Warsaw University of Technology

Faculty of Power and Aeronautical Engineering

Mechanics of Flight II

PROJECT NO. 12

Phugoid (Long Period) Motion of the Airplane

Student: Suleyman Ertugrul Sahin (309202)
Aircraft: Messerschmitt Bf-110
Group: P2
Teacher: Blesson Prakash (DOKT)

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Grade:

INTRODUCTION

The Phugoid or Long Period Motion is characteristic oscillations of the aircraft after a small disturbance of the steady flight (ie. Due to small horizontal control surface motion or the air gust). The airplane is traveling along the sinusoidal trajectory with small changes of the air speed and pitch angle.

Equations of Motion:

The initial assumptions are:

- The aircraft is initially flying in vertical plane with constant speed V_0 and with no rotation.
- After small disturbance of the light speed $V=V_0+v$ or pitch angle Θ, the airplane will always be in the vertical plane.
- The airplane has two degrees of freedom
- The wing angle of attack as well as the aerodynamic coefficients C_L and C_D are constant.

The equations of motion of the airplane developed by using Fernet coordinates are as shown below:

$$m\cdot\frac{dv}{dt}+m\cdot g\cdot\sin\theta+\frac{1}{2}\cdot\rho\cdot S_w\cdot\left(V_0+v\right)^2\cdot C_D=0\ ,$$
$$m\cdot V_0\frac{d\theta}{dt}+m\cdot g\cdot\cos\theta+\frac{1}{2}\cdot\rho\cdot S_w\cdot\left(V_0+v\right)^2\cdot C_L=0\ . \tag{1}$$

Dimensionless and linear form of equations derived from (1) is:

$$\frac{d\bar{v}}{d\bar{t}}+\frac{1}{2}\cdot C_L\cdot\theta+C_D\cdot\bar{v}=0\ ,$$
$$\frac{d\bar{\theta}}{d\bar{t}}+\frac{1}{2}\cdot C_D\cdot\theta-C_L\cdot\bar{v}=0\ , \tag{2}$$

Where:

$\bar{v}=\dfrac{v}{V_0}$ - non-dimensional small disturbance of flight speed,

$\bar{\theta}$ - small disturbance of the pitch angle (in radians),

$\bar{t}=\dfrac{t}{t_{aero}}$ - dimensionless time,

$t_{aero}=\dfrac{2\cdot m}{\rho\cdot S_w\cdot V_0}$ - aerodynamic time.

m	6750 [kg]
S_w	38.4 [m^2]
ρ	1.225 [kg/m^3]
V_0	72.0 [km/h]

Solution:

$$\begin{bmatrix} \bar{\lambda}+C_D & \frac{1}{2}\cdot C_L \\ -C_L & \bar{\lambda}+\frac{1}{2}\cdot C_D \end{bmatrix}\cdot\begin{bmatrix} \bar{v}_0 \\ \bar{\theta}_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad det\left(\begin{bmatrix} \bar{\lambda}+C_D & \frac{1}{2}\cdot C_L \\ -C_L & \bar{\lambda}+\frac{1}{2}\cdot C_D \end{bmatrix}\right) = 0$$

$$\bar{v}(\bar{t})=\bar{v}_0\cdot e^{\bar{\lambda}\cdot\bar{t}} \qquad \bar{\theta}(\bar{t})=\bar{\theta}_0\cdot e^{\bar{\lambda}\cdot\bar{t}}$$

$$\bar{\lambda}^2+\frac{3}{2}\cdot C_D\cdot\bar{\lambda}+\frac{1}{2}\cdot\left(C_D^2+C_L^2\right) = 0$$

$$\bar{\lambda}_{1,2} = \bar{\xi}_{1,2} \pm i\ \bar{\eta}_{1,2} = -\frac{3}{4}\cdot C_D \pm i\ \sqrt{\frac{1}{2}\cdot\left(C_D^2+C_L^2\right)-\left(\frac{3}{4}\cdot C_D\right)^2}$$

$$\bar{v}(\bar{t})=\bar{v}_1\cdot e^{\bar{\lambda}_1\cdot\bar{t}}+\bar{v}_2\cdot e^{\bar{\lambda}_2\cdot\bar{t}} = e^{\bar{\xi}_{1,2}}\cdot\left[\left(\bar{v}_1+\bar{v}_2\right)\cdot\cos\left(\bar{\eta}_{1,2}\cdot\bar{t}\right) + i\cdot\left(\bar{v}_1-\bar{v}_2\right)\cdot\sin\left(\bar{\eta}_{1,2}\cdot\bar{t}\right)\right]$$
$$\bar{\theta}(\bar{t})=\bar{\theta}_1\cdot e^{\bar{\lambda}_1\cdot\bar{t}}+\bar{\theta}_2\cdot e^{\bar{\lambda}_2\cdot\bar{t}} = e^{\bar{\xi}_{1,2}}\cdot\left[\left(\bar{\theta}_1+\bar{\theta}_2\right)\cdot\cos\left(\bar{\eta}_{1,2}\cdot\bar{t}\right) + i\cdot\left(\bar{\theta}_1-\bar{\theta}_2\right)\cdot\sin\left(\bar{\eta}_{1,2}\cdot\bar{t}\right)\right]$$

