Interference

Objectives

35.01 Using a sketch, explain Huygens' principle.

35.02 With a few simple sketches, explain refraction in terms of the gradual change in the speed of a wavefront as it passes through an interface at an angle to the normal.

35.03 Apply the relationship between the speed of light in vacuum c, the speed of light in a material v, and the index of refraction of the material n.

35.04 Apply the relationship between a distance L in a material, the speed of light in that material, and the time required for a pulse of the light to travel through L.

35.05 Apply Snell's law of refraction.

35.06 When light refracts through an interface, identify that the frequency does not change but the wavelength and effective speed do.

35.07 Apply the relationship between the wavelength in vacuum l, the wavelength ln in a material (the internal wavelength), and the index of refraction n of the material.

35.08 For light in a certain length of a material, calculate the number of internal wavelengths that fit into the length.

35.09 If two light waves travel through different materials with different indexes of refraction and then reach a common point, determine their phase difference and interpret the resulting interference in terms of maximum brightness, intermediate brightness, and darkness.

35.10 Apply the learning objectives of Module 17-3 (sound waves there, light waves here) to find the phase difference and interference of two waves that reach a common point after traveling paths of different lengths.

35.11 Given the initial phase difference between two waves with the same wavelength, determine their phase difference after they travel through different path lengths and through different indexes of refraction.

35.12 Identify that rainbows are examples of optical interference.

Key Points

- The three-dimensional transmission of waves, including light, may often be predicted by Huygens' principle, which states that all points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets
- The law of refraction can be derived from Huygens' principle by assuming that the index of refraction of any medium is n c/v, in which v is the speed of light in the medium and c is the speed of light in vacuum.
- The wavelength ln of light in a medium depends on the index of refraction n of the medium: $\lambda_n=\lambda/n$' in which l is the wavelength in vacuum.
- Because of this dependency, the phase difference between two waves can change if they pass through different materials with different indexes of refraction.

What Is Physics?

One of the major goals of physics is to understand the nature of light. This goal has been difficult to achieve (and has not yet fully been achieved) because light is complicated. However, this complication means that light offers many opportunities for applications, and some of the richest opportunities involve the interference of light waves—**optical interference**.

Nature has long used optical interference for coloring. For example, the wings of a *Morpho* butterfly are a dull, uninspiring brown, as can be seen on the bottom wing surface, but the brown is hidden on the top surface by an arresting blue due to the interference of light reflecting from that surface (Fig. 35- 1).

Moreover, the top surface is color-shifting; if you change your perspective or if the wing moves, the tint of the color changes. Similar color shifting is used in the inks on many currencies to thwart counterfeiters, whose copy machines can duplicate color from only one perspective and therefore cannot duplicate any shift in color caused by a change in perspective.

Figure 35-1 The blue of the top surface of a Morpho butterfly wing is due to optical interference and shifts in color as your viewing perspective changes.

Light as a Wave

The first convincing wave theory for light was in 1678 by Dutch physicist Christian Huygens. Mathematically simpler than the electromagnetic theory of Maxwell, it nicely explained reflection and refraction in terms of waves and gave physical meaning to the index of refraction.

Huygens' wave theory is based on a geometrical construction that allows us to tell where a given wavefront will be at any time in the future if we know its present position. Huygens' principle is:

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Here is a simple example.At the left in **Fig. 35-2,** the present location of a wavefront of a plane wave traveling to the right in vacuum is represented by plane *ab,* perpendicular to the page. Where will the wavefront be at time Δt later ? We let several points on plane ab (the dots) serve as sources of spherical secondary wavelets that are emitted at t_0. At time Δt the radius of all these spherical wavelets will have grown to c Δt where c is the speed of light in vacuum. We draw plane de tangent to these wavelets at time Δt .This plane represents the wavefront of the plane wave at time Δt ; it is parallel to plane ab and a perpendicular distance $c\,\Delta t$ from it.

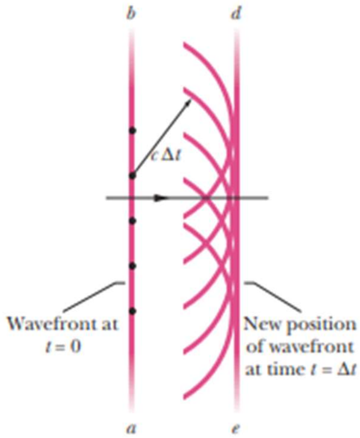
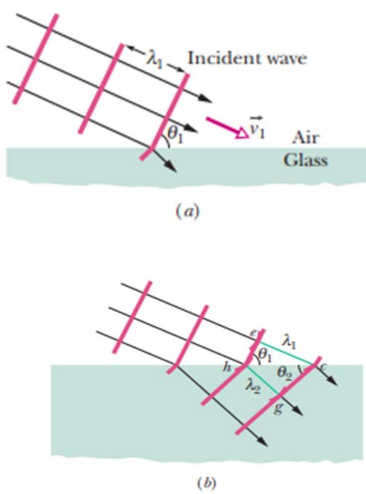


Figure 35-2 The propagation of a plane wave in vacuum, as portrayed by Huygens' principle.

