LINFO2263: Vector Semantics and Word Embeddings

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Outline

1. Vector semantics from co-occurrence matrices
2. Learning dense word embeddings

Outline

Lexical semantics

Distributional hypothesis

- The meaning of a word **is** its use in the language

  [Wittgenstein, 1953]

- Language use can be characterized by counting how often other words occur in the context of appearance of a specific word

Vector semantics and co-occurrence matrix

- Co-occurrence frequencies between a (target) word and other (context) words can be stored in a vector representing the target word meaning
- The context can be a document, a paragraph, a sentence or, simply, a local context before and after the target word

```
... lemon,  a [tablespoon of apricot jam,      a] pinch ...
               c1          c2    t      c3         c4
```

- Such information can be stored in a co-occurrence matrix
  - each row defines the vector associated to a specific target word
  - the number of columns (= the dimensionality of the vector space) depends on the number of contextual words, by default, the whole vocabulary for the task

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	
strawberry	0	...	0	0	1	60	19	
digital	0	...	1670	1683	85	5	4	
information	0	...	3325	3982	378	5	13	

Illustration from *Speech and Language Processing, Jurafsky and Martin, 3rd ed.*

Vector semantics: example

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	
strawberry	0	...	0	0	1	60	19	
digital	0	...	1670	1683	85	5	4	
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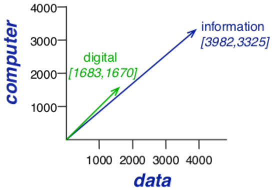


- 2-D representation here, the real space is $\mathbb{R}^d$ with $d \approx$ vocabulary size
- the similarity between word meanings can be **computed** from vector similarity

Cosine similarity

$$cos(\boldsymbol{v}, \boldsymbol{w}) = \frac{\boldsymbol{v} \cdot \boldsymbol{w}}{\|\boldsymbol{v}\| \|\boldsymbol{w}\|} = \frac{\sum_{i=1}^{d} v_i w_i}{\sqrt{\sum_{i=1}^{d} v_i^2} \sqrt{\sum_{i=1}^{d} w_i^2}}$$

$cos(\boldsymbol{v}, \boldsymbol{w}) = 1$ (is maximal)
$\Leftrightarrow$ the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is 0
$\Leftrightarrow$ the relative frequencies of **all** co-occurring words are the same

Illustration from *Speech and Language Processing, Jurafsky and Martin, 3rd ed.*

Weighted and normalized term frequencies

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	
strawberry	0	...	0	0	1	60	19	
digital	0	...	1670	1683	85	5	4	
information	0	...	3325	3982	378	5	13	

- Actual vector semantics are based on weighted and normalized frequencies because the raw counts are very skewed and not very discriminative
  - words that co-occur frequently (*e.g. pie* nearby *cherry*) look informative
  - yet, very frequent words (*e.g. the, it, they, ...*) co-occur with nearly every other word and are less informative
- Term-Frequency Inverse Document Frequency (TF-IDF) balances both
- TF-IDF comes from information retrieval where the various terms (= words) appear in sets of documents. Here, "documents" refer to the observed local contexts of a target word and the "terms" are the local context words c_j's

```
... lemon,  a [tablespoon of apricot jam,      a] pinch ...
               c1          c2    t      c3        c4
```

TF-IDF weighting applied to Vector Semantics

- $C(w_i, c_j)$ = number of times context word c_j occurs in the local contexts of target word w_i
- **Term frequency**
  The (smoothed) co-occurrence frequency on a log scale

$$tf_{i,j} = \log_{10}\left(C(w_i, c_j) + 1\right)$$

- **Inverse Document Frequency**

$$idf_j = \log_{10}\frac{N}{df_j}$$

  - df_j the number of contextual windows of any target word where this context word c_j occurs
  - N the total number of contextual windows for all target words
  - function words (*the, a, it, ...*) have a high document frequency $\Rightarrow$ a low inverse document frequency
- **TF-IDF** weighted frequency $w_{i,j} = tf_{i,j} \times idf_j$

