Viterbi Decoding for HMM, Parameter Learning

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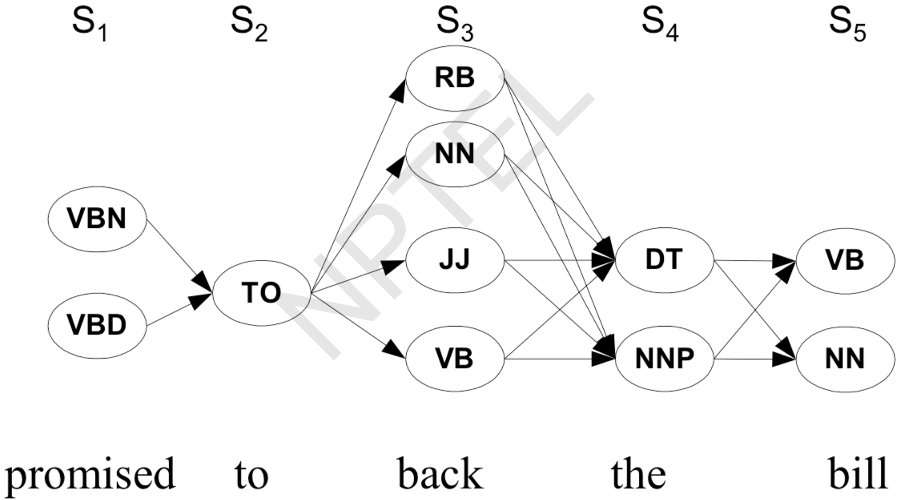
Week 4, Lecture 1

Walking through the states: best path



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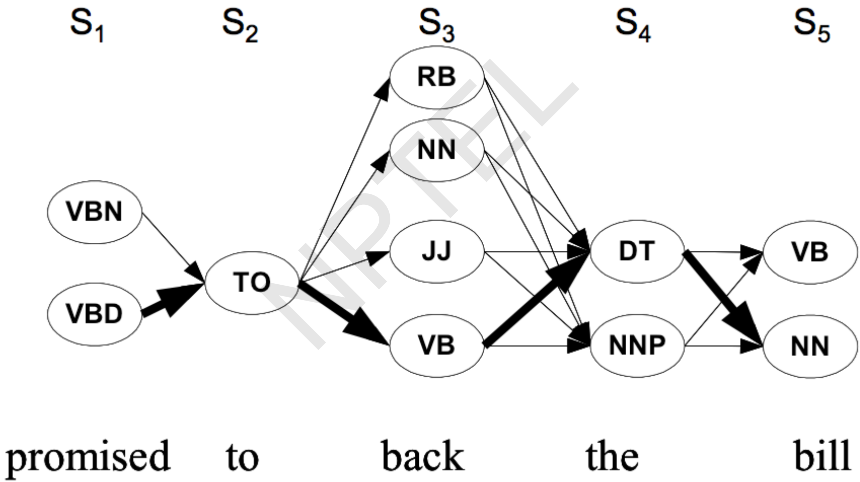

Finding the best path: Viterbi Algorithm

Intuition

Optimal path for each state can be recorded. We need

- Cheapest cost to state j at step s: $\delta_j(s)$
- Backtrace from that state to best predecessor $\psi_j(s)$

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Best final state is $argmax_{1 \leq i \leq N} \delta_i(|S|)$, we can backtrack from there

Practice Question

- Suppose you want to use a HMM tagger to tag the phrase, “the light book”, where we have the following probabilities:
- P(the|Det) = 0.3, P(the|Noun) = 0.1, P(light|Noun) = 0.003, P(light|Adj) = 0.002, P(light|Verb) = 0.06, P(book|Noun) = 0.003, P(book|Verb) = 0.01
- P(Verb|Det) = 0.00001, P(Noun|Det) = 0.5, P(Adj|Det) = 0.3, P(Noun|Noun) =0.2, P(Adj|Noun) = 0.002, P(Noun|Adj) = 0.2, P(Noun|Verb) = 0.3, P(Verb|Noun) = 0.3, P(Verb|Adj) = 0.001, P(Verb|Verb) = 0.1
- Work out in details the steps of the Viterbi algorithm. You can use a Table to show the steps. Assume all other conditional probabilities, not mentioned to be zero. Also, assume that all tags have the same probabilities to appear in the beginning of a sentence.