Where:

V_1, V_2, Θ_1, Θ_2 are dimensional amplitudes of oscillations (elements of two eigenvectors)

$\bar{\xi}_{1,2}=\mathrm{Re}\left(\bar{\lambda}_{1,2}\right) = -\frac{3}{4}\cdot C_D$ -Non-dimensional damping coefficient

$\bar{\eta}_{1,2}=\mathrm{Im}\left(\bar{\lambda}_{1,2}\right) = \sqrt{\frac{1}{2}\cdot\left(C_D^2+C_L^2\right)-\left(\frac{3}{4}\cdot C_D\right)^2}$ Non dimensional frequency of oscillations

$$\left(\frac{\bar{v}}{\bar{\theta}}\right)_{1,2} = -\frac{2\cdot\left(\bar{\lambda}_{1,2}+C_D\right)}{C_L} = a_{1,2}\pm i\cdot b_{1,2}$$

$$\bar{v}(\bar{t}) = e^{\bar{\xi}_{1,2}}\cdot\left[\left(\bar{v}_1+\bar{v}_2\right)\cdot\cos\left(\bar{\eta}_{1,2}\cdot\bar{t}\right) + i\cdot\left(\bar{v}_1-\bar{v}_2\right)\cdot\sin\left(\bar{\eta}_{1,2}\cdot\bar{t}\right)\right],$$
$$\bar{\theta}(\bar{t})= e^{\bar{\xi}_{1,2}}\cdot\left[\left(\frac{\bar{\theta}_1}{\bar{v}_1}\right)\cdot\bar{v}_1\cdot\cos\left(\bar{\eta}_{1,2}\cdot\bar{t}\right) - i\cdot\left(\frac{\bar{\theta}_2}{\bar{v}_2}\right)\cdot\bar{v}_2\cdot\sin\left(\bar{\eta}_{1,2}\cdot\bar{t}\right)\right].$$

$$\bar{t}=\bar{t}_0=0: \quad \bar{v}(0)=\bar{u}_0, \quad \bar{\theta}(0)=\bar{\theta}_0$$

$$\bar{v}(\bar{t}) = \frac{\bar{u}_0}{b_{1,2}}\cdot\sqrt{a_{1,2}^2+b_{1,2}^2}\cdot e^{\bar{\xi}_{1,2}}\cdot\cos\left(\bar{\eta}_{1,2}\cdot\bar{t}+\bar{\varphi}_0\right)$$
$$\bar{\theta}(\bar{t}) = \frac{\bar{u}_0}{b_{1,2}}\cdot\left(a_{1,2}^2+b_{1,2}^2\right)\cdot e^{\bar{\xi}_{1,2}}\cdot\sin\left(\bar{\eta}_{1,2}\cdot\bar{t}\right),$$

$$a_{1,2} = -2\cdot\frac{\bar{\xi}_{1,2}+C_D}{C_L} = -\frac{C_D}{2\cdot C_L} < 0, \qquad b_{1,2} = \frac{2\cdot\bar{\eta}_{1,2}}{C_L} > 0, \qquad \bar{\varphi}_0 = atan\frac{a_{1,2}}{b_{1,2}}$$

Assumptions:

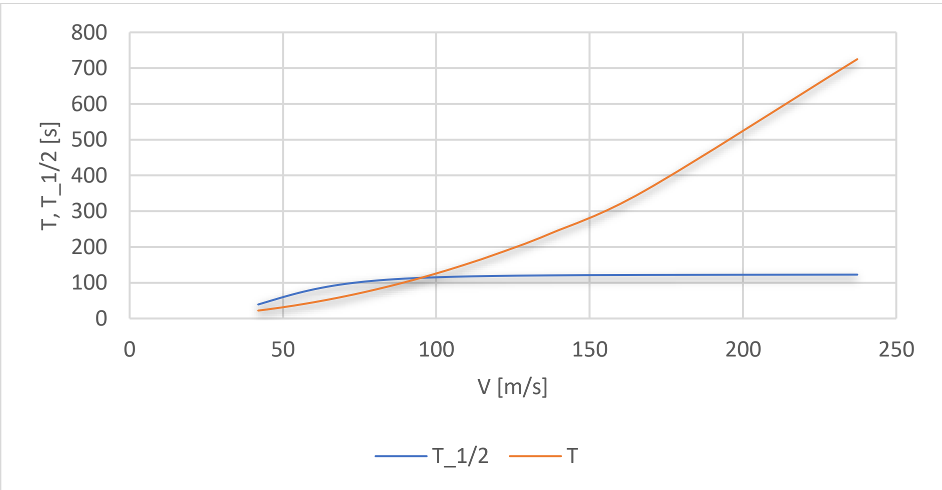
$T = \frac{2\pi t_0}{\eta}$ – period (in table)