Positive Pointwise Mutual Information (PPMI)

An alternative to TF-IDF

- **Pointwise Mutual Information** measures how much two words co-occur **more** than expected by chance
  For a word w and a context word c, $PMI(w,c) = \log_2 \frac{P(w,c)}{P(w)P(c)}$
- A negative PMI would mean w and c co-occur less than expected by chance but getting reliable estimates of this is difficult (huge corpora are required to estimate rare events reliably)
- **Positive PMI** replaces all negative PMI values by 0

$$PPMI(w,c) = \max\left(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0\right)$$

PPMI estimation

- $C(w_i, c_j)$ = number of times context word c_j occurs in the local contexts of target word w_i
- vocabulary V = the set of words
- set of context words C, possibly $C = V$
- additive smoothing hyper-parameter: $\varepsilon \approx \frac{1}{|V|}$ (e.g. $\varepsilon = 10^{-4}$)

$$\hat{P}(w_i, c_j) = \frac{C(w_i, c_j) + \varepsilon}{\sum_{i=1}^{V} \sum_{j=1}^{C} \left[C(w_i, c_j) + \varepsilon\right]}$$

$$\hat{P}(w_i) = \sum_{j=1}^{C} \hat{P}(w_i, c_j) \qquad \hat{P}(c_j) = \sum_{i=1}^{V} \hat{P}(w_i, c_j)$$

$$PPMI(w_i, c_j) = \max\left(\log_2 \frac{\hat{P}(w_i, c_j)}{\hat{P}(w_i)\hat{P}(c_j)}, 0\right)$$

Sparse word embeddings

- dimensions of co-occurrence based word embeddings (from TF-IDF or PPMI) have a direct interpretation = the identities of context words appearing in a local context of target words
- TD-IDF or PPMI define sparse word embeddings
  - the word meanings are embedded in a very high dimensional space with lots of zeros
  - a term frequency is 0 if a context word never occurs in the local context of a target word
  - a negative pointwise mutual information has been replaced by 0 (or the co-occurrence probability is exactly the one expected by chance)

	computer	data	result	pie	sugar
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

A PPMI matrix example

Illustration from *Speech and Language Processing, Jurafsky and Martin, 3rd ed.*

Outline

Learning dense word embeddings

- Use vector space of smaller dimensionality ($50 \ldots 300$) instead of the vocabulary size ($\approx 50,000$)
  - fewer parameters to represent word meanings may be enough
- **Learn** dense word embeddings rather than defining them from (weighted) co-occurrence counts
  - dimensions of the vector space no longer represent co-occurring words but rather abstract dimensions of meaning
  - dimensions *could* represent notions such as *positive/negative sentiment*, *trendy/old-fashioned concept*, ...
  - in practice, these abstract dimensions are automatically defined by a learning algorithm
- Word embeddings are learned to solve a specific NLP task: sentiment analysis, sentence completion (Shannon's game), ...and sometimes reused for another task: translate, ...
- Learned word embeddings are typically dense (mostly non-zeros)

Word2Vec

[Mikolov, et al., 2013]

1. Learn skip-grams embedding to solve a binary prediction task: is word *w* likely to occur in a context of the target word *t*?

```
... lemon,  a [tablespoon of apricot jam,     a] pinch ...
               c1         c2     t     c3       c4
```

   - Treat target words and neighboring context words (skip bi-grams) as positive examples
   - Randomly sample other words from the vocabulary to form negative examples

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

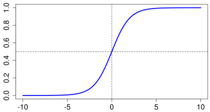
t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

2. Train a logistic regression as a classifier to discriminate $+/-$ examples
3. Use regression weights as word embeddings