Learning the Parameters

Two Scenarios

- A labeled dataset is available, with the POS category of individual words in a corpus
- Only the corpus is available, but not labeled with the POS categories

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Methods for these scenarios

- For the first scenario, parameters can be directly estimated using maximum likelihood estimate from the labeled dataset
- For the second scenario, *Baum-Welch Algorithm* is used to estimate the parameters of the hidden markov model.

Baum Welch Algorithm

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Week 4, Lecture 2

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Uses the well-known EM algorithm to find the maximum likelihood estimate of the parameters of a hidden markov model

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Parameters of HMM

Let X_t be the random variable denoting hidden state at time t, and Y_t be the observation variable at time T. HMM parameters are given by $\theta = (A, B, \pi)$ where

- $A = \{a_{ij}\} = P(X_t = j | X_{t-1} = i)$ is the state transition matrix
- $\pi = \{\pi_i\} = P(X_1 = i)$ is the initial state distribution
- $B = \{b_j(y_t)\} = P(Y_t = y_t | X_t = j)$ is the emission matrix

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Given observation sequences $Y = (Y_1 = y_1, Y_2 = y_2, \ldots, Y_T = y_T)$, the algorithm tries to find the parameters θ that maximise the probability of the observation.

The Algorithm

The basic idea is to start with some random initial conditions on the parameters θ, estimate best values of state paths X_t using these, then re-estimate the parameters θ using the just-computed values of X_t, iteratively.

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Intuition

- Choose some initial values for $\theta = (A, B, \pi)$.
- *Repeat the following step until convergence:*
- Determine probable (state) paths $\ldots X_{t-1} = i, X_t = j \ldots$
- Count the expected number of transitions a_{ij} as well as the expected number of times, various emissions $b_j(y_t)$ are made
- Re-estimate $\theta = (A, B, \pi)$ using a_{ij} and $b_j(y_t)$s.

A forward-backward algorithm is used for finding probable paths.

Forward-Backward Algorithm

Forward Procedure

$\alpha_i(t) = P(Y_1 = y_1, \ldots, Y_t = y_t, X_t = i|\theta)$ be the probability of seeing $y_1, \ldots, y_t$ and being in state i at time t. Found recursively using:

- $\alpha_i(1) = \pi_i b_i(y_1)$

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- $\alpha_i(1) = \pi_i b_i(y_1)$
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Backward Procedure

$\beta_i(t) = P(Y_{t+1} = y_{t+1}, \ldots, Y_T = y_T | X_t = i, \theta)$ be the probability of ending partial sequence $y_{t+1}, \ldots, y_T$ given starting state i at time t. $\beta_i(t)$ is computed recursively as:

- $\beta_i(T) = 1$

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- $\beta_i(T) = 1$
- $\beta_i(t) = \sum_{j=1}^{N} \beta_j(t+1) a_{ij} b_j(y_{t+1})$

Finding probabilities of paths

We compute the following variables:

- Probability of being in state i at time t given the observation Y and parameters θ

$$\gamma_i(t) = P(X_t = i|Y,\theta) = \frac{\alpha_i(t)\beta_i(t)}{\sum_{j=1}^{N} \alpha_j(t)\beta_j(t)}$$

- Probability of being in state i and j at time t and $t+1$ respectively given the observation Y and parameters θ

$$\zeta_{ij}(t) = P(X_t = i, X_{t+1} = j|Y,\theta) = \frac{\alpha_i(t)a_{ij}\beta_j(t+1)b_j(y_{t+1})}{\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i(t)a_{ij}\beta_j(t+1)b_j(y_{t+1})}$$

Updating the parameters

- $\pi_i = \gamma_i(1)$, expected number of times state i was seen at time 1
- $a_{ij} = \frac{\sum_{t=1}^{T} \zeta_{ij}(t)}{\sum_{t=1}^{T} \gamma_i(t)}$, expected number of transitions from state i to state j, compared to the total number of transitions away from state i
- $b_i(v_k) = \frac{\sum_{t=1}^{T} 1_{y_t = v_k} \gamma_i(t)}{\sum_{t=1}^{T} \gamma_i(t)}$ with $1_{y_t = v_k}$ being an indicator function, is the expected number of times the output observations are v_k while being in state i compared to the expected total number of times in state i.

Maximum Entropy Models

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Week 4, Lecture 3

Issues with Markov Model Tagging

Unknown Words

We do not have the required probabilities.

