \wedge v' \in Var' \wedge name(v) = name(v')\}$ and vice versa.

Concrete State Delta Given two concrete states $\sigma \in \Sigma_P, \sigma' \in \Sigma_{P'}$ and a variable correspondence VC, we define the concrete state delta $\triangle_S(\sigma, \sigma')$ as the part of the state σ where corresponding variables do not agree on values (with respect to σ'). Formally: $\triangle_S(\sigma, \sigma') \triangleq \{(var, val) | (var, var') \in VC \wedge \sigma(var) = val \neq \sigma'(var')\}$. In case there is no observable difference in state we get that $\triangle_S(\sigma, \sigma') = \emptyset$. As state delta is directional, we notice that unless it is empty, $\triangle_S$ is not symmetric and we use the notation $\triangle_S^-$ for $\triangle_S(\sigma, \sigma')$ and $\triangle_S^+$ for $\triangle_S(\sigma', \sigma)$.

For instance, given two states $\sigma : \{x = 1, y = 2, z = 3\}$ and $\sigma' : \{x' = 0, y' = 2, w' = 4\}$ and assuming our default VC then $\triangle_S^- = \{x = 1\}$ since x and x' match and do not agree on value, y and y' agree (thus are not in delta) and z' is not in VC_{EQ}. Respectively, $\triangle_S^+ = \{x' = 0\}$.

We defined a notion for difference between states but this is insufficient to describe difference between whole runs of programs i.e. traces. Naturally, we are only interested in traces that originate from **the same input** and every mention of trace differentiation will assume the traces agree on input. The way to differentiate traces is by differentiating their states, but which states? this is not a trivial question since traces can vary in length and order of states. We need a state correspondence for choosing the states to be differentiated within the two traces. We define it as following:

Trace Diff Points Given two traces π and π', we define a trace index correspondence relation named trace diff points denoted DP_π as a matching of indexes specifying states in where concrete state delta should be computed. Formally: $DP_\Pi \equiv \{(i, i') | i \in 0..|\pi|, i' \in 0..|\pi'|\}$. The question of supplying this matching, in a way that results in meaningful delta, is not a trivial one, we delay this discussion until we define the trace delta.

Now that we have a way of matching states to be compared between two traces, we define the notion of trace differentiation:

Trace Delta Given traces π, π' of programs P, P' respectively, and a state correspondence DP_Π we define the trace delta $\triangle_T(\pi, \pi')$ as state differentiations between all corresponding states in π and π'. Formally, for every $(i, i') \in DP$ such that $\triangle_S(\sigma_i, \sigma'_{i'}) \neq \emptyset$, $\triangle_T$ will contain the mapping $i \mapsto \triangle_S(\sigma_i, \sigma'_{i'})$, thus the result will map certain states in π to their state delta with the corresponding state' in π' (deemed interesting by DP_Π). We define $\triangle_T^+$ and $\triangle_T^-$ in a similar way.

One possible choice for DP_Π would be the endpoints of the two traces $\{(fin, fin')\}$ (assuming they are finite) meaning differentiating the final states of the executions or formally: $\triangle_{in}^- \equiv \triangle_{Fin}(\pi, \pi') = \{fin \mapsto \triangle_S(\sigma_{fin}, \sigma'_{fin'})\}$ (we will also be interested in $\triangle_{Fin}^+$). It is clear that the delta is not sufficient for truly describing the difference between said traces as it will only compare

```
void foo(unsigned x) {
    unsigned i = 0;
lab:guard g = (i >= x);
    if (g) return;
    ...
    i++;
    goto lab;
}
```

```
void foo'(unsigned x) {
    unsigned i = 0;
lab:guard g = (i >= 2*x);
    if (g) return;
    ...
    i++;
    goto lab;
}
```

Figure 4: P, P' differentiation candidates

final state values and will miss out on what happened during the execution. This can be overcome by instrumenting the semantics such that the state will contain all "temporary" values for a variable along with the trace index (program location is not sufficient here as a trace can loop over a certain location) where the values existed by, for instance, adding temporary variables, allowing for a complete differentiation at the end point. This may substantially complicate the selection of VC it will require a matching between all of these temporary, indexed, variables that will somehow produce meaning. Also the number of variables here can range up to the length of the trace.

Defining the DP over any two traces is a daunting task since traces of separate (although similar) programs can vastly differ. If we take a look at two versions of a program depicted in Fig. **??** and the following traces generated from the input $x = 2$: $\pi = \{x = 2, i = 0\}\{i = 1\}\{i = 2\}$ and $\pi' = \{x' = 2, i' = 0\}\{i' = 1\}\{i' = 2\}\{i' = 3\}\{i' = 4\}$ (we omit labels and guard values and only mention parts of the trace where variable values change), we see that even in this simple program, finding a correlation based on traces alone is hard. However, one can get the sense that using program location as a means of correlation, one can produce a meaningful result. For example, if we look at all the possible values for i in label lab and differentiate them (as a set) from the values in the patched version (in the same location), we may get a meaningful result that i in the patched version can range from $x+1$ up to $2x$. We will discuss differentiation os sets of states later on as we describe the collecting semantics.