$\frac{T}{2} = \frac{ln(2t_0)}{\xi}$ (in table)

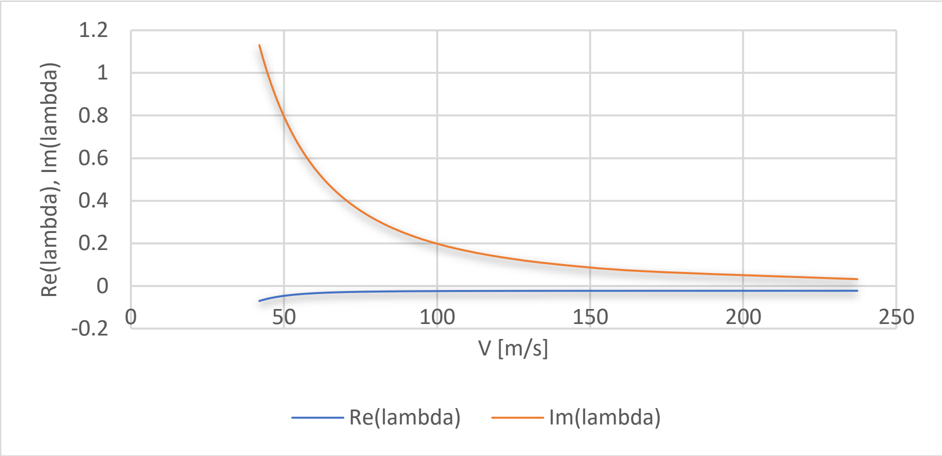
u = 5%

t_aerody	3.98597
U_0	0.00694

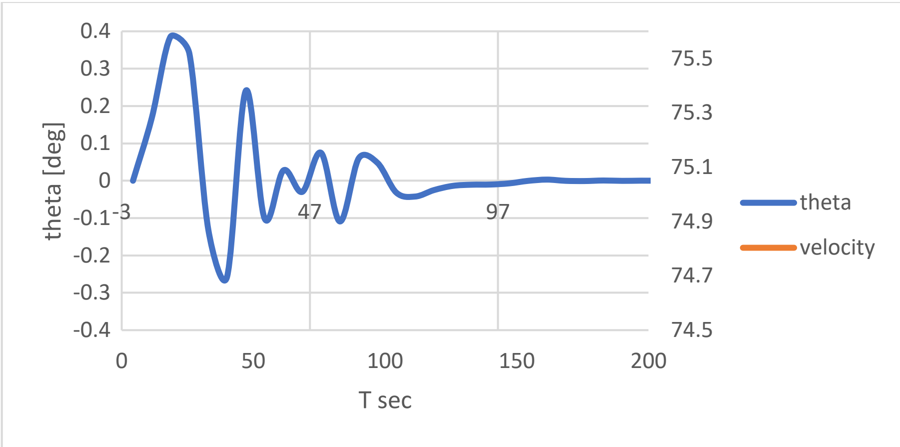
t [sec]	C_Lplane	C_Dplane	Re 1,2	Im 1,2	T [sec]	T_1/2[sec]	a	b	t_aero[sec]	v [m/s]	Θ deg	V_fli[m/s]	V [m/s]
0	0.05	0.03006	-0.0225	0.032	724.936	122.539	0.30063	1.28004	0	0.00694	0	72.5	237.292
5	0.1	0.03025	-0.0227	0.06907	356.227	121.779	0.15125	1.38148	1.2544	0.00666	0.16812	72.4795	167.791
10	0.15	0.03056	-0.0229	0.10496	236.738	120.544	0.10187	1.39946	2.5088	0.00621	0.38602	72.4468	137
15	0.2	0.031	-0.0233	0.14057	177.358	118.833	0.0775	1.40569	3.7632	0.00532	0.33997	72.3827	118.646
20	0.25	0.03156	-0.0237	0.17607	141.815	116.724	0.06312	1.40857	5.0176	0.0037	-0.1297	72.2663	106.12
25	0.3	0.03225	-0.0242	0.21152	118.147	114.227	0.05375	1.41012	6.272	0.00122	-0.2575	72.088	96.874
30	0.35	0.03306	-0.0248	0.24693	101.252	111.428	0.04723	1.41106	7.5264	-0.0018	0.24102	71.869	89.6879
35	0.4	0.034	-0.0255	0.28233	88.586	108.348	0.0425	1.41166	8.7808	-0.0045	-0.1016	71.6775	83.8953
40	0.45	0.03506	-0.0263	0.31771	78.7374	105.072	0.03896	1.41207	10.0352	-0.0053	0.02784	71.6169	79.0973
45	0.5	0.03625	-0.0272	0.35309	70.8601	101.623	0.03625	1.41235	11.2896	-0.0033	-0.0298	71.7628	75.0383
50	0.55	0.03756	-0.0282	0.38846	64.4159	98.0782	0.03415	1.41256	12.544	0.0009	0.07457	72.0645	71.5462
55	0.6	0.039	-0.0293	0.42382	59.0462	94.4569	0.0325	1.41272	13.7984	0.00425	-0.109	72.3061	68.5002
60	0.65	0.04056	-0.0304	0.45917	54.5031	90.8239	0.0312	1.41284	15.0528	0.0035	0.06005	72.2517	65.8129
65	0.7	0.04225	-0.0317	0.49452	50.6092	87.191	0.03018	1.41292	16.3072	-0.001	0.04772	71.9315	63.4189
70	0.75	0.04406	-0.033	0.52987	47.2347	83.6091	0.02937	1.41299	17.5616	-0.0039	-0.0318	71.7214	61.2685
75	0.8	0.046	-0.0345	0.56522	44.2821	80.083	0.02875	1.41304	18.816	-0.0012	-0.0423	71.9128	59.3229