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Possible solutions:

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Possible solution: Use higher order model, combine various n-gram models to avoid sparseness problem

Maximum Entropy Modeling: Discriminative Model

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  - Whether it is the first word in the article
  - Whether the next word is *to*
  - Whether one of the last 5 words is a preposition, etc.
- MaxEnt combines these features in a probabilistic model

Maximum Entropy: The Model

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- x denotes an observed datum and y denotes a class

What is the form of the features?

Features in Maximum Entropy Models

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- Context x is taken around the word w, for which a tag y is to be predicted
- Features are binary values functions, e.g.,

$$f(x,y) = \left\{ \begin{array}{ll} 1 & \text{if } isCapitalized(w) \& y = NNP \\ 0 & otherwise \end{array} \right\}$$

Example Features

Example: Named Entities

- LOCATION (in Arcadia)
- LOCATION (in Québec)
- DRUG (taking Zantac)
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Example Features

- $f_1(x,y) = [y = LOCATION \wedge w_{-1} = \text{“}in\text{”} \wedge isCapitalized(w)]$
- $f_2(x,y) = [y = LOCATION \wedge hasAccentedLatinChar(w)]$
- $f_3(x,y) = [y = DRUG \wedge ends(w, \text{“}c\text{”})]$

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- The context x_i also includes previously assigned tags for a fixed history.
- Beam search is used to find the most probable sequence

Beam Inference

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- Extend each sequence in each local way
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But what is a MaxEnt model?

Let's go to the basics now!

Maximum Entropy Model

Intuitive Principle

Model all that is known and assume nothing about that which is unknown.

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Model all that is known and assume nothing about that which is unknown.
Given a collection of facts, choose a model which is consistent with all the facts, but otherwise as uniform as possible.

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- Collect a large sample of instances of the expert's decisions
- **Goal**: extract a set of facts about the decision-making process (first task) that will aid in constructing a model of this process (second task)

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First clue: list of allowed translations

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- Infinite number of models p for which this identity holds, the most intuitive model?
- *allocate the total probability evenly among the five possible phrases* → most uniform model subject to our knowledge.
- *Is it the most uniform model overall?* → No, that would grant an equal probability to every *possible* French phrase.

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More clues from the expert's decision

- **Second clue:** Suppose the expert chose either '*dans*' or '*en*' 30% of the time.

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- **Third clue:** In half of the cases, the expert chose either '*dans*' or '*á*'

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How do we measure uniformity of a model?

As we add complexity to the model, we face two difficulties:

- What exactly is meant by "uniform"?
- How can one measure the uniformity of a model?

Maximum Entropy Modeling

Entropy: measures the uncertainty of a distribution.

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Entropy: expected surprise (over p)

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Coin Tossing

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Maximize entropy H, subject to feature-based constraints:

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Solution

Maximize entropy H, subject to feature-based constraints:

$$E_p[f_i] = E_{\tilde{p}}[f_i]$$

Adding constraints

- Lowers maximum entropy
- Brings the distribution further from uniform and closer to data

Maximum Entropy Principle

Given n feature functions f_i, we would like p to lie in the subset C of P defined by

$$C = \{p \in P | p(f_i) = \tilde{p}(f_i), i \in \{1, 2, \ldots, n\}\}$$

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Model expectation of a feature

$$p(f_i) = \sum_{x,y} \tilde{p}(x) p(y|x) f_i(x,y)$$

Select the distribution which is most uniform (conditional probability):

$$p^* = argmax_{p \in C} H(p) = H(Y|X) \approx -\sum_{x,y} \tilde{p}(x) p(y|x) log p(y|x)$$

Maximum Entropy Principle

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Constraint Optimization

Introduce a parameter λ_i for each feature f_i. Lagrangian is given by

$$\wedge(p,\lambda) = H(p) + \sum_i \lambda_i (p(f_i) - \tilde{p}(f_i))$$

Solving, we get

$$p_\lambda(y|x) = \frac{1}{Z_\lambda(x)} exp\left(\sum_i \lambda_i f_i(x,y)\right)$$

where $Z_\lambda(x)$ is a normalizing constant given by

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Maximum Entropy Models

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Week 4, Lecture 4

Practice Question

Consider the maximum entropy model for POS tagging, where you want to estimate $P(tag|word)$. In a hypothetical setting, assume that *tag* can take the values *D*, *N* and *V* (short forms for Determiner, Noun and Verb). The variable *word* could be any member of a set V of possible words, where V contains the words *a*, *man*, *sleeps*, as well as additional words. The distribution should give the following probabilities

- $P(D|a) = 0.9$
- $P(N|man) = 0.9$
- $P(V|sleeps) = 0.9$
- $P(D|word) = 0.6$ for any word other than *a*, *man* or *sleeps*
- $P(N|word) = 0.3$ for any word other than *a*, *man* or *sleeps*
- $P(V|word) = 0.1$ for any word other than *a*, *man* or *sleeps*

It is assumed that all other probabilities, not defined above could take any values such that $\sum_{tag} P(tag|word) = 1$ is satisfied for any word in V.