4.2 Differencing at Program Labels

Trace Delta using Program Label Given two traces π, π' and two program labels l, l' we define a trace delta based on all traces locations (states) that are labeled l, l'. First we define π_l as a subsequence of π where only states that are labeled l were chosen ($\pi'_{l'}$ is defined similarly). Next, we denote $\triangle_L(\pi_l, \pi'_{l'})$ as a means for comparing these sequences. As $\pi_l, \pi'_{l'}$ may vary in length and order, we cannot simply define it as applying $\triangle_S$ on each pair of states in $(\pi_l, \pi'_{l'})$ by order. In fact, $\triangle_L$ can be defined in different way to reflect different concepts of difference, for instance, it can be defined as the differentiating the last states of π_l and $\pi'_{l'}$ (assuming they are both finite) to reflect we are only interested in the final values in that location. We chose to define $\triangle_L$ as the difference between *the set of states* which appear in π_l against the set of those in $\pi'_{l'}$. Formally: $\triangle_{L_{set}}(\pi_l, \pi'_{l'}) \triangleq \{\sigma \in ran(\pi_l) | \neg\exists\sigma' \in ran(\pi'_{l'}) s.t. \triangle_S(\sigma, \sigma') = \emptyset\}$. For example, if we look at Fig. **??**, for π, π' that originate from $x = 2$ then $\triangle_{L_{set}}(\pi_{lab}, \pi'_{lab'}) = \{\}$ and $\triangle_{L_{set}}(\pi'_{lab'}, \pi_{lab}) = \{\{i' = 3\}, \{i' = 4\}\}$. We see that this notion of $\triangle$ indeed captures a useful description of difference.

The problem of choosing DP is now reduced to the matching of labels as the trace indexing correspondence DP_Π defined in Definition 4.1 is no longer needed here. as we need to differentiate sets of states belonging to a certain program label. We require a correspondence of *labels* and therefore we define the label diff points correspondence.

Label Diff Points Given two programs P, P' and their sets of program labels Lab, Lab', we define a label correspondence relation named label diff points denoted DP_{Lab} as a matching of labels between programs. Formally: $DP_{Lab} \equiv \{(l, l') | l \in Lab, l' \in Lab'\}$. From this point on any mention of the diff-points correspondence DP will refer to label diff-points DP_{Lab}. We address the question of selection of DP_{Lab} in a meaningful way in **??**.

Now, we will move past the concrete semantics towards a *collecting semantics* as a step towards abstraction. This is required as it is unfeasible to describe difference based on traces. However, we need to adjust our concrete semantics before we can correctly define this as the collecting semantics based on individual traces **will not allow us to correlate traces that originate from the same input**. This is the first formal indication of how a separate abstraction, that considers each of the programs by itself, cannot succeed.

4.3 Concrete Correlating Semantics

In this section we shortly define, based on previous definitions, the correlating state and trace which bind the executions of both programs, P and P', together and we define the notion of delta there. Essentially, these will be states and traces of the product program $P \times P'$ but only traces that originate from the same input are considered. This will allow us to then define the correlating collecting semantics which is key for successful differencing.

Correlating Concrete State A correlating concrete state $\sigma_\times$ is a pair of concrete states (σ, σ') belonging to a pair of programs P and P'. The set of all possible correlating states is denoted $[\![P \times P']\!]$.

Correlating Concrete Trace A correlating trace $\pi_\times$, is a sequence of correlating states ..., (σ_i, σ'_j), ... describing a dual execution of the two programs where at every point each of the program can perform a single step. The $label_\times$, $last_\times$ and $pre_\times$ operations are defined similarly.

Correlating Collecting Semantics of a Program Label Given a paired label $l_\times = (l, l') \in Lab_\times$ in the product program we define the correlating collecting semantics $CS(l_\times) \subseteq [\![P \times P']\!]$ as all the correlating states which are "possible" at that paired label (i.e. exist in some trace reaching that label), grouped by trace. Formally:

1. $at_\times : (Lab_\times \times \Pi_\times) \rightarrow 2^{[\![P \times P']\!]^*})$ for a given label $l_\times$ and trace $\pi_\times$, is the set of prefixes of $\pi_\times$ that end in a state labeled $l_\times$ formally: $at(l_\times, \pi_\times) \triangleq \{\pi^*_\times | \pi^*_\times \in pre_\times(\pi_\times) \wedge last_\times(\pi^*_\times) = l_\times\}$.
2. $CS(l_\times) \triangleq \{\{last_\times(at_\times(l_\times, \pi_\times))\} | \pi_\times \in \Pi_\times\}$.

As mentioned, distinguishing groups of states that arrived from different traces is imperative here, otherwise we could be comparing states that originate from different inputs.

Now we have a collecting semantics that collects dual states from correlating executions only, and distinguishes them by input.