The Law of Refraction

We now use Huygens' principle to derive the law of refraction, **Eq. 33-40** (Snell's law). Figure **35-3** shows three stages in the refraction of several wavefronts at a flat interface between air (medium 1) and glass (medium 2). We arbitrarily choose the wavefronts in the incident light beam to be separated by λ_1 the wavelength in medium 1. Let the speed of light in air be v_1 and that in glass be v_2 .We assume that $v_2 < v_1$ which happens to be true.



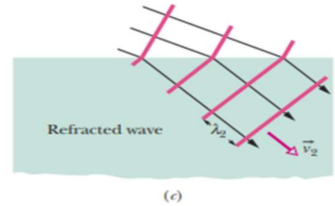


Figure 35-3 The refraction of a plane wave at an air–glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.

Angle θ_1 in **Fig. 35-3a** is the angle between the wavefront and the interface; it has the same value as the angle between the normal to the wavefront (that is, the incident ray) and the normal to the interface.Thus, θ_1 is the angle of incidence

As the wave moves into the glass, a Huygens wavelet at point e in Fig. 35-3b will expand to pass through point *c*, at a distance of λ_1 from point e. The time interval required for this expansion is that distance divided by the speed of the wavelet, or λ_1/v_1 . Now note that in this same time interval, a Huygens wavelet at point h will expand to pass through point g, at the reduced speed v_2 and with wavelength λ_2 . Thus, this time interval must also be equal to λ_2/v_2 . By equating these times of travel, we obtain the relation.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \qquad \textbf{(35-1)}$$

which shows that the wavelengths of light in two media are proportional to the speeds of light in those media. By Huygens' principle, the refracted wavefront must be tangent to an arc of radius λ_2 centered on h, say at point g. The refracted wavefront must also be tangent to an arc of radius λ_1 centered on e, say at c. Then the refracted wavefront must be oriented as shown. Note that θ_2, the angle between the refracted wavefront and the interface, is actually the angle of refraction.

For the right triangles hce and hcg in **Fig. 35-3b** we may write

$$\sin\theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle hce)}$$

and

$$\sin\theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle hcg)}$$

Dividing the first of these two equations by the second and using **Eq. 35-1**, we find

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \qquad \textbf{(35-2)}$$

We can define the index of refraction n for each medium as the ratio of the speed of light in vacuum to the speed of light v in the medium. Thus,

$$n = \frac{c}{v} \text{ (Index of refraction)} \qquad (35\text{-}3)$$

In particular, for our two media, we have

$$n_1 = \frac{c}{v_1} \text{ and } n_2 = \frac{c}{v_2}$$

We can now rewrite Eq. 35-2 as :

$$\frac{sin\theta_1}{sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$n_1 sin\theta_1 = n_2 sin\theta_2 \text{(law of refraction)} \qquad (35\text{-}4)$$

Wavelength and Index of Refraction

We have now seen that the wavelength of light changes when the speed of the light changes, as happens when light crosses an interface from one medium into another. Further, the speed of light in any medium depends on the index of refraction of the medium, according to Eq. 35-3. Thus, the wavelength of light in any medium depends on the index of refraction of the medium. Let a certain monochromatic light have wavelength λ and speed c in vacuum and wavelength λ_n n and speed v in a medium with an index of refraction n. Now we can rewrite Eq. 35-1 as :

$$\lambda_n = \lambda\frac{v}{c} \qquad (35\text{-}5)$$

Using Eq. 35-3 to substitute 1/n for v/c then yields

$$\lambda_n = \frac{\lambda}{n} \qquad (35\text{-}6)$$

This equation relates the wavelength of light in any medium to its wavelength in vacuum:A greater index of refraction means a smaller wavelength. Next, let f_n represent the frequency of the light in a medium with index of refraction n.Then from the general relation of Eq. 16-13 (v= λf), we can write

$$f_n = \frac{v}{\lambda_n}$$

Substituting Eqs. 35-3 and 35-6 then gives us

$$f_n = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f$$

where f is the frequency of the light in vacuum. Thus, although the speed and wavelength of light in the medium are different from what they are in vacuum, the *frequency of the light in the medium is the same as it is in vacuum.*

Phase Difference. The fact that the wavelength of light depends on the index of refraction via Eq. 35-6 is important in certain situations involving the interference of light waves. For example, in Fig. 35-4, the waves of the rays (that is, the waves represented by the rays) have identical wavelengths λ and are initially in phase in air (n≈ 1). One of the waves travels through medium 1 of index of refraction n_1 and length L. The other travels through medium 2 of index of refraction n_2 and the same length L.When the waves leave the two media, they will have the same wavelength—their wavelength λ in air. However, because their wavelengths differed in the two media, the two waves may no longer be in phase.