80	0.85	0.04806	-0.036	0.60056	41.677	76.6504	0.02827	1.41308	20.0704	0.00297	-0.025	72.2137	57.5517
85	0.9	0.05025	-0.0377	0.6359	39.3614	73.3098	0.02792	1.41311	21.3248	0.00164	-0.0137	72.1183	55.9302
90	0.95	0.05256	-0.0394	0.67124	37.2897	70.0879	0.02766	1.41313	22.5792	-0.0025	-0.0107	71.8231	54.4385
95	1	0.055	-0.0413	0.70657	35.4251	66.9785	0.0275	1.41314	23.8336	-0.0011	-0.0102	71.9239	53.0601
100	1.05	0.05756	-0.0432	0.7419	33.7381	63.9996	0.02741	1.41315	25.088	0.0023	-0.007	72.1653	51.7813
105	1.1	0.06025	-0.0452	0.77723	32.2046	61.1422	0.02739	1.41315	26.3424	-0.0002	-0.0004	71.9889	50.5908
110	1.15	0.06306	-0.0473	0.81256	30.8044	58.4177	0.02742	1.41315	27.5968	-0.0017	0.00306	71.8781	49.4788
115	1.2	0.066	-0.0495	0.84789	29.5209	55.8154	0.0275	1.41314	28.8512	0.00132	-0.0002	72.0954	48.437
120	1.25	0.06906	-0.0518	0.88321	28.3401	53.3423	0.02762	1.41313	30.1056	0.00014	-0.0008	72.0099	47.4584
125	1.3	0.07225	-0.0542	0.91853	27.2502	50.9871	0.02779	1.41312	31.36	-0.0011	0.00064	71.9221	46.5368
130	1.35	0.07556	-0.0567	0.95385	26.241	48.7535	0.02799	1.41311	32.6144	0.00105	-0.0004	72.0755	45.6668
135	1.4	0.079	-0.0593	0.98916	25.3039	46.6306	0.02821	1.41309	33.8688	-0.0005	0.00019	71.9657	44.8439
140	1.45	0.08256	-0.0619	1.02447	24.4314	44.6199	0.02847	1.41307	35.1232	-1E-04	-9E-05	71.9929	44.064
145	1.5	0.08625	-0.0647	1.05978	23.6171	42.7109	0.02875	1.41304	36.3776	0.00042	1.3E-05	72.0305	43.3234
150	1.55	0.09006	-0.0675	1.09509	22.8554	40.904	0.02905	1.41302	37.632	-0.0005	1.8E-05	71.9636	42.6189
155	1.6	0.094	-0.0705	1.13039	22.1413	39.1896	0.02938	1.41299	38.8864	0.00045	-7E-06	72.0322	41.9477



Period T and damping time of amplitude to half $T_{1/2}$



Non-dimensional eigenvalues of the phugoid oscillations



Aircraft Response to small flight speed perturbation