Collecting Semantics Delta As discussed in **??**, we now calculate difference in locations based on a correspondence of labels DP in P and P'. This matches our collecting semantics perfectly as our collecting semantics is defined over paired labels. We compute the collecting semantics delta by applying delta on each $CS(l_\times)$ for each matched pair of label $(l, l') = l_\times$. This is very similar to the label-based trace delta as now we handle *sets* of correlating states (instead of a series of regular states) and we are encountered with the same difficulty. For example, When we compute the collecting semantics for label (lab, lab') in the example from Fig. **??**, we are resulted in $CS(lab, lab') = \{\{< (x = 0, i = 0), (x' = 0, i' = 0) >\}, \{< (x = 1, i = 0), (x' = 1, i' = 0) >, < (x = 1, i = 1), (x' = 1, i' = 0) >, < (x = 1, i = 0), (x' = 1, i' = 1) >, < (x = 1, i = 1), (x' = 1, i' = 1) >, < (x = 1, i = 0), (x' =$

$1, i' = 2) >, < (x = 1, i = 1), (x' = 1, i' = 2) >\}, ...\}$ which indeed groups together all possible states at the desired label according to originating input, but we are still left with the question of how do we calculate delta on each of these groups (for instance the group originating from input $x = 1$). Again, this decision is determined by the kind of difference we want to capture. As before, we choose to define differentiation according to sets of states: we will divide each group of dou-states into two sets containing P and P' states, and try to match each of the P states with a single P' state (using VC of course) and vice versa. Formally:

- for each group of correlating states $\Sigma^i_\times \in CS(l_\times)$ (which originated from the same input) we break it into $\Sigma^i = \{\sigma = first(\sigma_\times) | \sigma_\times \in \Sigma^i_\times\}$ and $\Sigma'^i = \{\sigma' = second(\sigma_\times) | \sigma_\times \in \Sigma^i_\times\}$.
- as $\triangle_{CS}$ is directional as well, we denote $\triangle^+_{CS}(l_\times)$ as states of P' at label l that do not exist in P at label P' (grouped by originating input) i.e. added states and formally: $\triangle^+_{CS} \triangleq \{\Sigma'^i \setminus \Sigma^i | \Sigma^i_\times \in CS(l_\times)\}$. We take notice that the $\setminus$ operation uses VC for comparison.
- Similarly: $\triangle^-_{CS} = \{\Sigma^i \setminus \Sigma'^i | \Sigma^i_\times \in CS(l_\times)\}$ for "removed" states.

In the previous example $CS(lab, lab')$, the $x = 1$ group will be broken into $\Sigma = \{(x = 1, i = 0), (x = 1, i = 1)\}$ and $\Sigma = \{(x' = 1, i' = 0), (x' = 1, i' = 1), (x' = 1, i' = 2)\}$ which will then be compared to produce $\triangle^+_{CS}[x = 1] = \{(x' = 1, i' = 2)\}$ and $\triangle^-_{CS}[x = 1] = \emptyset$ which successfully describes the difference here as in P' a new state is added.

We must remember however, that the sets of states to be compared are potentially unbounded which means that the delta we compute may be unbounded too. Therefore we must use an abstraction over the collecting semantics that will allow us to represent the collecting semantics in a bounded way.

5. Abstract Correlating Semantics

In this section, we introduce our correlating abstract domain which allows bounded representation of product program state while focusing on maintaining equivalence between correlated variables. This comes at the cost of an acceptable lose of precision of other state information. We represent variable information using standard relational abstract domain. As our analysis is path sensitive, we allow for a set of abstract sub-states, each adhering to a certain path in the product program. To assure we only allow correct paths, that truly abstracts our collecting semantics of correlated executions, our abstraction will initially assume equality on all inputs. This power-set domain records precise state information but does not scale due to exponential blow-up of number of paths. We describe minimization techniques, or canonization, that reduce the super-state size by joining part of its sub-states together (using the sub-domain lossy join operation). We select these sub-states according to criteria that allows separation of equivalence preserving paths thus achieving better precision. We start off by abstracting the collecting semantics in **??**.

In the following, we assume an abstract relational domain $RD = \langle RC, \sqsubseteq_{RD} \rangle$ equipped with operations $\sqcap_{RD}$, $\sqcup_{RD}$ and ∇_{RD}, where RC is a set of relational constraints over the variables in Var, and do not go into further details about the particular abstract domain as it is a parameter of the analysis. We also assume that the sub-domain RD allows for a sound over-approximation of the concrete collecting semantics (given a sound interpretation of program operations).