Illustration from *Speech and Language Processing, Jurafsky and Martin, 3rd ed.*

Word2Vec: logistic regression

- Given a target word t (*e.g.* `apricot`) and a candidate context word c (*e.g.* `jam` or `aardvark`), define the probability $P(+|t,c)$ that c is a positive example (c is a real context word for t)
- $P(+|\boldsymbol{t},\boldsymbol{c})$ is a function of the vectors $\boldsymbol{t}$ and $\boldsymbol{c}$ representing t and c
  - $\boldsymbol{t} \cdot \boldsymbol{c}$ measures the similarity between t and c (an unscaled cosine)
  - this measure is squashed between $[0,1]$ by a sigmoid function
    $P(+|\boldsymbol{t},\boldsymbol{c}) = \sigma(\boldsymbol{t} \cdot \boldsymbol{c})$ with $\sigma(x) = \frac{1}{1+\exp^{-x}} = \frac{\exp^x}{\exp^x + 1}$



Sigmoid a.k.a **Logistic** function

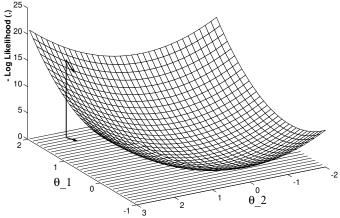
- $P(-|\boldsymbol{t},\boldsymbol{c}) = 1 - P(+|\boldsymbol{t},\boldsymbol{c})$
- Start from **randomly chosen vectors** in d dimensions (an hyper-parameter fixed by the designer)
- Greedily **optimize them** to better fit the training data

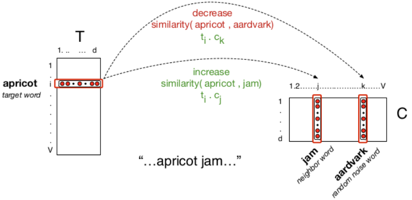
Learning Word2Vec embeddings

- **Model parameters** θ: vectors $\boldsymbol{t}$ and $\boldsymbol{c}$ for all target/context words
- **Log-likelihood**:

$$LL(\theta) = \sum_{t,c \in +} \log P(+|\boldsymbol{t}, \boldsymbol{c}) + \sum_{t,c \in -} \log P(-|\boldsymbol{t}, \boldsymbol{c})$$

  - measures the fit to the positive and negative training examples
  - look for the parameters maximizing $LL(\theta)$ or, equivalently, minimizing $-LL(\theta)$ called a **loss**
  - standard optimization relies on gradient descent





- **Final result**: d-dimensional embeddings for T and C
  - keep just T, or add them $t_i + c_i$, or concatenate them (a $2d$ solution)

Semantic properties of word embeddings

Popular word embeddings, such as **Word2Vec** [Mikolov, et al., 2013] or **GloVe** [Pennington et al., 2014], are effective ways to represent word meanings by vectors

- vector arithmetic allows to combine meanings!!

  $\overrightarrow{king} - \overrightarrow{man} + \overrightarrow{woman} \approx \overrightarrow{queen}$

  $\overrightarrow{Paris} - \overrightarrow{France} + \overrightarrow{Italy} \approx \overrightarrow{Rome}$

- but word embeddings include gender or racist biases, most probably present in the corpora they are trained from

  $\overrightarrow{computer\_scientist} - \overrightarrow{man} + \overrightarrow{woman} \approx \overrightarrow{homemaker}$

  (*ménagère, femme au foyer*...)

Summary

- The meaning of a word is its use in a language $\Rightarrow$ co-occurring words
- Word meanings can be represented by vectors
  - constructed from weighted co-occurrence counts
    $\Rightarrow$ sparse word embeddings (TF-IDF, Pointwise Mutual Information)
  - automatically learned to get abstract dimensions
    $\Rightarrow$ dense word embeddings (Word2Vec, GloVe, . . .)

Further Reading

Jurafsky D. and Martin J.H.
Speech and Language Processing, 3rd edition (draft)
chapters (5 and) 6.

Mikolov, T., Sutskever, I., Chen, K., Corrado, G. S., and Dean, J.
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Advances in Neural Information Processing Systems (NIPS), pp. 3111—3119, 2013.

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GloVe: Global Vectors for Word Representation
Empirical Methods in Natural Language Processing (EMNLP), pp. 1532–1543, 2014.

Wittgenstein, L.
Philosophical Investigations. (Translated by Anscombe, G.E.M.).
Blackwell, 1953