- Define the features of your maximum entropy model that can model this distribution. Mark your features as f_1, f_2 and so on. Each feature should have the same format as explained in the class. [**Hint:** 6 Features should make the analysis easier]
- For each feature f_i, assume a weight λ_i. Now, write expression for the following probabilities in terms of your model parameters
  - $P(D|cat)$
  - $P(N|laughs)$
  - $P(D|man)$
- What value do the parameters in your model take to give the distribution as described above. (i.e. $P(D|a) = 0.9$ and so on. *You may leave the final answer in terms of equations*)

Features for POS Tagging (Ratnaparakhi, 1996)

The specific word and tag context available to a feature is

$$h_i = \{w_i, w_{i+1}, w_{i+2}, w_{i-1}, w_{i-2}, t_{i-1}, t_{i-2}\}$$

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The specific word and tag context available to a feature is

$$h_i = \{w_i, w_{i+1}, w_{i+2}, w_{i-1}, w_{i-2}, t_{i-1}, t_{i-2}\}$$

Example: $f_j(h_i, t_i) = 1$ if $suffix(w_i) = \text{``}ing\text{''} \,\&\, t_i = VBG$

Example Features

Condition	Features	
w_i is not rare	$w_i = X$	& $t_i = T$
w_i is rare	X is prefix of w_i, $\|X\| \leq 4$	& $t_i = T$
	X is suffix of w_i, $\|X\| \leq 4$	& $t_i = T$
	w_i contains number	& $t_i = T$
	w_i contains uppercase character	& $t_i = T$
	w_i contains hyphen	& $t_i = T$
$\forall\ w_i$	$t_{i-1} = X$	& $t_i = T$
	$t_{i-2}t_{i-1} = XY$	& $t_i = T$
	$w_{i-1} = X$	& $t_i = T$
	$w_{i-2} = X$	& $t_i = T$
	$w_{i+1} = X$	& $t_i = T$
	$w_{i+2} = X$	& $t_i = T$

Example Features

Word:	the	stories	about	well-heeled	communities	and	developers
Tag:	DT	NNS	IN	JJ	NNS	CC	NNS
Position:	1	2	3	4	5	6	7

Example Features

Word:	the	stories	about	well-heeled	communities	and	developers
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$w_i =$ about & $t_i =$ IN
$w_{i-1} =$ stories & $t_i =$ IN
$w_{i-2} =$ the & $t_i =$ IN
$w_{i+1} =$ well-heeled & $t_i =$ IN
$w_{i+2} =$ communities & $t_i =$ IN
$t_{i-1} =$ NNS & $t_i =$ IN
$t_{i-2}t_{i-1} =$ DT NNS & $t_i =$ IN

$w_{i-1} =$ about & $t_i =$ JJ
$w_{i-2} =$ stories & $t_i =$ JJ
$w_{i+1} =$ communities & $t_i =$ JJ
$w_{i+2} =$ and & $t_i =$ JJ
$t_{i-1} =$ IN & $t_i =$ JJ
$t_{i-2}t_{i-1} =$ NNS IN & $t_i =$ JJ
prefix(w_i)=w & $t_i =$ JJ
prefix(w_i)=we & $t_i =$ JJ
prefix(w_i)=wel & $t_i =$ JJ
prefix(w_i)=well & $t_i =$ JJ
suffix(w_i)=d & $t_i =$ JJ
suffix(w_i)=ed & $t_i =$ JJ
suffix(w_i)=led & $t_i =$ JJ
suffix(w_i)=eled & $t_i =$ JJ
w_i contains hyphen & $t_i =$ JJ

Search Algorithm

Conditional Probability

Given a sentence $\{w_1, \ldots, w_n\}$, a tag sequence candidate $\{t_1, \ldots, t_n\}$ has conditional probability:

$$P(t_1, \ldots, t_n | w_1 \ldots, w_n) = \prod_{i=1}^{n} p(t_i | x_i)$$

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A *Tag Dictionary* is used, which, for each known word, lists the tags that it has appeared with in the training set.