Correlating Abstract State A correlating abstract program state $\sigma^\natural_\times$, is a tuple $\langle l_\times, S^{RD} \rangle \in \Sigma^\natural$, where $l_\times$ is a label in the product program and S_{RD} is a set of abstracts from the sub-domain RD, such that each $rd \in S_{RD}$ will abstract a certain **path** in the product program. We achieve that by instrumenting each $\sigma_\times = last(pre(\pi_\times))$ in the collecting semantics $CS(l_\times)$ with the sequence of branches taken up to $\sigma_\times$ in $pre(\pi_\times)$. This is easily achieved by using *guard* values as they are recorded along the trace and we denote $path(\sigma_\times)$ as the sequence of branches $b_0, ..., b_n$ i.e. guard values ($b_i =< g, T/F >$), that led up to $\sigma_\times$. Therefore we define for every path p_i in the product program the group of concrete correlating states that exist at the end of that path denoted $\Sigma^{p_i}_\times \triangleq \{\sigma_\times | path(\sigma_\times) = p_i\}$. Finally we define the abstract state itself as $\sigma^\natural_\times \triangleq rd | rd = ABS_{RD}(\Sigma^{p_i}_\times) \wedge p_i isapathinP \times P)$. For example, the abstraction of Fig. **??** collecting semantics at labels (lab, lab') will first collect together all trace prefixes that iterate the loop 0 times: $\Sigma^{p_0}_\times = \{(x = 0, i = 0, x' = 0, i' = 0), (x = 1, i = 0, x' = 1, i' = 0), (x = 2, i = 0, x' = 2, i' = 0), ...\}$ and abstract them using the relational sub-domain to get $rd_0 = \{x = x', i = i' = 0\}$ which correctly expresses the fact that equivalence is kept in case none of the loops iterate. The next path to be abstracted would be where each loop iterate once i.e. $p_1 =< g, F >, < g', F >$ that also maintains equivalence and will be abstracted as $rd_1 = \{x = x', i = i' = 1, x \geq 1\}$. The next path however, which iterates the P' loop twice but only once for the P loop, $p_2 =< g, F >, < g', F >, < g, T >, < g', F >$ will not maintain equivalence as it will abstract $\Sigma^{p_2}_\times = \{(x = 1, i = 1, x' = 1, i' = 2), (x = 2, i = 2, x' = 2, i' = 2), (x = 3, i = 2, x' = 3, i' = 2), ...\}$ as $rd_2 = \{x = x', 2 \leq i \geq 1, i' = 2\}$. We mention that the abstract rd ignores the separation of concrete states of P and P' and abstracts both variable data together. In fact, the ability to maintain direct relationships between the two sets of variables (and specifically those matched by VC) is crucial for maintaining equivalence.

Every path in the product program will be represented by a single abstract of the sub-domain. As a result, all **traces prefixes** that follow the same path to $l_\times$ will be abstracted into a single sub-state of the underlying domain. This abstraction fits semantics differencing well, as inputs that follow the same path display the same behavior and will usually either keep or break equivalence together, allowing us to separate them from other behaviors (it is possible for a path to display both behaviors as in Fig. **??** and we will discuss how we are able to manipulate the abstract state computed at the end to separate equivalent behaviors from the ones that offend equivalence). Another issue to be addressed is that fact that our state is again potentially unbound as there may be an infinite number of paths in the program (due to loops).

Computing Abstract State with Our Analysis We compute the correlating abstract state at each label $\sigma^\natural_\times$ by using means of abstract interpretation and program analysis. We interpret our correlating program (described in Section 6) which is a semantic preserving reduction over the much more arbitrary product program, using a standard fixed-point analysis algorithm, and apply interpreted operations on the set of sub-states of the current state S_{RD}. Our initial state explicitly assumes equality on all input variables (by adding a $v = v'$ constraint for every input variable v where $v' = VC(v)$) to allow for sound checking of equivalence. This also allows for pruning of unfeasible paths where inputs diverge (though we may still interpret some as feasible due to incompleteness of program operations representation, for example bitwise operations). To adhere to the representation of traces in the abstract state, our join operation is in fact set union, thus indeed every prefix that reaches a certain program label will be awarded its own rd. Let us explicitly define the operations of the domain in our analysis:

- $S^{RD}_1 \sqsubseteq S^{RD}_2 \iff \forall rd_1 \in G_1 \exists rd_2 \in G_2 : rd_1 \sqsubseteq_{RD} rd_2$

```
int f(int x) {    int f'(int x) {
    return x;         return 2*x;
}                 }
```

- $S_1^{RD} \sqcap S_2^{RD} \equiv rd_1 \sqcap_{RD} rc_2 | rd_1 \in G_1 \wedge rd_2 \in G_2$
- $S_1^{RD} \sqcup S_2^{RD} \equiv S_1^{RD} \cup S_2^{RD}$

Next we define two more operations: Canonization (denoted $\odot$) - which allows for minimization of state size according to equivalence criteria and Widening (denoted ∇) - which allows reaching a fixed-point when handling loops.

One advantage of our correlating domain over using two separate domains, is the ability to preserve equivalence in the face of non-linear operations - this argument may be too thin to include .

Canonization We can see from our motivating example that it is not feasible to allow our correlating domain to keep diverging and double in size with every conditional as it will exponentially blow up the analysis run-time and memory. Instead, we employ an equivalence conserving canonization technique such that at every canonization point will partition the sub-states according to the set of variables to which they hold equivalence for.

Widening

5.1 Correlating Abstract State Differencing

Given a state in our correlating domain, we want to determine whether equivalence is kept and if so under which conditions it is kept (for partial equivalence) or determine there is difference and characterize, as much as possible, this difference. As our state may hold several sub-states, each holding different equivalence data, we can provide a much more verbose answer regarding whether equivalence holds. We partition our sub-states according to the set of variables they hold equivalence for and report the state for each equivalence partition class. Since we instrument our correlating program to preserve initial input values, for some of these states we will also be able to report input constraints thus informing the user of the input ranges that maintain equivalence. In the cases where equivalence could not be proved, we report the offending states and apply a differencing algorithm for extraction of the delta. Fig. **??** shows an example of where our analysis is unable to prove equivalence (as it is sound), although part of the state does maintain equivalence (specifically for $x = 0$). This is due to the abstraction being to coarse. We describe an algorithm that given a sub-state $rd \in RD$, separates the sub-state into parts according to equivalence by treating the relational constraints in our domain as geometrical objects and formulating delta based on that.