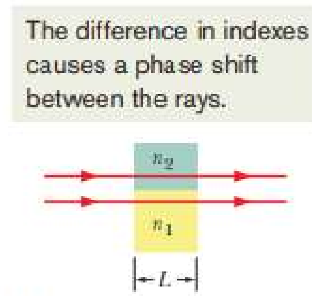


Figure 35-4 Two light rays travel through two media having different indexes of refraction.

As we shall discuss soon, this change in the phase difference can determine how the light waves will interfere if they reach some common point. To find their new phase difference in terms of wavelengths, we first count the number N1 of wavelengths there are in the length L of medium 1. From Eq. 35-6, the wavelength in medium 1 is $\lambda_{n1} = \lambda/n_1$; so

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda} \qquad (35\text{-}7)$$

Similarly, we count the number N_2 of wavelengths there are in the length L of medium 2, where the wavelength is $\lambda_{n2} = \lambda/n_2$

$$N_2 = \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda} \qquad (35\text{-}8)$$

To find the new phase difference between the waves, we subtract the smaller of N_1 and N_2 from the larger. Assuming $n_2 < n_1$, we obtain

$$N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1) \qquad (35\text{-}9)$$

Suppose Eq. 35-9 tells us that the waves now have a phase difference of 45.6 wavelengths. That is equivalent to taking the initially in-phase waves and shifting one of them by 45.6 wavelengths. However, a shift of an integer number of wavelengths (such as 45) would put the waves back in phase; so it is only the decimal fraction (here, 0.6) that is important. A phase difference of 45.6 wavelengths is equivalent to an effective phase difference of 0.6 wavelength.

A phase difference of 0.5 wavelength puts two waves exactly out of phase. If the waves had equal amplitudes and were to reach some common point, they would then undergo fully destructive interference, producing darkness at that point. With a phase difference of 0.0 or 1.0 wavelength, they would, instead, undergo fully constructive interference, resulting in brightness at the common point. Our phase difference of 0.6

wavelength is an intermediate situation but closer to fully destructive interference, and the waves would produce a dimly illuminated common point.

We can also express phase difference in terms of radians and degrees, as we have done already. A phase difference of one wavelength is equivalent to phase differences of 2π rad and 360°

Path Length Difference. As we discussed with sound waves in Module 17- 3, two waves that begin with some initial phase difference can end up with a different phase difference if they travel through paths with different lengths before coming back together. The key for the waves (whatever their type might be) is the path length difference ΔL, or more to the point, how ΔL compares to the wavelength λ of the waves. From Eqs. 17-23 and 17-24, we know that, for light waves, fully constructive interference (maximum brightness) occurs when

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \quad \text{(fully constructive interference)}, \quad (35\text{-}10)$$

and that fully destructive interference (darkness) occurs when

$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \quad \text{(fully destructive interference)}. \quad (35\text{-}11)$$

Intermediate values correspond to intermediate interference and thus also illumination.

Refraction Rainbows and Optical Interference

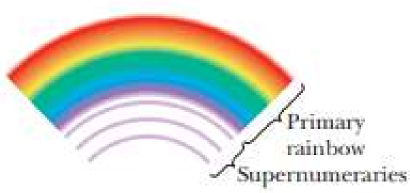


Figure 35-5 A primary rainbow and the faint supernumeraries below it are due to optical interference.

In Module 33-5, we discussed how the colors of sunlight are separated into a rainbow when sunlight travels through falling raindrops. We dealt with a simplified situation in which a single ray of white light entered a drop. Actually, light waves pass into a drop along the entire side that faces the Sun. Here we cannot discuss the details of how these waves travel through the drop and then emerge, but we can see that different parts of an incoming wave will travel different paths within the drop. That means waves will emerge from the drop with different phases. Thus, we can see that at some angles the emerging light will be in phase and give constructive interference. The rainbow is the result of such constructive interference. For example, the red of the rainbow appears because waves of red light emerge in phase from each raindrop in the direction in which you see that part of the rainbow. The light waves that emerge in other directions from each raindrop have a range of different phases because they take a range of different paths through each drop. This light is neither bright nor colorful, and so you do not notice it.

If you are lucky and look carefully below a primary rainbow, you can see dimmer colored arcs called supernumeraries (Fig. 35-5). Like the main arcs of the rainbow, the supernumeraries are due to waves that emerge from each drop approximately in phase with one another to give constructive interference. If you are very lucky and look very carefully above a secondary rainbow, you might see even more (but even dimmer) supernumeraries. Keep in mind that both types of rainbows and both sets of supernumeraries are naturally occurring examples of optical interference and naturally occurring evidence that light consists of waves.