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Let $W = \{w_1, \ldots, w_n\}$ be a test sentence, s_{ij} be the jth highest probability tag sequence up to and including word w_i.

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- Generate tags for w_1, find top N, set $s_{1j}, 1 \leq j \leq N$, accordingly.
- Initialize $i = 2$
  - Initialize $j = 1$
  - Generate tags for w_i, given $s_{(i-1)j}$ as previous tag context, and append each tag to $s_{(i-1)j}$ to make a new sequence
  - $j = j + 1$, repeat if $j \leq N$

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- $i = i + 1$, repeat if $i \leq n$
- Return highest probability sequence s_{n1}

A Good Reference

Berger et al., *A Maximum Entropy Approach to Natural Language Processing*, Computational Linguistics, Vol. 22, No. 1.

Conditional Random Fields

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Week 4, Lecture 5

Practice Question

Suppose you want to use a MaxEnt tagger to tag the sentence, "the light book". We know that the top 2 POS tags for the words *the*, *light* and *book* are $\{Det, Noun\}$, $\{Verb, Adj\}$ and $\{Verb, Noun\}$, respectively. Assume that the MaxEnt model uses the following history h_i (context) for a word w_i:

$$h_i = \{w_i, w_{i-1}, w_{i+1}, t_{i-1}\}$$

where w_{i-1} and w_{i+1} correspond to the previous and next words and t_{i-1} corresponds to the tag of the previous word. Accordingly, the following features are being used by the MaxEnt model:

- f_1: $t_{i-1} = Det$ and $t_i = Adj$
- f_2: $t_{i-1} = Noun$ and $t_i = Verb$
- f_3: $t_{i-1} = Adj$ and $t_i = Noun$
- f_4: $w_{i-1} = the$ and $t_i = Adj$
- f_5: $w_{i-1} = the \& w_{i+1} = book$ and $t_i = Adj$
- f_6: $w_{i-1} = light$ and $t_i = Noun$
- f_7: $w_{i+1} = light$ and $t_i = Det$
- f_8: $w_{i-1} = NULL$ and $t_i = Noun$

Assume that each feature has a uniform weight of 1.0.
Use Beam search algorithm with a beam-size of 2 to identify the highest probability tag sequence for the sentence.

Problem with Maximum Entropy Models

Per-state normalization

All the mass that arrives at a state must be distributed among the possible successor states

Problem with Maximum Entropy Models

Per-state normalization

All the mass that arrives at a state must be distributed among the possible successor states

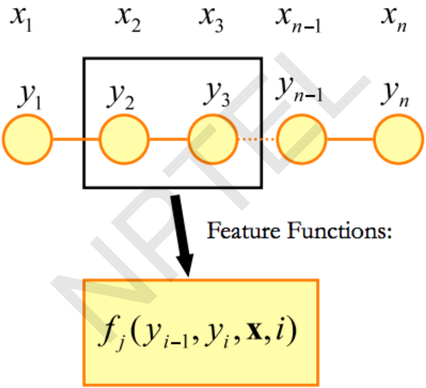
This gives a 'label bias' problem

Let's see the intuition (on paper)

Conditional Random Fields

- CRFs are conditionally trained, undirected graphical models.
- Let's look at the linear chain structure

Conditional Random Fields: Feature Functions



Express some characteristic of the empirical distribution that we wish to hold in the model distribution

$$f_j(y_{i-1}, y_i, \mathbf{x}, i) = \begin{cases} 1 \;\; \text{if } y_{i-1} = IN \text{ and } y_i = NNP \text{ and } x_i = September \\ 0 \;\; \text{otherwise} \end{cases}$$

Conditional Random Fields: Distribution

Label sequence modelled as a normalized product of feature functions:

$$P(\mathbf{y} \mid \mathbf{x}, \boldsymbol{\lambda}) = \frac{1}{Z(\mathbf{x})} \exp \sum_{i=1}^{n} \sum_{j} \lambda_j f_j(y_{i-1}, y_i, \mathbf{x}, i)$$

$$Z(\mathbf{x}) = \sum_{\mathbf{y} \in Y} \sum_{i=1}^{n} \sum_{j} \lambda_j f_j(y_{i-1}, y_i, \mathbf{x}, i)$$

CRFs

- Have the advantages of MEMM but avoid the label bias problem
- CRFs are globally normalized, whereas MEMMs are locally normalized.
- Widely used and applied. CRFs have been (close to) state-of-the-art in many sequence labeling tasks.