Correlating Abstract State Delta Given two abstract sub-states and a correspondence VC, the correlating state delta $\triangle_A(rd)$, computes abstract state differentiations between $\sigma^\natural$ and $\sigma'^\natural$. The result is an abstract state $\sqsubseteq \sigma^\natural$ approximating all concrete values possible in P but not in P' (regarding variables that match in VC). Formally, the delta is simply $\sigma^\natural \setminus \sigma'^\natural$ but since this concept is vague to the reader and furthermore, does not exist in most domain implementation (and specifically in the ones we used) we break it down to a simpler multi-step operation as following:

1. $U \equiv \sigma^\natural \sqcap \sigma'^\natural$ is the joint state of the original and patched program. No precision is lost while joining states as they operate on different variables. This state, as well as the ability to cleanly separate variables, is achieved by symbolically executing a *union program* as defined in Section **??**.
2. R is a state abstracting the concrete states shared by the original and patched program. It is achieved by computing: $R \equiv U|_{V=V'} \equiv U \sqcap \bigwedge\{v = v' | VC(v) = v'\}$.
3. $R|_V$ is the projected state where all the variables from Var' are eliminated from R.
4. $\overline{R|_V}$ is the negated state i.e. $D \setminus R|_V$ and it is computed by negating $R|_V$ (as mentioned before, all logical operations, including negation, are defined on our representation of an abstract state).
5. Eventually: $\triangle(\sigma^\natural, \sigma'^\natural) \equiv \sigma^\natural \sqcap \overline{R|_V}$ meaning it is part of the original program state $\sigma^\natural$ that does not appear in $\sigma'^\natural$ i.e. appears in the negation of R (which is the intersection of both abstract states).

Algorithm 1: Compute Abstract Difference.

Input: Abstract state rd
Output: Abstract difference - $\triangle(rd)$
$R \leftarrow rd \sqcap \bigwedge\{v = v' | VC(v) = v'\}$
$NR \leftarrow \neg R = \{\neg c | c \in R\}$
Foreach $nr_i \in NR$: $\triangle(rd) \leftarrow \triangle(rd) \cup (rd \sqcap nr_i)$
return $\triangle(rd)$

A geometrical representation of $\triangle_A$ calculation can be seen in Fig. **??**

From this point forward any mention of 'delta' (denoted $\triangle$) will refer to the correlating abstract state delta (denoted $\triangle_A$). We claim that $\triangle(\sigma^\natural, \sigma'^\natural)$ is a correct abstraction for the concrete state delta which allows for a scalable representation of difference we aim to capture.

6. Semantics of Union Program

6.1 General Product Program

A simple approach for a joint analysis is to construct a product program $P \times P'$ where at every point during the execution we can perform a program step (as defined in Definition 3) of either programs. The product program has a duo-state (σ, σ') and each step updates (σ, σ') accordingly. The product program can also be seen as a concurrent run $P||P'$ where every interleaving is possible. The product program emphasizes the fact that, as described in Section 2, the notion of $\triangle$ is unclear without an established variable and label correspondence. Choosing the location where $\triangle$ is checked is a key part of identifying differences. Consider Fig. **??**, which presents a product automata of the simple program with itself, we see that even in this trivial program, although it is clear that $\triangle = \emptyset$, checking for difference in any of the non-correlating states will result in a false difference being reported. As this example demonstrates, selecting a correct label correspondence is crucial for a meaningful delta, we will elaborate on our approach for choosing DP in **??**.

```
1  void foo() {
2      int x = 0;
3  }
```

Figure 5: Program P

6.2 Program Correspondence and Differential Points

Selecting the point where $\triangle$ is computed is vital for precision. As mentioned, a natural selection for diff points would be at the endpoints of traces but that loses meaning under the collecting semantics. A possible translation of this notion under the collecting semantics would be to compute delta between *all* the endpoints of the two programs i.e. $DP = \{(fin, fin') | fin \in exit(P), fin' \in exit(P')\}$ somehow differentiating the final states of the programs. This approach is problematic for two reasons:

1. Comparing all endpoints results in a highly imprecise delta. This is shown by the simple exercise of taking program with 2 endpoints and comparing it with itself.
2. This choice for DP may result in missing key differences between versions. If at some point during the calculation existed a delta that failed reaching the final state - it will be ignored. An interesting example for this is an array index receiving different bounds after a patch (but later overwritten so that it is not propagated to some final state).

Alternatively, the brute force approach where we might attempt to capture more potential diffs by selecting a diff-point after every line, will result in a highly inaccurate result as, for instance in Fig. **??**, many diffs will be reported although there is no difference. Finally, we must be careful with the selection of DP as it affects the soundness of our analysis: we might miss differences if we did not correctly place diff-points in locations where delta exists. **Our approach employs standard syntactic diff algorithm ?? for producing the correlation. This selection for DP assures soundness** . The Diff approach works well since two versions of the same software (and especially those that originate from subsequent check-ins to a code repository) are usually similar. Another important factor in the success of the diff is the guarded instruction format for our programs (as defined in Definition 3). Transforming both programs to a our format helps remove a lot of the "noise" that a patch might introduce yet it is superior to low lever intermediate representation as it retains many qualities (such as variable names, conditions, no temporaries, etc.). **See Appendix ?? for examples illustrating said benefits and qualities** . There are alternate ways for creating the correspondence such as **graph equivalence, etc.** , this could be a subject of future research. Calculating delta according to DP over the product automata is a complex task as it allows both programs to advance independently. We formulate the *correlating program* as a restricted product automata where we advance the programs while keeping the correlation allowing for a superior calculation of delta using our correlating abstract domain later defined in Section 5.

6.3 The Correlating Program $P \cup P'$

We will generally describe the process of constructing the correlating program. A more elaborate and formal description of the algorithm can be found in Algorithm **??**. The correlating program is an optimized structure where not all pairs of (σ, σ') are considered, but only pairs that result from a controlled execution, where correlating instructions (according to DP) in P and P' will execute together. This will allow for superior precision. As said, the main idea is to create one program which contains both versions. The correlating starts out as (exactly) the older version P (after being converted to our guarded instruction form). Afterwards a syntactic diff with P' (also transformed to guarded mode) is computed (the programs are not combined just yet). In fact, this is the point where DP is created as the diff supplies us with the correlation between labels we desire. Then P''s instructions are interleaved into the guarded P while maintaining the correlation found by the diff (matched instructions will appear consequentially). Just before a patched instruction is interleaved into the correlating, all variables that appear in it are tagged, as to make sure that the patched instructions will only affect patched variables. Thus we maintain the semantics of running both programs correctly while achieving a new construct that will allow us to analyze change more easily and precisely. **Fig. ?? holds a complete correlating program of the program in Fig. ?? and it's patched version, a graphic description as a controlled automata is shown in Fig. ??** . Note that if we view the general correlating program as a concurrent program, then this optimized program can be viewed as a partial-order reduction applied over the concurrent program. One final observation regarding the correlating program is that it is a legitimate program that can be run to achieve the effect of running both versions. This ability allows us to use dynamic analysis and testing techniques such as fuzzing **??** and directed automated testing **??** which may produce input that lead to states approximated by $\triangle$.

6.4 Analyzing Correlating Programs

Analyzsis of a guarded correlating program has certain caveats. In

```
1  l:  guard g = (i>0);
2      if (g) i--;
3      if (g) goto l;
```

Figure 6: example program illustrating guard analysis caveat

order to correctly analyze the program in Fig. **??** we need our analysis to assume $(i > 0)$ whenever taking the true branch on the `if (g)` instruction and $(i <= 0)$ when taking the false branch. However, since the `i-` instruction invalidates this assumption we would need to update the guard assumption to $(i > -1)$ which would complicate the analysis as we would need to consider updating the guard assumption while widening etc. Our solution simply incorporates the guard's assumption the first time it encounters the guard and allows it to flow to the rest of the nodes. We are not in danger of losing the assumption during the following join as our join employs a partition-by-equivalence strategy and will not join the two states where $g, i > 0$ and $\neg g, i <= 0$.

7. Evaluation

We mainly tested our tool on the GNU core utilities, differencing versions 6.10 and 6.11. This benchmark included 40 patches where most of the patches (35) were a one-line patch aimed at updating the version information string in the code. Our analysis easily showed equivalence for these programs. About 10 of these patches included actual changes to numerical variables and we were able to precisely describe the difference. We also tested our tool on a few handpicked patches taken from the Linux kernel and the Mozilla Firefox web browser.

We implemented a union compiler named *ucc* which creates union programs from any two C programs as well as a differencing oriented dataflow analysis solver for analyzing union programs, both tools use the LLVM and CLang compiler infrastructure. We analyze C code directly thus benefiting from a low number of variables as there are no temporary values as there might appear in an intermediate representation. We also benefit from our delta being computed over original variables. As mentioned in Section **??**, we normalize the input programs before unifying them for a simpler analysis.

Analysis of some of our benchmarks required the use of widening. We applied a basic widening strategy which widens all cfg blocks once reaching a certain threshold. All of our experiments were conducted running on a Intel(R) Core-i7(TM) processor with 4GB.

7.1 Results

Tab. 1 summarizes the results of our analysis. The columns indicate the benchmark name and description, lines of code for the analyzed program, the number of lines added and removed by the patch, the number of diff-points generated, the numerical domain used, and the number of differences found in the analysis (i.e. number of diff-points where $\triangle \neq \varnothing$). In our benchmarks, we focused on computing intra-procedural difference between the two versions of procedures. Procedure calls presented difficulty as they potentially

Table 1: Experimental Results

Name	Domain	#Added	#Deleted	#DiffPoints	#Diffs
dd.i	ppl	52	54	87	4
id.i	ppl	15	6	26	0
pr.i	ppl	10	3	13	1
su.i	ppl	2	2	2	0
env.i	ppl	2	2	3	0
seq.i	ppl	10	10	15	4
nice.i	ppl	3	3	36	0
test.i	ppl	3	3	14	0
chmod.i	ppl	3	3	7	0
nohup.i	ppl	2	2	18	0
paste.i	ppl	24	17	4	0
rmdir.i	ppl	3	0	3	0
users.i	ppl	3	4	30	0
chroot.i	ppl	3	3	22	0
md5sum.i	ppl	15	7	32	28
runcon.i	ppl	2	2	3	0
lbracket.i	ppl	3	3	14	0
setuidgid.i	ppl	7	4	34	12
chown-core.i	ppl	3	3	10	0

```
1  static bool
2  bsd_split_3 (char *s, size_t s_len, unsigned char **hex_digest, char **
   size_t i;
4
5  if (s_len == 0) return false; // Patch
6
7  *file_name = s;
8
9  /* Find end of filename. The BSD 'md5' and 'sha1' commands do not esc
10    filenames, so search backwards for the last ')'. */
11
12 i = s_len - 1;
13 while (i && s[i] != ')')
14   i--;
15
16 if (s[i] != ')') return false;
17
18 s[i++] = '\0';
19
20 while (ISWHITE (s[i])) i++;
21
22 if (s[i] != '=') return false;
23 i++;
24
25   while (ISWHITE (s[i])) i++;
26
27   *hex_digest = (unsigned char *) &s[i];
28   return true;
29 }
```

Figure 7: md5sum.c bsd_split_3 function patch

change global variables and local variables through pointers. We overcame this by either (i) assuming equivalence (alone) once we encounter a call to a procedure we already established as equivalent or (ii) warn that all results regarding variables touched by the procedure is un-sound. **In the majority of our benchmarks we identified calls only to library and system procedures thus we could omit their effect as they do not change variables beyond those given as parameter or those being assigned the return value.** All differences reported describe, in constraints over variables, an existing delta at that program point.

Identifying delta down the line One advantage of our analysis method is the ability to identify differences in variables that were not directly affected by the patch. Fig. **??** shows part of the `bsd_split_3` function that was patched by the line marked by a comment. Note that although the patch directly restricts the value range of `s_len` to above zero, our analysis is able to identify the effects on the index variable `i` and also report a lost program state of `i>-1` further down the line...

Non-convex delta

Maintaining equivalence in loops

8. Related Work

Bounded symbolic execution in CLang As prior work we used the CLang infrastructure [1] static analysis graph reachability engine in order to perform a simple and bounded state differentiation exploration. We used the existing infrastructure and it's abstract representation facilities to simply record every location where the 2 versions of the variables differ. This of course was not sufficient since it only presents a bounded solution and we will show the limitations of this method by example.

Existing work on patch-based exploit generation Brumley, Poosankam, Song and Zheng [**?**] is the prominent work addressing patch-based analysis. We differ from this work in the following aspects:

1. First, the problem definition in said work is different from our own. They aim to find an $exploit$ for vulnerabilities fixed by a certain patch. Furthermore, this exploit is defined in relevance to a $securitypolicy$ which can differ. While our goals are similar to those of [**?**], we achieve them by solving 2 extended problems of a) recording the delta between new variable values and old ones and b) producing input from said values. These problems are a superset of the problem described in [**?**] and solving them has the potential for a much more complete and sound result.
2. We aim to find differentiation between every variable changed by the patch and analyze that differentiation while they concentrate on input sanitation alone. Thus if in the patched program some variable has changed in a way that does not involve input validation, it will be disregarded: for instance if an array index variable i to a buffer B is patched by adding an assignment $i = sizeof(B)-1$, it will be ignored in the previous work while we will record that the old version of i can no longer have values greater than $sizeof(B) - 1$ and may use it for exploit generation.
3. We perform our analysis on the source code of the program and patch instead of the binary. Working on a higher level gives us much more data thus potentially allowing for more results.

Kroening and Heelan [**?**] main focus focus was producing an exploit from given input that is known to trigger a bug. No patch is involved in the process. Our goal is to produce said input from the corrected software thus [**?**] can be used to create an exploit from our results.

Song, Zhang and Sun [**?**] also relate to the patch-based exploit generation problem but their main focus is on finding similarities between versions of the binary to better couple functions from the original program with their patched counter-part, a problem that was not addressed in [**?**]. Also their method of recognizing possible exploits is degenerate and relies on identifying known input validation functions that were added to a certain path - a method that could be easily overcome.

Oh [**?**] presented a new version for the DarunGrim binary diffing tool aimed at better reviewing patched programs and specifically finding patches with security implications. The goal of the tool is to help researches who manually scan patches for the purpose of producing intrusion prevention system signatures. The tool relies mainly on syntactic analysis of patterns to produce a security

implication score for procedures patches making them a candidate for manual inspection. [?] used the DarunGrim binary diffing tool EBDS for their experiment.

Person, Dwyer, Elbaum and Pasareanu [7] introduced an extension and application of symbolic execution techniques that computes a precise behavioral characterization of a program change called differential symbolic execution. As we also implemented bounded symbolic execution as our preliminary work we will discuss this method in comparison to our own.

Godlin and Strichman [?] developed a method for proving the equivalence of similar C programs under certain restrictions based on and existing functional verification tool. This was a basis for future work regarding equivalence and we intend to base our work upon these advances.

Kawaguchi, Lahiri and Rebelo [6] defined the concept of *conditional equivalence* meaning under which conditions (inputs) are 2 different versions of a program equivalent (i.e. produce the same output). Their goal is to keep software changes from breaking procedure contracts and changing module behavior too drastically and they achieve this by computing the conditions under which the behavior is preserved. This work indirectly addresses our problem and we believe we can leverage their techniques for producing the inputs that break the equivalence while focusing on bug triggering rather than contract breaking.

[?]

Determining corresponding components As suggested in [?], one possibility is to rely on the editing sequence that creates the new version from the original one. Another option is using various syntactic differencing algorithms as a base for computing correspondence tags.

their idea for computing correspondence, is to minimize the "size of change". They have two different notions of size of change.

[?] introduced a correlating heap semantics for verifying linearizability of concurrent programs. In their work, a correlating heap semantics is used to establish correspondence between a concurrent program and a sequential version of the program at specific linearization points.

A. Appendix

A.1 Algorithm 2 : Convert P To Guarded Instruction Format

The algorithm is constructive i.e. it takes a procedure P and outputs the new lines for a guarded version of P. The original P is not part of the output.

- *Stage 0:* Output P's signature.
- *Stage 1:* Convert all `while` constructs to `if` and `goto` constructs.
- *Stage 2:* For each non branch instruction I:

  If I is a declaration, output it under a new block.

  Otherwise collect all branch conditions C under which I executes. Produce the code line: `if (`$\bigwedge C$`) I;`

We take a small but sufficient example to demonstrate the algorithm:

```
void foo(char Out[], int n) {
    char In[42];
    int i;
    if (n >= 42) n = 41;
    while (i < n) {
        int j = i + 1;
        In[i] = Out[j];
        i++;
    }
}
```

Stage 1 Output:

```
void foo(char Out[], int n) {
    char In[42];
    int i;
    if (n >= 42) n = 41;
l:  if (i < n) {
        int j = i + 1;
        In[i] = Out[j];
        i++;
        goto l;
    }
}
```

Stage 2 Output:

```
void foo(char Out[], int n) {
    char In[42];
    int i;
    if (n >= 42) n = 41;
    {
        int j;
l:      if (i < n) j = i + 1;
        if (i < n) In[i] = Out[j];
        if (i < n) i++;
        if (i < n) goto l;
    }
}
```

Figure 8: Algorithm 1 application example

Note that as required, the loop is implemented in means of branches and goto and that the branches do not exceed the 1 level of nesting allowed.

A.1.1 Algorithm 2 Assumptions

1. We only deal with `while` loop statements.
2. Logical statement (branch conditions) do not have side-effects i.e. they do not change variable values.

A.2 Algorithm 1 Example

We run algorithm 1 on the previous example with the following patch to line 4:

```
- if (n >= 42) n = 41;
+ if (n >= 42) n = 40;
```

We start off from the output of algorithm 2 i.e. a "guarded" program that performs simple buffer handling:

```
void foo(char Out[], int n) {
    char In[42];
    int i;
    if (n >= 42) n = 41;

    {
        int j;
l:      if (i < n) j = i + 1;
        if (i < n) In[i] = Out[j];
        if (i < n) i++;
        if (i < n) goto l;
    }

}
```

The result:

```
void foo(char Out[], int n, char Out'[], int n') {
    char In[42], In'[42];
    int i, i';

    if (n >= 42) n = 41;
    if (n' >= 42) n' = 40;

    {
        int j,j';
l:      if (i < n) j = i + 1;
l':     if (i' < n') j' = j' + 1;
        if (i < n) In[i] = Out[j];
        if (i' < n') In'[i'] = Out'[j'];
        if (i < n) i++;
        if (i' < n') i'++;
        if (i < n) goto l;
        if (i' < n') goto l';
    }
}
```

Figure 9: Algorithm 1 Application Example

B. Worklist

B.1 Points to Hammer

1. a special kind of self composition, where correlated steps are kept together. This is particularly important when handling loops.

B.2 TODO

1. run on uc-klee examples.
2. Consider describing each sub-state as a single (possibly looping) path of execution in both programs that originated from the same input.

B.3 Questions

1. how come you don't need the "product program"?
2. what are the theorems that you provide? (no reason to have definitions if there are no theorems).
3. what abstract domains can we use as "underlying domains" for our abstraction? Do we have any particular requirements from the abstract domains (one requirement is being relational).
4. what makes a "patched version of a program" different from just saying "a different program"? In other words - what are the requirements on the difference between P and P'?
5. can we claim that our abstraction "forgets" paths along which equivalence is established, but keeps apart paths along which the is a difference, hoping that it will re-converge later?
6. (intuition only) what if we correlate badly and lose soundness? one can propose a 2 "trick" programs that correlating them our way gives a result that loses difference.
7. WHY DO WE CHECK DIFFERENCE ABOVE THE SUB-STATE LEVEL? doesn't that mean we compare different paths? isn't that bad?

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